2011

Bounded Rationality and the Emergence of Simplicity Amidst Complexity

Cassey Lee
University of Wollongong, casseyl@uow.edu.au

Bounded Rationality and the Emergence of Simplicity Amidst Complexity

Abstract

The purpose of this essay is to explore the relationship between the simple and the complex in economics by anchoring our analysis on bounded rationality. Much of the conventional literature focuses on 'un-bounded rationality' of the rationality-as-consistency variety. Theorizing of bounded rationality tends to assume that the problem to be solved is independent of the nature of bounded rationality. Following the insights from the works of Herbert Simon and contributions from outside economics, both bounded rationality and the environment are inextricably linked. The boundaries between bounded rationality and its environment can shift. The form in which bounded rationality is found depends on the complexity of the environment. Furthermore, if local interactions between bounded rational agents result in the formation of hierarchies – the complexity of the collective system will change. Whether this will occur depends on the nature of bounded rationality at the individual level.

Bounded Rationality and the Emergence of Simplicity Amidst Complexity[1]

Cassey Lee

School of Economics

University of Wollongong

NSW 2522, Australia

E-mail: casseylee@gmail.com

10 November 2010

Abstract

The purpose of this essay to explore the relationships between the simple and the complex in economics by anchoring our analysis on bounded rationality. Much of the conventional literature focuses on "un-bounded rationality" of the rationality-as-consistency variety. Theorizing of bounded rationality tends to assume that the problem to be solved is independent of the nature of bounded rationality. Following the insights from the works of Herbert Simon and contributions from outside economics , both bounded rationality and the environment are inextricably linked. The boundaries between bounded rationality and its environment can shift. The form in which bounded rationality is found depends on the complexity of the environment. Furthermore, if local interactions between bounded-rational agents result in the formation of hierarchies - the complexity of the collective system will change. Whether this will occur depends on the nature of bounded rationality at the individual level.

Bounded Rationality and the Emergence of Simplicity Amidst Complexity

Cassey Lee

"You cannot successfully use your technical knowledge unless you are a fairly educated person, and, in particular, have some knowledge of the whole field of the social sciences as well as some knowledge of history and philosophy. Of course real competence in some particular field comes first. Unless you really know your economics or whatever your special field is, you will be simply a fraud. But if you know economics and nothing else, you will be a bane to mankind, good, perhaps, for writing articles for other economists to read, but for nothing else" F.A.Hayek (1991, p.38)

"We feel clearly that we are only now beginning to acquire reliable material for welding together the sum total of all that is known into a whole; but on the otherhand, it has become next to impossible for a single mind fully to command more than a small specialized portion of it ... I can see no other escape from this dilemma than that some of us should venture to embark on a synthesis of facts and theories, albeit with second-hand and incomplete knowledge of some of them - and at the risk of making fools of ourselves" Erwin Schrödinger (1967, p.1)

1   Introduction

The notion of rationality and the way in which it is assumed and applied in economics is a much debated topic within the discipline itself and beyond. The vast body of literature under the headings of 'behavioral economics' and 'economics and psychology' have attempted to make sense of the extent to which and the manner in which rationality in reality differs from rationality as it is assumed in economics. The term 'bounded rationality', which can be traced back to Herbert Simon's influential contributions in the 1950s, has been used by many when referring to departures from the conceptualization of rationality as consistency or rationality as maximization in mainstream economic theory. Today, the notion of bounded rationality has a permanent place in economics. Its impact has been profound in terms of our theoretical and empirical understanding of decision making and judgement, markets, organizations and institutions.

Parallel to the current research in mapping the boundaries of bounded rationality (to paraphrase Kahneman's Nobel lecture title) is an increasingly influential line of research that attempts to transform economics, this time into

a social science that embraces complexity theory. A core element within this research programme is its focus on the emergence of complex structures from micro-level interactions between relatively simple parts/elements/agents. An analysis of how bounded rationality and complexity is related should be of great interests to economists. Most economists are likely to agree on the bounded rational nature of the human species as well as the complex nature of the economy. Yet, these two aspects have often been considered and researched separately with a few exceptions even though both are inextricably linked. This is a key aspect of Herbert Simon's ideas.

The purpose of this essay to explore the relationships between the simple and the complex in economics by anchoring our analysis on bounded rationality. The point of view taken in this paper is that bounded rationality and the complexity of environment are both inextricably linked - that the emergence of complex social structures would not possible without interactions between bounded-rational agents, vice-versa. Furthermore, the bounded-rational nature of agents is in itself a consequence of a complex environment. What this implies in terms of a broader vision of economics is a topic worth exploring.

