Multi-frequency and multiple phase-shift sinusoidal fringe projection for 3D profilometry

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Abstract: In this paper, we report on a laser fringe projection set-up, which can generate fringe patterns with multiple frequencies and phase shifts. Stationary fringe patterns with sinusoidal intensity distributions are produced by the interference of two laser beams, which are frequency modulated by a pair of acousto-optic modulators (AOMs). The AOMs are driven by two RF signals with the same frequency but a phase delay between them. By changing the RF frequency and the phase delay, the fringe spatial frequency and phase shift can be electronically controlled, which allows high-speed switching from one frequency or phase to another thus makes a dynamic 3D profiling possible.

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OCIS codes: (230.1040) Acousto-optical devices; (120.3180) Interferometry; (110.6880) Three-dimensional image acquisition.

References and links

1. Introduction

Non-contact and full-field optical surface profilometry is becoming an important measurement tool in both scientific research and industrial applications including automated manufacturing, solid modeling, computer graphics, machine vision systems, etc. Different methods have been investigated and developed, among them those based on fringe projection and triangulation have attracted a lot of attention and have been widely applied in practical applications due to their superior performance (such as full-field and 3D fast measurement) as well as simple implementation [1-5]. In the fringe projection based profilometry, a set of fringes with a preferably sinusoidal intensity distribution is projected onto the surface to be measured and a CCD camera is positioned at a location with a viewing angle to record the distorted fringes. From the recorded images, one can calculate the phase map of the distorted fringes by using different methods, such as Fourier transform or phase shifting methods. The 3D profile of the measured object can be obtained from the phase map by using a transform determined by the geometry of the optical set-up. However, since the phase calculation involves the arctangent operation, the phase map calculated from the detected intensity distributions through either Fourier transform or phase shifting methods has principal values ranging from $-\pi$ to $\pi$, and has discontinuities with $2\pi$ phase jumps. Therefore, a so-called phase unwrapping process is necessary to remove the phase jumps and to obtain continuous phase distribution. The phase unwrapping process is normally carried out using the so-called spatial unwrapping algorithm, which compares neighboring pixels and adjusts their relative phases by addition of integral multiples of $2\pi$ until the difference lies within the range of ($-\pi$, $\pi$). However, the spatial unwrapping algorithm suffers from the drawback that large phase errors can propagate across the image. Moreover it could be impossible to unwrap the phases correctly when the surfaces of interest contain discontinuities or the recorded images contain speckle noise [6]. Intensive studies have been devoted to develop reliable and robust phase unwrapping methods and algorithms, and a review on this topic can be found in [7].

Among the proposed phase unwrapping methods, the one based on multi-frequency fringe projection can be applied to the objects with discontinuities [8, 9]. In this method, multiple images are recorded at fringe patterns with different periods (from long to short) or frequencies (from low to high) in time series, and the phase at each pixel is unwrapped along the time axis. The longest fringe period is selected so that the phase across the whole object is within ($-\pi$, $\pi$) range. The phase map obtained from a lower fringe frequency will be used to guide the phase unwrapping process at a higher fringe frequency. In order to realize this phase unwrapping process, a fringe projection set-up which can generate fringe patterns with variable and controllable frequencies is desirable. On the other hand, in the phase shifting profilometry (PSP), fringe patterns with sinusoidal intensity distribution and multiple phase shifts are also necessary.

Multi-frequency and multiple phase-shift fringe patterns can be generated by using mechanical devices, such as a translation stage [10]. Due to hysteresis, backlash and wear, inaccurate shift such as unequal or incomplete shift often results, leading to undesirable errors. Therefore, it is still a challenging task to develop methods which can generate sinusoidal fringe patterns with controllable frequencies and phase shifts. In a paper published in 2000, a fringe projector using an acousto-optic modulator (AOM) has been demonstrated [11]. In this method, an AOM driven by two RF frequencies generates two sets of gratings inside the acousto-optic crystal and diffracts an incident laser beam into two first-order beams which are

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separated by a angle determined by the frequency difference of the two RF driving signals. The two first-order beams are brought together and expended by a lens. As the two diffracted beams are of different frequency shifts, the fringe pattern generated by the interference of the two first-order beams are moving with a velocity determined by the frequency difference. To make the fringes look stationary, a laser diode which amplitude is modulated by a signal synchronized with the RF driving signals is employed as the laser source. By controlling the frequency difference between the two RF driving signals, the period of the generated fringe pattern can be changed. Phase shifting is realized by controlling the time when the laser diode is switched on.

