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# DILATIONAL SLIP ANGLE OF REBAR BOLTS UNDER AXIAL LOADING

Chen Cao, Jan Nemcik, Naj Aziz and Ting Ren

**ABSTRACT:** Mechanical interlock is an important component in load transfer capacity in the rockbolting system. It is in turn dominated by the rebar bolt profile configuration. To gain a deeper understanding of their interactions, rebar bolt units under axial loading were studied. Two kinds of failure mode have been identified, namely parallel shear and dilational slip failures. Based on some assumptions made, a universal upper limit of slipping angle of dilation slip failure can be found as the complementary angle of the grout material internal friction angle, which is also the minimum value of the bolt rib face angle. This theory can explain similar performances of rock bolts in pull-out tests while their face angles are large. In addition, once the geometric parameters of a rebar bolt profile are provided, more narrow slip angles range can be figured out via simple plots. As a result, the grout between the bolt profiles can be recognised as three sections, one works as part of the bolt profile, one carries out shear failure and the bottom part keeps intact.

## INTRODUCTION

Steel bolts have been widely used for rock reinforcement in civil and mining engineering for several decades. Bolts reinforce rock mass through restraining the deformation within the rock. To improve bolt loading capacity through the steel rebar, it is necessary to have a good understanding of the rock bolt behaviour in deformed medium. This can be acquired through analytical studies, laboratory tests and numerical modelling.

Monitoring of load transfer between the bolt, resin and rock strata indicates that the bolt profile plays an important role in generation of shear strength between the bolt and the surrounding rock. The short encapsulation pull out test of roof bolts indicates a significant increase of load transfer at a particular bolt rib profile spacing. Other variables such as profile rib angle, shape and size are also important parameters contributing to the load capacity of rock bolt systems.

The load transfer capacity of the bolt is governed by the shear strengths developed between the rock/grout and the grout/bolt interface interaction. Grout/rock interface failure can rarely occur in lab pull-out tests and in practice. As a result the bonding strength at the grout/bolt interface dominates the effect of rockbolting (Aziz, *et al.*, 2006; 2008).

It is commonly accepted that the bonding strength has three components: cohesion, friction and interlock. Singer (1990) demonstrated that there is no adhesion between the grout to bolt interface but in most cases reported, there is very little adhesion between grout-bolt, Aziz and Webb (2003).

The frictional component can be catalogued into the dilational slip, shear failure of surrounding medium and bolt unscrewing, Hyett *et al.* (1995). Each of them depends on the pressure generated at the bolt-grout interface, which in turn depends on the internal reaction forces of the three phase materials.

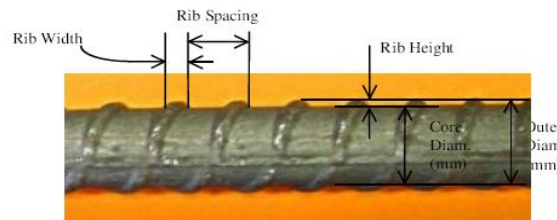
The mechanical interlock component plays an important role in bonding capacity and load transfer in the rockbolting system. It, in turn, is influenced by the bolt profile configuration. The profile configuration is defined by the rib profile shape, profile height, angle of wrap and spacing between the ribs, as shown in Figure 1.

In traditional rockbolting mechanism analysis, the effect of mechanical interlocking is often integrated into the analytical model in various manners, but do not concern with the rib geometry. In the so-called 'Interfacial Shear Stress' (ISS) model, the deformation of surrounding materials is lumped into a zero thickness interface, which is assigned with specific stress-strain behaviour to simulate the mechanical interlocking observed in pull-out tests. For example, Farmer (1975) has shown that exponential decay of the axial stress of the bolt occurs in the case of perfect bonding and elastic deformation. Li and Stillborg