2 Rationality

The notion of rationality occupies a central position in modern economic theory. Blaug (1992, p.230) opines that neoclassical economists regard the rationality postulate as part of the Lakatosian 'hard core' in their research programme. Within choice theory, many have characterized rationality in terms ofpreferences conforming to a set of axioms such as completeness, reflexivity, transitivity and continuity.[1]. Walsh (1996) has labeled this approach as 'rationality as consistency'. Others have argued that the predominant view of rationality is that of 'rationality as maximization' e.g. utility (or profit) maximization [2]. The two are related, that is, a 'perfectly' rational agent is one exhibiting "consistent maximization of a well-ordered function" (Becker, 1962, p.1) or alternatively (and more generally), one who "makes decisions consistently in pursuit of his own objectives" (Myerson, 1991, p.2). To scholars from other disciplines, such views may seem to be a very narrow view of human rationality. For the purpose of exploring the nature and implications of departures from perfect rationality, it is perhaps useful to note that such views were not always the predominant ones in economics.

---

[1]See for example, the formal exposition in Varian (1992), pp.94-95.

[2]See Arrow, 1986, pp.S388-390 and Simon, 1976, p.67

In terms of ancestral views and visions, scholars point to a more 'pragmatic' notion of rationality adopted by Adam Smith in terms of "preferring more to less" (Arrow, 1986, p.S388) and satisficing behavior described as "practical behavior of reasonable persons" (Simon, 2000, p.27). Elsewhere, Simon (1997, p.6) suggests that rationality, in so far as it can be inferred from Adam Smith's works, can be interpreted to mean "having reasons for what you do". An interesting and related issue in discussions on rationality is the role of self-interest. Simon (1997, p.7) provides such an interpretation: "...the economic actors are certainly behaving rationally - that is, pursing what is they suppose to be their self-interest." In this regard, Sen (1977) has argued for a need to go beyond this conceptualization of rationality as self-interest. This brings in the question of ethics and morality and their relationship to rationality. Walsh (1996), for example, cites Hilary Putnam's observation that "our values of equality, intellectual freedom, and rationality, are deeply connected".[3]

Beyond the classical economics period or rather at the tail end of it, it can perhaps be argued that the notion of rationality underwent further narrowing in the form of utilitarianism in the 1850s and 1860s and later in the form of the marginal utility doctrine via the works of Stanley Jevons, Carl Menger and Leon Walras in the 1870s.[4] The next significant development was the formulation of demand/preference theory based upon an ordinal interpretation of utility - indifference curves via Fisher and Pareto in the 1890s, consumer behavior based on ordinal utility via Hicks and Allen in 1934 and Samuelson's weak axiom of revealed preference in the late 1930s [5]. These developments were crucial in establishing the basic axioms underlying the rationality-as-consistency approach in choice theory.

The 1930s also saw the beginnings of a dramatic transformation of economics of a different sort, namely the axiomatization of economic theory via the works of John von Neumann, Oskar Morgenstern and Abraham Wald. After the Second World War, the axiomatic method gained a permanent foothold in economic theory through general equilibrium analysis and social choice theory in the 1950s.[6] One interpretation of these developments (hopefully not a too naive one), is that the sanctity or unassailability of the rationality-as-

---

[3]The discussions in Smith (2003) seem to hint at the possible gains from further examining, first the broad views held by Scottish philosophers such as David Hume and Adam Smith, and second, more specifically, an integrated view of Smith's visions on rationality inferred from the *Wealth of Nations* and the *Moral Sentiments*.

[4]We rely on Blaug (1985) for this and the following narrative.

[5]See Blaug (1985, Ch.9) and Blume and Easley (2008, p.2)

[6]See Ingrao and Israel (1990) and Giocoli (2003).

consistency view in economics was further entrenched by the axiomatization of economic theory.

Apart from these developments, two other developments that are usually mentioned in discussing rationality in modern economics, namely, game theory and rational expectations. Formal treatments of game theory dates as far back as 1912 in the form of Ernst Zermelo's work followed by the 'subjective' approach by von Neumann and Nash in the 1940s-1050s (Vellupilai, 2009, p.1411). Rational expectations was first proposed by John Muth in 1961 and developed later by Robert Lucas and Thomas Sargent in the 1970s. What are the notions of rationality associated with these developments?

The discussions on the notion of rationality in game theory have centered around a number of issues. These relate to whether the axioms underlying a given game characterize a rational person e.g. agents maximize or minimaxing payoffs and the efficient division of the game's surplus (Simon, 1991a, p.2 and Samuelson, 1996, p.19). Rationality in game theory has also been related to the ability of agents to undertake long backward-induction calculations in extensive form games (Simon, 1991a, p.24). Not only are agents (players) in game theory usually assumed to be perfectly rational, such rationality is assumed to be common knowledge (known to all players).[7] In contrast, evolutionary game theory does away with any assumption of maximizing but this problem is not really addressed as the players in such theories do not correspond to individual players (Aumann, 1997, p.5). Overall, the notion of rationality remains an open problem within game theory. Many scholars have advocated the usefulness of incorporating bounded rationality in game theory albeit there are disagreements about how this should be accomplished. For example, commenting on a preliminary draft of Ariel Rubinstein's book on modeling bounded rationality, Herbert Simon advocates an approach anchored "careful observation and experimentation" rather than "casual observations" (Rubinstein, 1998, p.188).