In this paper, we propose a fringe projection set-up also based on acousto-optic modulation. We demonstrate that by using two AOMs, a stationary fringe pattern with variable periods and phase shifts can be generated. In the following section, the principle of the proposed fringe projector is first introduced. Section 3 presents a theoretical analysis of the process of the fringe formation, followed by a description of the experiments carried out and the experimental results. Conclusions are given in Section 5.

2. Experimental set-up and principle of operation

A schematic diagram of the projector proposed in this study is shown in Fig. 1. A beam from a CW laser is first split by a beam-splitter into two parallel beams with equal intensity. The two parallel beams then enter two acousto-optic modulators (AOMs) respectively. The AOMs are driven by two RF signals with identical frequency but different initial phases. The AOMs are arranged in such a way that diffractions generated by both AOMs occur in the horizontal plane and the diffracted beams are symmetrical about the optical axis of the system. The first-order diffracted beams from the AOMs are combined by the second beam-splitter (as a beam-combiner) and focused by a lens. Since the two beams are of the same frequency shift, a set of stationary interference fringes are produced in the intersecting area of the two beams as the result of laser interference. The interference fringes are projected onto the surface to be measured by a microscope objective. The AOMs are normally designed to work in the so-called Bragg regime where only the first-diffraction order is generated. In practice, however, the zero- and higher-order diffracted beams always exist. In order to filter out the beams other than the first orders, two pinholes can be used. Normally the AOMs need to work at a frequency higher than 40 MHz to achieve a high diffraction efficiency. In this case, the diffracted angles generated at such a frequency could be larger than the angles needed for the lowest fringe frequency. To adjust the angle between the first-order diffracted beams before combining them, two wedge prisms with a small angle are used.

When the frequency of the RF driving signals is varied, the diffraction angles of the first-order beams will change correspondingly. As a consequence, the period (or spatial frequency) of the fringe pattern generated by the interference will be changed. The phase shifts can be realized by introducing phase delay between the two RF driving signals, which can be done by using the well-developed phase shifting or time delay electronic circuitry.
As shown in Fig. 1, the optical layout of the projector is symmetrical about its optical axis. Therefore, the influence of the environmental conditions on the fringe patterns will be minimized. As demonstrated below, the intensity distribution of the fringe pattern generated by the interference of laser beams is sinusoidal.

3. Analysis

Suppose that the beam from the laser is a fundamental mode Gaussian beam. Mathematically the electric field of a Gaussian beam can be described by [12]

\[
E(x, y, z, t) = E_0 \frac{w_0}{w(z)} \exp\left(-\frac{x^2 + y^2}{w^2(z)}\right) \exp\left\{-i[kz - \omega t - \tan^{-1}\left(\frac{z}{z_0}\right) + k \frac{x^2 + y^2}{2R(z)}]\right\}, \quad (1)
\]

where, \(E_0\) is the amplitude of the electric field of the beam; \(k\) is the propagation constant; \(w_0\) is the radius of the beam waist; \(z\) is the distance along the beam propagation direction; \(\omega\) is the frequency of the laser beam. Other parameters are defined as follows:

\[
z_0 = \frac{\pi w_0^2}{\lambda}, \quad (2)
\]

\[
w(z) = w_0[1 + (z / z_0)^2]^{1/2}, \quad (3)
\]

\[
R(z) = z[1 + (z_0 / z)^2]. \quad (4)
\]

When a Gaussian beam is well collimated and the optical path is short (as in the case considered in this study), it can be simplified to a plane wave with Gaussian intensity distribution, ie

\[
E(x, y, z, t) = E_0 \exp\left(-\frac{x^2 + y^2}{w_0^2}\right) \exp\left\{-i[kz - \omega t]\right\}. \quad (5)
\]

