Analytical solutions for a single vertical drain with time-dependent vacuum combined surcharge preloading in membrane and membraneless systems

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Abstract. Vertical drains combined with vacuum pressure and surcharge preloading are widely used to accelerate the consolidation process of soft clay in order to decrease the pore pressure as well as to increase the effective stress. Currently there are two types of vacuum preloading systems commercially available; (a) membrane system with an airtight membrane over the drainage layer and, (b) membraneless system where a vacuum system is connected to individual drain. Their effectiveness varies from site to site depending on the type of soil treated and the characteristics of the drain-vacuum system. This study presents the analytical solutions of vertical drains with vacuum preloading for both membrane and membraneless systems. According to the field and laboratory observations, the vacuum in both of the membraneless and membrane system was assumed to be decreasing along the drain whereas in the membrane system, it was maintained at a constant level. This model was verified by using the measured settlements and excess pore pressures obtained from large-scale laboratory testing and case studies in Australia. The analytical solutions improved the accuracy of predicting the dissipation of pore water pressure and the associated settlement. The effects of the permeability of the sand blanket in a membrane system and the possible loss of vacuum were also discussed.

1. Introduction
The booming population and associated development in coastal and metropolitan areas have necessitated the use of previously undeveloped low lying areas for construction purposes. Most of the Australian coastal belt contains very soft clays up to significant depth. The low bearing capacity and high compressibility of these deposits affect the long term stability of infrastructure. Hence, it is imperative to stabilize these soils before commencing construction to prevent unacceptable differential settlement. The system of vertical drains with a combined vacuum and surcharge preloading is an effective method for promoting radial flow to accelerate soil consolidation, which are widely used in modern ground improvement practices.
Prefabricated Vertical Drains (PVDs) are installed to short the length of the drainage path (radial flow), which reduces the time for consolidation [1-9]. Negative pressure along the drain increases the radial hydraulic gradient towards the drain, which in turn prevents the build up of excess pore water pressure in the soil and reduces the risk of failure [10]. Besides, applying the vacuum pressure allows the height of the surcharge embankment to be reduced to prevent any instability and lateral movement in the soil. There are two types of vacuum preloading systems commercially available (Figure 1), but the field effectiveness of the two systems is shown to vary widely with the type of soil treated and the nature of drains installed. Based on Teraghi’s one-dimensional consolidation theory, Mohamedelhassan and Shang [11] initially developed a combined one-dimensional vacuum and surcharge consolidation model. Subsequently, preliminary analytical and numerical models capturing the vacuum consolidation mechanisms were introduced by Indraratna et al. [12]. All the above solutions assume that the surcharge loading is applied instantaneously and kept constant during consolidation (step loading), and any change of the vacuum pressure during application is ignored.

In this paper, analytical solutions which reflect the difference between the vacuum preloading systems (membrane and membraneless system) under time-dependent surcharge and vacuum preloading are presented. The smear zone, well resistance of the drain and the loss of the vacuum pressure along the drain length are also considered.

![Figure 1. Vacuum-assisted preloading system: a) membrane system; (b) membraneless system (adopted from Indraratna et al. [13])](image)

2. Governing equations and solutions

The unit cell is used to approach the single drain (Figure 2). For the membrane system, a sand blanket is placed on top of the PVDs, and then an airtight membrane is placed on it where the edge of the membrane is submerged in a bentonite trench to prevent any air leaks. After that, a pre-determined vacuum pressure is applied through the horizontal drainage pipe located within the sand blanket (Figure 1a and Figure 2a). The three-dimensional flow in the sand blanket beneath the membrane \((0 \leq z \leq L_w)\) can be expressed as:

\[
-\frac{k_{bl}}{\gamma_w} \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_i}{\partial r} \right) \right) - \frac{k_{c1}}{\gamma_w} \frac{\partial^2 u_i}{\partial z^2} = -m_{el} \left( \frac{\partial u_i}{\partial t} - \frac{dq}{dt} \right) \quad r_w \leq r \leq r_c
\]

(1)

\[
\frac{\partial^2 u_{i1}}{\partial z^2} = - \left. \frac{2k_{bl}}{r_w k_{c1}} \frac{\partial u_i}{\partial r} \right|_{r=r_w}
\]

(2)

\[
\bar{u}_i = \frac{1}{\pi (r_c^2 - r_w^2)} \int_{r_w}^{r_c} 2\pi ru_i \, dr
\]

(3)

The main difference between a membrane and membraneless system is the boundary conditions. In the membraneless system, a vacuum pump is connected directly to the individual PVD’s through a system of horizontal pipes (Figure 1b and Figure 2b). The governing equations and initial conditions...
of underlying soil improved by PVDs are the same as for the membrane system. The vacuum pressure \( p(t) \) along the length of the drain was considered to vary linearly from \( p(t) \) at the top of the drain to \( \eta p(t) \) at the bottom, where \( \eta \) varies between 0 and 1.

