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A skew-Hadamard matrix of order 92

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Abstract

Previously the smallest order for which a skew-Hadamard matrix was not known was 92. We construct such a matrix below.

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A skew-Hadamard matrix of order 92

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Previously the smallest order for which a skew-Hadamard matrix was not known was 92 . We construct such a matrix below. The orders < 200 which are now undecided are 100, 116, 148, 156, 172, 188, 196; see [2], [3]. The existence of any Hadamard matrix of order 92 was unknown until 1962 [1].

We construct a skew-Hadamard matrix of Williamson-type by using the matrix

$$W = \begin{matrix} A & B & C & D \\ -B & A & D & -C \\ -C & -D & A & B \\ -D & C & -B & A \end{matrix} .$$

Then if A is a $(1, -1)$ skew-type cyclic matrix of order 23 (that is $a_{i+1,j+1} = a_{i,j}$ where the subscripts are taken modulo 23), B, C, D are $(1, -1)$ anticyclic matrices of order 23 having symmetrical first rows (that is $b_{i,j} = b_{i+1,j-1}$, $b_{11} = 1$, $b_{1j} = b_{1,25-j}$ and so on, subscripts modulo 23) and

$$AA^T + BB^T + CC^T + DD^T = 92I_{23} ,$$

W is a skew-Hadamard matrix of order 92 .

Suitable first rows for the blocks A, B, C, D are

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A : 1 -1 -1 -1 -1 -1 -1 -1 1 1 -1 1 -1 1 -1 -1 1 1 1 1 1 1 1

B : 1 1 -1 -1 1 -1 -1 1 1 1 1 -1 -1 1 1 1 1 -1 -1 1 -1 -1 1

C : 1 1 -1 -1 -1 1 -1 1 -1 1 -1 1 1 -1 1 -1 1 -1 1 -1 -1 -1 1

D : 1 -1 -1 -1 -1 1 -1 -1 1 -1 -1 1 1 -1 -1 1 -1 -1 1 -1 -1 -1 -1

If $W = U + I$ is a skew-Hadamard matrix of order 92 where I is the identity matrix then

$$\begin{matrix} U+I & U+I \\ U-I & -U+I \end{matrix}$$

is a skew-Hadamard matrix of order 184 .

References

[1] Leonard Baumert, S.W. Golomb and Marshall Hall, Jr, "Discovery of an Hadamard matrix of order 92", *Bull. Amer. Math. Soc.* 68 (1962), 237-238.

[2] Jennifer Wallis, "(v, k, λ) configurations and Hadamard matrices", *J. Austral. Math. Soc.* 11 (1970), 297-309.

[3] Albert Leon Whiteman, "An infinite family of skew Hadamard matrices", (to appear).

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