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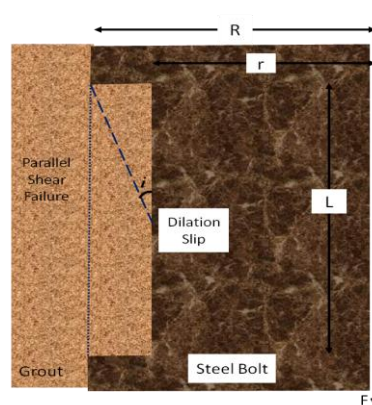
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(1999) developed an ISS model for predicting the behaviour of rock bolts in pullout tests, in uniformly deformed rock mass and when subjected to opened joints. Ivnovic and Neilson (2009) developed a lumped parameter model with varying shear load failure properties along the fixed anchor length to analyse the bolt behaviour under static or dynamic loads. More recently, a tri-linear bond-slip model with residual strength at the grout-bolt interface is adopted and closed-form solutions are obtained for the prediction of full range behaviour of fully grouted rockbolts under axial load, Ren, *et al.* (2009) and Martin, *et al.* (2011).



**Figure 1 - Terminology of the rib profile of steel bolt.**

How the bolt profile interacts with the grout material under axial loading is the focus of this study. It is a structurally based approach to reasoning the performances of the rock bolt when pull tested. A rib profile, shown in Figure 2, is only one unit of the bolt profile system which is the subject of discussion in this paper.



**Figure 2 - Schematic diagram of one bolt unit. The load  $F$  is resultant force in axial direction;  $r$  is the core radius of the steel bolt;  $R$  equals to  $r$  plus rib height;  $L$  is rib spacing except rib itself; the dotted line indicates parallel shear failure surface; dashed line indicates the dilational slipping surface and the angle  $i$  is the slipping angle of the bolt unit.**

The derived mathematical expressions of slipping angle,  $i$ , describe the initial orientation of dilational slip failure of the bolt unit; it is also the “real” face angle for the steel bolt. The aim of the theoretical predictions is to provide an understanding of the initial grout failure and to offer a new tool for research into the best profile geometries to reach optimum shear strength between the bolt and the surrounding medium.

### FAILURE MODES OF ROCK BOLTS UNDER AXIAL LOADING

Failure modes of rock bolting subjected to axial loading are the major concern of the load transfer mechanism. Two kinds of failure modes can be identified via pull-out tests, namely direct parallel shear failure (indicated by dotted line in Figure 2) and dilational slip (indicated by the dashed line in Figure 2).

Direct parallel shear failure is characterized by a cylinder shaped failure surface. It is a characteristic failure pattern, which occurs for smooth surface bar (without profiles) along the bolt-grout contact, and for very closely spaced rebar (like a screw) along the rib tips of the bar. For one unit of a smooth bolt, the resultant axial load can be expressed as

$$F = 2\pi rL\tau \quad (1)$$

Where  $\tau$  is the shear stress and once failure occurs, the shear stress can be calculated via Coulomb's failure criterion as:

$$\tau = c_{\text{grout-steel}} + p \cdot \tan\phi_{\text{grout-steel}} \quad (2)$$

Here  $c$  and  $\phi$  are grout/steel interface properties and  $p$  represents the confining pressure when failure occurs.

In another situation, if a bolt has closely spaced ribs, parallel shear failure between the rib peaks will always occur. It can be found that during such failure, the grout between the profiles will remain as if it was part of the steel bar. In fact, a threaded bar can be thought of as a smooth bar with little larger diameter. The mechanical behaviour of threaded bar can be expressed using similar equations but replacing grout-steel properties with grout property only, thus:

$$F = 2\pi RL\tau \quad (3)$$

$$\tau = c_{\text{grout}} + p \cdot \tan\phi_{\text{grout}} \quad (4)$$

The load capacity of direct parallel shear failure is apparently lower if compared with ribbed steel bar of the same core diameter (Aydan, 1989; Ito, *et al.*, 2001; Aziz, *et al.*, 2008; Kilic, *et al.*, 2002). The force  $F$  is normally large and the resin is much softer than steel. Therefore, the direct parallel shear failure, without any dilatation can rarely occur for a common rebar bolt. That is, under normal circumstances dilatation will always occur, more or less, around the rebar bolt surface. If the confinement material is stiff, or say in hard rock, the initial dilatation can be depressed by increasing confinement pressure. In some cases where rock is very soft, the rib will push the surrounding material radially outwards and slip out from the initial flute.