The governing equations for the underlying soil \( (L_w \leq z \leq H) \), can be written as:

\[
-\frac{k_z}{\gamma_w} \left( \frac{\partial^2 u_{z2}}{\partial r^2} + \frac{\partial^2 u_{z2}}{\partial r \partial z} \right) - \frac{k_z}{\gamma_w} \frac{\partial^2 u_{z2}}{\partial z^2} = -m_{v2} \left( \frac{\partial u_{z2}}{\partial t} - \frac{dq}{dt} \right) \quad r_w \leq r \leq r_s \quad \text{(4)}
\]

\[
-\frac{k_z}{\gamma_w} \left( \frac{\partial u_{z2}}{\partial r} \right) - \frac{k_z}{\gamma_w} \frac{\partial^2 u_{z2}}{\partial z^2} = -m_{v2} \left( \frac{\partial u_{z2}}{\partial t} - \frac{dq}{dt} \right) \quad r_s \leq r \leq r_e \quad \text{(5)}
\]

\[
\frac{\partial^2 u_{z2}}{\partial z^2} = \left. \frac{2k_z}{r_s k_w} \frac{\partial u_{z2}}{\partial r} \right|_{r=r_w}
\]

\[
\overline{u}_z = \frac{1}{\pi (r^2 - r_e^2)} \left( \int_{r_e}^{r_w} 2\pi r u_{z2} dr + \int_{r_s}^{r_e} 2\pi r u_{z2}^r dr \right)
\]

The boundary conditions for both the radial and vertical directions for a membrane system are as follows:

\[
r = r_s: \frac{\partial u_{z2}}{\partial r} = 0, \quad \frac{\partial u_i}{\partial r} = 0
\]

\[
r = r_e: k_{z2} \frac{\partial u_{z2}}{\partial r} = k_{s2} \frac{\partial u_{z2}}{\partial r}
\]

\[
r = r_w: u_{z2} = u_{w2}, \quad \overline{u}_i = u_{w1}
\]

\[
z = 0: u_{w1} = p(t), \quad \overline{u}_i = p(t)
\]

\[
z = H: \frac{\partial u_{z2}}{\partial z} = 0, \quad \frac{\partial u_{z2}}{\partial z} = 0
\]

The boundary conditions for a membraneless system are:

\[
z = 0: u_w = p, \quad \frac{\partial u}{\partial z} = 0
\]

\[
z = H: \frac{\partial u_w}{\partial z} = \frac{\eta - 1}{H} p, \quad \frac{\partial u}{\partial z} = 0
\]

Continuity for the membrane system at the interface between the sand blanket and underlying layer of soil \( (z = L_w) \) may be then expressed by:

\[
z = L_w: u_{w1} = u_{w2}
\]

\[
z = L_w: \overline{u}_i = \overline{u}_2
\]

\[
z = L_w: k_{w1} \frac{\partial u_{w1}}{\partial z} = k_w \frac{\partial u_{w2}}{\partial z}
\]

\[
z = L_w: k_{w1} \frac{\partial \overline{u}_i}{\partial z} = k_{v1} \frac{\partial \overline{u}_2}{\partial z}
\]

The initial condition is:

At \( t = 0 \), \( \overline{u}_i = \overline{u}_2 = u_{i0}(z) = q_0 \)

where \( i \) is the index number of arbitrary layer, \( (i = 1, 2) \), \( r_s \) is the radius of smear zone, \( r_e \) is the radius of influence zone, \( r \) is the radial coordinate, \( z \) is the vertical coordinate, \( t \) is the time, \( m_{v2} \) is the
coefficient of volume compressibility of soil, \( k_v \) is the horizontal coefficient of permeability of soil, \( k_h \) is the vertical coefficient of permeability of the soil, \( k_w \) is the coefficient of permeability of the vertical drain, \( \bar{u}_i \) is the average pore pressure, \( u_d \) is the pore pressure at any point in the smear zone, \( u_w \) is the pore pressure at any point in the natural soil zone, \( u_w \) is the excess pore water pressure within the vertical drain, \( q(t) \) is the time-dependent surcharge preloading, \( q_0 \) is the initial value of preloading, \( L_w \) is the thickness of the sand layer, \( H \) is the thickness of the whole layer (i.e., for the membrane system (both sand blanket and clay layer, and for the membraneless system, only the clay layer), and \( p(t) \) is the vacuum pressure.