We assume that the acoustic waves inside the crystals of the AOMs are

\[
A_1 = A_{10} \exp\left[-i(K_1 X_1 - \Omega t + \Phi_1)\right], \quad (6)
\]

\[
A_2 = A_{20} \exp\left[-i(K_2 X_2 - \Omega t + \Phi_2)\right]. \quad (7)
\]

where, \(A_{10}\) and \(A_{20}\) are the amplitudes of the acoustic waves inside the crystals; \(K_1\) and \(K_2\) are the propagation constants of the acoustic waves along the \(X_1\) - and \(X_2\) -directions respectively; \(\Omega\) is the frequency of the acoustic waves and is equal to the frequency of the RF driving signals; \(\Phi_1\) and \(\Phi_2\) are the initial phases of the RF driving signals.

The electric fields of the first-order diffracted beams before leaving the acousto-optic crystals are

\[
E_1'(x_1, y_1, z_1, t) = E_{10} \exp\left(-\frac{x_1^2 + y_1^2}{w_0^2}\right) \exp\left\{-i[k_1 z_1 - (\omega_0 + \Omega) t + \Phi_1]\right\}, \quad (8)
\]

\[
E_2'(x_2, y_2, z_2, t) = E_{20} \exp\left(-\frac{x_2^2 + y_2^2}{w_0^2}\right) \exp\left\{-i[k_2 z_2 - (\omega_0 + \Omega) t + \Phi_2]\right\}, \quad (9)
\]

where, \(k_1\) and \(k_2\) are propagation constants of the first-order diffracted beams produced by AOM1 and AOM2 respectively.
The propagation directions of the diffracted beams are adjusted by the wedge prisms. After the beam combiner, the beams are brought together by the focusing lens. At the focal point of the lens, two beams intersect with an angle, $\theta(\Omega)$, which is determined by the focal length of the focusing lens, $F$, and the driving frequency of the AOMs, $\Omega$. The microscope objective converts the input beams into spherical waves. In the space after the microscope objective, interference between two point sources located at $(d(\Omega)/2, 0, 0)$ and $(-d(\Omega)/2, 0, 0)$ happens. $d(\Omega)$ is determined by the focal lengths of the focusing lens and the microscope objective, and the AOM driving frequency, $\Omega$.

The electric field at a point $(x, y, z)$ in the fringe area is

$$E(x, y, z, t) = E_{10} \frac{w_0}{w(z)} \exp\left(-\frac{r^2}{w^2(z)}\right) \exp\left[-i\left(k_z + \frac{r^2}{2R_1} - (\omega_0 + \Omega)t + \Phi_1\right)\right]$$

$$+ E_{20} \frac{w_0}{w(z)} \exp\left(-\frac{r^2}{w^2(z)}\right) \exp\left[-i\left(k_z + \frac{r^2}{2R_2} - (\omega_0 + \Omega)t + \Phi_2\right)\right], \quad (10)$$

where,

$$r^2 = x^2 + y^2,$$

$$R_1 = [(x - d(\Omega)/2)^2 + y^2 + z^2]^{1/2},$$

$$R_2 = [(x + d(\Omega)/2)^2 + y^2 + z^2]^{1/2},$$

and $w_0$ is the beam waist radius after the focusing lens and the microscope objective;

$$w'(z) = w_0 [1 + \left(\frac{\lambda z}{n w_0^2}\right)^2]^{1/2}.$$

The interference intensity at $(x, y, z)$ is given by

$$I(x, y, z) = [E(x, y, z, t) \cdot E(x, y, z, t)^*]. \quad (11)$$

As well known and implied by the above equations, the interference patterns produced by the two point sources are planes parallel to the $y$-axis and the fringe spacing in a plane perpendicular to the $z$-axis linearly increases with the $z$-distance. The fringe spacing is also a function of the distance between the two point sources, $d(\Omega)$, meaning that the fringe spacing can be varied by changing the AOM driving frequency, $\Omega$. At a point $(x, y, z)$, the phase of the fringe pattern is determined by the phase difference between the two driving signals, $\Phi_2 - \Phi_1$. Therefore, one can introduce a phase shift to the fringe pattern by simply controlling the phase delay between the AOM driving signals.