## THEORETICAL RANGE OF DILATIONAL SLIP DIRECTIONS

### Problem of description and assumptions

From the failure modes analysis, the mechanism of dilational slip failure is the major concern of rebared rock bolting system. For one bolt unit, the initial failure is characterised by propagation of micro-cracks along a specific surface. Once the mobilised shear stress along this surface reaches a critical value, relative movement will take place. Then the post-failure behaviour will be governed by more complicated interaction as the initial confining pressure will be re-distributed.

In this paper, the post-failure behaviour will not be discussed, and the focus will be on the slipping angle analysis. Firstly, the following assumptions are made;

- (1) Coulomb failure criterion is used to locate the failure surface within the grout material
- (2) The initial confining pressure,  $p$ , is compressive and universal.
- (3) The cohesion and friction at the resin-bolt contact are neglected.

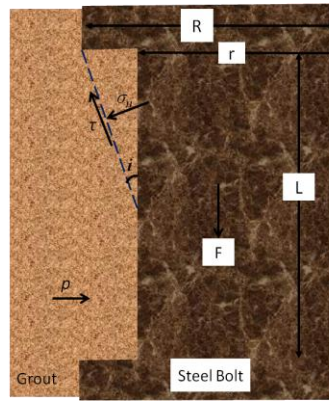
### Dilational slip failure of one unit bolt

Before slipping occurs, the axial force,  $F$ , cannot affect the initial horizontal stress field because they are perpendicular to each other. That is,  $p$  will keep its magnitude while  $F$  increases until relative movement takes place. In this procedure (shown in Figure 3):

$$\sigma_n = p \cos i - \frac{F}{A} \sin i \quad (5)$$

$$\tau = \frac{F}{A} \cos i - p \sin i \quad (6)$$

Where  $A = \pi(R^2 - r^2)/\sin i$  is the conical area of the failure surface.



**Figure 3 - Dilational slip just occurs along the dashed line**

Once the relative movement along the supposed surface takes place, according to assumption (1), then the dilational slip failure follows

$$\tau = c + \sigma_n \cdot \tan\phi \quad (7)$$

### Initial slipping angle analysis

Eq. (3) and (4) can be re-arranged as

$$F/A = \sigma_n \sin i + \tau \cos i \quad (8)$$

$$p = \sigma_n \cos i - \tau \sin i \quad (9)$$

Substitute shear stress,  $\tau$ , in Eq. (7) and using Eq. (5),

$$p = \sigma_n \cos i - (c + \sigma_n \cdot \tan\phi) \sin i$$

This is the expression of confining pressure before it begins to change. The confining pressure is always positive (compression), so

$$\sigma_n \cos i - (c + \sigma_n \cdot \tan\phi) \sin i \geq 0$$

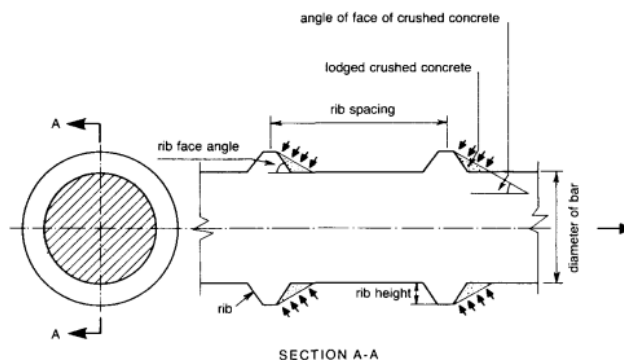
$$\text{As } c \geq 0, \text{ and } i \leq \frac{\pi}{2} - \phi \quad (10)$$

This is the expression of the upper limit of the dilational slip angle. It demonstrates that the upper limit of slipping angle is solely dependent on the grout internal friction angle. For example, if the resin internal frictional angle =  $35^\circ$ , the dilation slip angle must be less than  $55^\circ$ , no matter what the real bolt face angle is.

The dilational slip angle reaches its maximum value ( $\pi/2 - \phi$ ) when  $c=0$  and  $p=0$ . This is a kind of spontaneous dilational slip which is obviously not real for a rockbolting system. Thus the slipping angle will always be under this upper limit.