![Figure 2. Analysis schemes of unit cell with vertical drain: (a) membrane system; and (b) membraneless system.](image)

Using Laplace transform, the excess pore water pressure \( \hat{u}_w \) and average pore water pressure \( \bar{u}_i \) in the Laplace frequency domain can be given. The solutions to the excess pore water pressure \( u_w \) and average pore water pressure \( \bar{u}_i \) \((i = 1, 2)\) were obtained using the inverse Laplace transform (Durbin [14]):

\[
u_w(Z, S) = \frac{1}{2\pi I} \int_{a-I \sigma}^{a+I \sigma} \hat{\nu}_w(Z, S) e^{ST} dS \quad (i = 1, 2) \tag{9}
\]

\[
\bar{u}_i(Z, S) = \frac{1}{2\pi I} \int_{a-I \sigma}^{a+I \sigma} \hat{\bar{u}}_i(Z, S) e^{ST} dS \quad (i = 1, 2) \tag{10}
\]

where, \( I = \sqrt{-1} \), \( Z = \frac{z}{H} \), \( \hat{\nu}_w(Z, S), \hat{\bar{u}}_i(Z, S) \), are the Laplace transform of \( u_w(Z, S) \), \( \bar{u}_i(Z, S) \), respectively. \( T_h = \text{Time factor} \left( T_h = \frac{c_h^2}{d_c^2} \right\cdot i = 1 \) means the sand layer; \( i = 2 \) means the underlying soil layer.

The settlement of the soil is given by:

\[
s(t) = \int_{L_w}^{t} \varepsilon_s dz \tag{11}
\]
in which, \( \varepsilon_2 \) is the soil vertical strain.

3. Computed results
A large scale radial drainage consolidation test was conducted by Indraratna et al. [10] to study the soil improved using a vacuum with a membrane system, and to examine the soil response when the vacuum was re-applied. In order to simulate the membrane system, a large scale consolidometer was modified and used to examine vacuum preloading in conjunction with conventional surcharge loading. Several series of tests were performed to examine the effect of vacuum and surcharge preloading and the vacuum distribution along the drain. Approximately 0.14 m\(^3\) of soil was required for each sample. Moruya clay from NSW was used for the tests, and it consists of approximately 40%-50% of clay particles (<2 \( \mu \)m), and approximately 60% of particles smaller than 6 mm. The cell used for this test was 450 mm in diameter and 850mm high. A 1.5 mm thick sheet of Teflon was placed at the bottom of the cell and around the internal periphery to reduce friction, and a 50 mm thick layer of sand was placed at the top to represent the membrane system explained earlier. The selected geotechnical properties of a typical specimen are shown in Table 1.

Table 1. Soil properties of the reconstituted Moruya clay sample (adapted from Indraratna et al [12]).

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay Content (%)</td>
<td>40-50</td>
</tr>
<tr>
<td>Silt Content (%)</td>
<td>45-60</td>
</tr>
<tr>
<td>Water Content w (%)</td>
<td>38</td>
</tr>
<tr>
<td>Liquid Limit, wL (%)</td>
<td>57</td>
</tr>
<tr>
<td>Plastic Limit, wp (%)</td>
<td>17</td>
</tr>
<tr>
<td>Unit Weight, (t/m3)</td>
<td>1.81</td>
</tr>
<tr>
<td>Specific Gravity, Gs</td>
<td>2.56</td>
</tr>
<tr>
<td>Undrained shear strength (kPa)</td>
<td>7.3</td>
</tr>
</tbody>
</table>

An average vacuum of 100 kPa was applied to the PVD and soil surface through a hole in centre of the rigid piston. A subsequent surcharge load of 50 and 100 kPa, respectively was respectively applied instantaneously in two stages, with a 14 day gap between each stage. Over this 28 day period, the vacuum was released for short periods in two stages to investigate the effects of unloading and reloading. The suction measured by the piezometers showed a vacuum loss of up to 20% towards the bottom of the drain (in this case, \( \eta = 0.8 \) which represents the vacuum loss along the drain as). The settlement of the soil predicted by the membrane system model (based on the equation (11)) and the laboratory data plotted in Figure 3 shows an acceptable agreement.