By using the equations given above, one can calculate the fringe patterns produced by the projector. Shown in Fig. 2 is an example of the calculated fringe pattern projected on a convex surface with a plane as reference.
Fig. 2. Simulated fringe pattern projected on a convex surface with a plane as reference.

4. Experiments

An experimental set-up was designed and constructed to test the fringe projector as shown in Fig. 1 and to demonstrate the controllability of the fringe spacing and the phase of the fringe pattern generated. In the experiments, a laser diode with an output power of 10 mW at 670 nm was used. Two identical integrated prisms were used as the beam-splitter and beam-combiner. The beam spacing is 25 mm. The crystals used in the AOMs are PbMoO$_4$. The AOMs have a central frequency of 65 MHz and a bandwidth of 20 MHz. The focal length of the focusing lens is 10 mm, and the power of the microscope objective is 20X.

![Fig. 3. Schematic diagram of the electronics developed for driving the AOMs, and for controlling the fringe spacing and the phase shift.](image)

Figure 3 shows a schematic diagram of the electronics developed for driving the AOMs, and for controlling the fringe spacing and the phase shift. A sinusoidal signal was generated by a voltage-controlled oscillator (VCO), and split into two channels. One of them was amplified to the level required by the AOM, and another was fed into a phase delay circuit.
where the input signal was phase shifted. Four fixed phase delays were generated with the current design. Using a multi-channel switching IC, one of the signals with different phases was selected by a digital control. The phase-shifted signal was amplified before feeding to the second AOM. The control of the VCO frequency was realized by either a DC voltage or a D/A converter.

![Fig. 4. Image recorded by a CCD camera when the fringe pattern was projected on a dome-shaped object with a reference plane.](image)

Shown in Fig. 4 is an image recorded by a CCD camera when the fringe pattern was projected on a dome shape with a reference plane. The viewing angle was 30 degrees, corresponding to the simulated fringe pattern as given in Fig. 2.

In order to test the phase shift function of the projector, the CCD camera with imaging lens removed was directly positioned in the fringe area. Depicted in Fig. 5 is the fringes recorded at two states with a nominal phase shift of 180 degrees. In the experiments, the phase delays between the two RF driving signals to the AOMs were monitored and measured by using a digital oscilloscope.

![Fig. 5. Fringes recorded with a nominal phase shift of 180 degrees.](image)
The developed projector was tested in a 3D measurement of a rectangle block. Shown in Fig. 6 are the deformed fringe patterns with three different fringe periods. The reconstructed 3D surface is given in Fig. 7.

![Fig. 6. Recorded fringe patterns with three different periods projected on a rectangle block](image1)

![Fig. 7. Reconstructed 3D surface plot of the rectangle block](image2)

5. Conclusion

We have proposed and demonstrated a laser fringe projection method, which can generate fringe patterns with multiple frequencies and phase shifts. Stationary fringe patterns with sinusoidal intensity distributions can be produced by the interference of two laser beams, which are frequency shifted by using two AOMs. By controlling the frequency shifts, phase delays and the pointing directions of the two laser beams, the fringe spacing and the phase of the fringe pattern can be controlled electronically at a fast switching speed. Thus, precision mechanical motions can be replaced with precision time delays of electronic signals. No moving part is necessary.

The proposed AOM projector is an enabling technique to realize dynamic 3D profile measurement. Because the interference happens in whole intersection area, the AOM projector does not have the defocusing problem suffered by other grating projectors. In another word, it has unlimited depth of focus. The AOM projector can be applied to both large-scale and micro-scale objects.

The AOM projector will be further tested in our ongoing projects. By constructing a 3D profile measurement system, we will evaluate the overall performance of the system including the accuracy and resolution. This work is currently under way, and the results will be reported in a separate paper.

Acknowledgements

The work was supported by Natural Science Foundation of China (NSFC) under the grant No. 60275012 and No. 60472107, and partly supported by Natural Science Foundation of China.
Gaungdong province (No. 031804) and Science & Technology Bureau of Shenzhen (No. 200341). The financial support from the Australian Research Council (ARC) is also acknowledged. The authors would like to thank the reviewers of the manuscript for their suggestions for revising the manuscript of the paper.