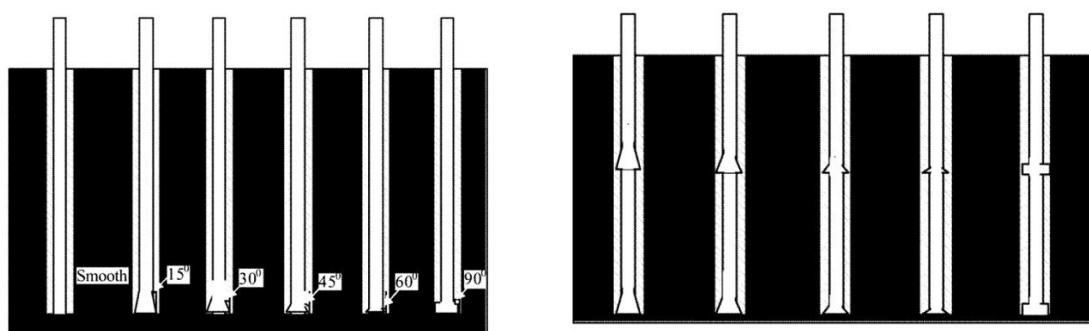
When the rib face angle is greater than the upper limit of slipping angle, the material between them will remain because there is no relative movement. In other words, grout material in this area will become part of the bolt profile, and in most cases, forever.

There are several research works of study on the bolt face angle. According to Tepfers (1973), slip on rebar can occur in two ways: (1) the rib can split the concrete by wedging action, and (2) the rib can crush the concrete. Tepfers also pointed out that a rib with a face angle between  $40^\circ$  and  $105^\circ$  produced about the same movement. Angle of the rib is defined in Figure 4. In addition, when concrete was crushed to a compacted powder, it became lodged in front of the ribs. However, Tepfers did not give any explanation of this phenomenon. The theory of upper limit of slipping angle derived in this paper is able to reason this experiment observation. If the internal friction angle of concrete is about  $30^\circ$ , then the bolt with face angle greater than  $60^\circ$  will produce the same performance.



**Figure 4 - The geometry of a deformed reinforcing bar and the mechanical interaction between the bar and the concrete, after Tepfers (1973)**

Kilic, *et al.*, (2002) studied the effect of different shaped lugs on the steel bolt, by pullout tests. These were smooth bars, thread bars, single conical lugged bars and double conical lugged bars. The schematic illustration of two groups of their tests is shown in Figure 5; and the result of these tests is shown in Table 1.



**Figure 5 - Schematic illustration of two groups tests conducted by Kilic, *et al.*, (2002)**

**Table 1 - Pull out test results for different rock bolt types, after Kilic, *et al.*, (2002)**

Bolt type	Yield load (kN)	Elastic displacement (mm)	Failure load (kN)
10° Single conical <sup>a</sup>	56	0.67	67.2
15° Single conical <sup>a</sup>	56	0.60	65.5
20° Single conical <sup>a</sup>	48	0.62	62.4
25° Single conical <sup>a</sup>	48	0.51	61.7
30° Single conical <sup>a</sup>	48	0.43	60.8
60° Single conical <sup>a</sup>	48	0.36	59.1
90° Single conical <sup>a</sup>	48	0.29	57.6
Smooth surfaces <sup>b</sup>	16	1.10	17.6
15° Single conical <sup>b</sup>	64	0.99	68.4
30° Single conical <sup>b</sup>	64	0.95	65.8
45° Single conical <sup>b</sup>	56	0.82	63.5
60° Single conical <sup>b</sup>	56	0.78	60.5
90° Single conical <sup>b</sup>	56	0.74	59.3
15° Double conical <sup>b</sup>	88	1.48	92.8
30° Double conical <sup>b</sup>	88	1.42	89.7
45° Double conical <sup>b</sup>	80	0.88	87.7
60° Double conical <sup>b</sup>	80	0.82	86.1
90° Double conical <sup>b</sup>	80	0.80	84.6

Their experimental results show a strong influence of the bolt profile on the load bearing capacity and deformational behaviour of the reinforcement system. It can also be found that the 60° bolts has very close performance with the 90° bolts (highlighted in Table 1). These results can be understood by, and also used to support to, the upper limit slipping angle theory conducted in this paper.