In order to simulate a membraneless system, another large scale consolidometer was used. Apart from there being no sand at the top drainage layer, every other aspect was the same as the membrane system described earlier. The sample preparations and testing procedures have been explained by Indraratna et al. [12]. The tests showed that there was about 15-20% of vacuum loss at the bottom of the drain (\( \eta \) varies between 0.85-0.8). The preconsolidation pressure was 20 kPa, applied vacuum pressure was 40 kPa and then additional 30 kPa surcharge pressure was applied. Figure 4 shows the comparison between the calculated and measured average pore water pressure. It can be seen that the excess pore water pressure agrees well with the predicted results.
Figure 3. Predicted and measured settlement at the top of the consolidometer cell (adapted from Indraratna et al 2004): (a) vacuum load model; (b) predicted and measured settlement.

Figure 4. Comparison between the measured and calculated excess pore water pressure dissipation: (a) Measured 0.79 m from bottom; (b) Measured 0.47 m from bottom; (c) Measured 0.15 m from bottom.

4. Sensitivity analysis
The assumed parameters were: \( r_w = 0.05 \text{m} \), \( r_c = 0.5 \text{m} \), \( r_s = 0.15 \text{m} \), \( k_{s1}/k_{v1} = 1.0 \), \( k_{s2}/k_{v2} = \) from \( 10^{-1} \) to \( 10^{-6} \), \( k_{s2}/k_{v2} = 2.0 \), \( k_{s2}/k_{v2} = 10^{-6} \), \( k_{s2}/k_{v2} = 4 \), \( L_s = 2 \text{m} \), \( H_{clay} = \) from 10m to 40m, \( m_{v1} = m_{v2} = 0.25 m^2 / MN \), \( q = 80 kPa \) and \( p = -80 kPa \). They are similar to the typical values obtained from previous case studies (i.e. Tianjin Port, China (Rujikiatkmajorn et al., 2008) and the laboratory experiment results (Indraratna et al. [12]).
Figure 5 presents a variation of the time dependent settlement curves. Normalised settlement is defined by the settlement at a given time divided by the ultimate settlement due to surcharge preloading alone, \( \frac{s_t}{s_\infty} \). If there is no loss of vacuum, the difference in the normalised settlement between a membrane system and membraneless system is negligible.

The permeability of a sand blanket in a membrane system controls the speed at which the vacuum pressure can propagate from a horizontal drainage system placed within the sand to PVD’s, and the interface between the bottom of the sand blanket and the upper layer of clay. A sand blanket generally varies from 0.3 to 2 metres in thickness. In this analysis, \( L_m \) was assumed to be 2 metres. Figure 5 illustrates the effect of the permeability of a sand blanket in a membrane system. As expected, when permeability decreases, consolidation takes longer. When the PVD’s are relatively short, say less than 10 m (Figure 5a), the permeability of the sand blanket should not be less than 0.01 times the permeability of the PVD and \( 10^4 \) times the permeability of the clay to maintain the desired time to achieve a degree of consolidation of 90%. With longer drains (Figure 5b), the ratios of the permeability of the sand blanket to that of the PVD, and the ratio of the permeability of the sand blanket to that of the clay layer should be more than 0.1 and \( 10^5 \), respectively.

![Figure 5. Normalized settlement-time factor curves for varying \( K_s \) (for membrane system) and \( \eta \) (for membraneless system): (a) the thickness of the clay is 10 m; (b) the thickness of the clay is 40 m.](image)

**5. Conclusions**

The application of PVDs combined with vacuum and surcharge preloading has become common practice, and is now considered to be one of the most effective ground improvement techniques. Currently there are two types of vacuum preloading systems commercially available: (a) membrane system; and (b) membraneless system. The effectiveness of the two systems is shown to vary widely with the type of soil treated and the nature of PVDs installed. This study presented the analytical solutions of vertical drains with vacuum preloading for both membrane and membraneless systems. According to the field and laboratory observations, the vacuum in both of the membrane and membraneless system was assumed to be decreasing along the drain. This model was verified by using the measured settlements and excess pore pressures obtained using large-scale laboratory testing. It was found that the permeability of the sand blanket can significantly affect the consolidation in a membrane system. The permeability ratio of the PVD to the sand blanket for relatively short PVD’s (< 10 m) should be less than 100 to accelerate the vacuum pressure propagation, thereby minimising the effect on consolidation (the degree of consolidation \( U \) up to 90%), but this ratio for deeper soft clay (longer PVDs) should be less than 10. Therefore, for deep soft clay, a membrane system with a high...
permeability sand blanket layer will be more appropriate to achieve the designed degree of consolidation within a desired period of time.

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