### Slipping angle range for known rebar bolt

It should be noted that, according to previous analysis, the upper limit of the slipping angle is independent of bolt profile. In another word, it is universal whenever dilational slipping failure occurs for a bolt unit. For a known bolt however, the angle range of possible slipping can further be narrowed. Combine Eq. (5), (6) and (7), eliminate  $\sigma_n$  and  $\tau$ , the axial load which causes dilational slip to occur is

$$F_{\text{dila}} = \frac{\pi(R^2 - r^2)}{\sin i} \left[ \frac{\cos \phi}{\cos(i + \phi)} c + p \tan(i + \phi) \right]$$

And Eq. (1) is the expression of the axial load capacity of the unit bolt for shear failure. So, if the dilational slip does occur, it will always have

$$F_{\text{shear failure}} \geq F_{\text{dilational slip}}$$

Combining with Eq. (5) leads to

$$2\pi RL(c + p \tan \phi) \geq \frac{\pi(R^2 - r^2)}{\sin i} \left[ \frac{\cos \phi}{\cos(i + \phi)} c + p \tan(i + \phi) \right] \quad (11)$$

To avoid cumbersome math, simply let  $p=0$ , then the equation can be easily solved as

$$\sin(\phi + 2i) \geq \frac{R^2 - r^2}{RL} \cos \phi + \sin \phi \quad (12)$$

The domain satisfying this equation will be the theoretical slipping angle range for this bolt.

Here is an example. The bolt shown in Figure 1 is a commercial bolt of average rib spacing 12.5 mm, core diameter is measured to be 21.5 mm. After taking the average to the rib cross section, the average ribs height is found to be 1.35 mm and the average rib width is 2.8 mm. This leads to  $R=12.2$  mm,  $r=10.8$  mm,  $L=9.7$  mm. Let the internal friction angle of the resin to be  $35^\circ$ , and put these parameters into Eq. (8) and solve, getting:

$$9^\circ \leq i \leq 46^\circ$$

The initial possible dilational slip direction for this bolt can be limited to a range of  $[9^\circ, 46^\circ]$ , as shown in Figure 6. Consequently, resin between the bolt profile can be divided into three divisions in case of dilational slipping. The resin above face angle  $46^\circ$  (indicated as I in Figure 6) will move with the bolt and can be thought of as part of the bolt profile. Section II will experience shear failure, hence micro-cracks in this area are inevitable. The resin below  $9^\circ$  will keep intact with resin in the annulus and its strength not expected to change very much.

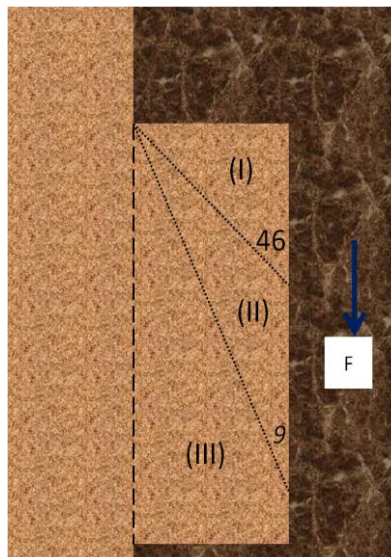


Figure 6 - The slipping angle range for rebar bolt in Figure 1



## CONCLUSIONS

Mechanical interlock is an important component in load transfer capacity in rockbolting systems. It is dominated by the rebar bolt profile configuration which includes rib height, rib spacing, rib face angle and shape of rib cross section. When an axial load is applied to cause failure of the rock bolt, two major failure modes can be identified as parallel shear failure and dilational slip. After introducing Coulomb's shear failure criterion, a universal upper limit of slipping angle can be calculated as the complementary angle of the grout internal friction angle. In addition, once the geometric parameters of a rebar bolt profile have been given, more accurate result can be achieved via simple plots. Consequently, the grout between the bolt profiles can be divided into three parts, one will be part of the bolt profile, one will undergo shear failure and the bottom part will remain unaffected.

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