In rational expectations, discussions on the notion of rationality center around how agents confront uncertainty about the future. In this regard, the standard assumptions are ones in which "people behave in ways that maximize their utility (their enjoyment of life) or profits" and "outcomes do not differ systematically (i.e., regularly or predictably) from what people expected

---

[7]A number of paradoxes arise out of the common knowledge assumption, the solution to which lies in distinguishing between outcome of a game and the assumptions on agents' behavior. See Dekel and Gul (1997) and the Autumn issue of the Journal of Economic Perspectives, 1992.

them to be".[8] The former suggests that agents in rational expectations are fully rational in the sense of maximizing. The latter implies that uncertainty is removed from agents by assuming that all of them know the correct model of the economy (Simon, 1991a, p.8).[9] The consequence of this is that agents are able to predict the future with accuracy, thus enabling them to behave in a substantively rational manner (Simon, 1976, p.79).

3   Bounded Rationality

3.1   Bounded Rationality: Origins

Within economics, there have been some interests in departing from the notion of 'perfect rationality' (either in the form of rationality as consistency or rationality as maximization). To date, there is no consensus on the form in which such departures *should* assume in theoretical models (a normative issue). This is encapsulated by Frank Hahn remarks that "there is only one way to be perfectly rational, while there are an infinity of ways to be partially rational ... where do you set the dial of rationality?".[10] To further muddle this debate, the theoretical and empirical responses to this challenge in terms of departures from rationality have been variously labeled as nonrational, irrational and bounded rational.[11]   Amongst these terms, the most often used one is that of bounded rationality.

The notion of 'bounded rationality' can be traced back to the pioneering work of Herbert Simon beginning in the 1950s. In an early work, Simon (1955) embarked on an attempt to drastically revise the concept of economic man by paying attention to the limits in the information and computational capacities of an economic man:

---

[8]Thomas J. Sargent. "Rational Expectations." The Concise Encyclopedia of Economics. 2008. Library of Economics and Liberty. Retrieved October 4, 2010 from the World Wide Web: http://www.econlib.org/library/Enc/RationalExpectations.html

[9]This is similar to the common knowledge assumption in game theory. The correct model of the economy is "common knowledge" amongst agents in the macroeconomy.

[10]See Waldrop (1992, p.92).

[11]Do these terms refer to different things? Gigerenzer and Selten (2001) defines irrationality in terms of discrepancies between a norm and human judgement (e.g. in terms of optimization, probability and utilities - elements of what is known as substantive rationality). In contrasts with irrationality, models of bounded rationality dispenses with these types of human judgement. In fact, bounded rationality is considered to be related to nonrationality.

"Broadly stated, the task is to replace the global rationality of economic man with a kind of rational behavior that is compatible with the access to information and the computational capacities that are actually possessed by organisms, including man, in the kinds of environments in which such organisms exist." (p.99)

Thus, the term 'bounded rationality' can be interpreted to mean rational choice under computational constraints (Simon, 1955, p.101). Within this interpretation, satisficing, in Simon's words, are 'approximating procedures' or "simplifications the choosing organism may deliberately introduce into its model of the situation in order to bring the model within the range of its computing capacity" (ibid, p.100). In the paper, in addition to the incorporation of simpler (discrete) pay-off functions, Simon introduced information gathering which improves the precision of behavior-outcome mapping. The notion of 'aspiration level' was also introduced with the view that this can change depending on the ease of discovering satisfactory alternatives (ibid, p.111).[12]

Simon's emphasis on the computational foundations of decision making were probably further reinforced after 1955 when he became more involved in research on problem-solving in cognitive psychology and computer science (artificial intelligence).[13] By the 1970s, the term 'procedural rationality' was used to denote a concept of rationality which focused on "the effectiveness of the procedures used to choose actions" (Simon, 1978, p.9) where the process of choice is important (ibid, p.2 and Simon, 1976, p.131). This is different from the notion of 'substantive rationality' found in concepts such as 'rationality as maximization' or 'rationality as consistency' where choice is entirely determined by the agent's goals subject to constraints (and consistency requirements). Since the goals of agents are either assumed (maximizing utility) or are embedded in the axioms of preferences, the focus of substantive rationality lies in the results of rational choice. In discussing the concept of procedural rationality, Simon (1976, pp.72-73) also highlighted the importance of using heuristics as means of selectively searching the "immense tree of move possibilities". Furthermore, Simon (1978, p.12) attempted to relate the two concepts of satisficing and heuristics to the theories of computational complexity (which emerged in the 1960s) and heuristic search:

---

[12]It is interesting to note that at this juncture Simon used the term 'approximate rationality' in his concluding remarks. Klaes and Sent (2005, p.37) suggests that the term 'bounded rationality' is likely to have been first used in Simon (1957).

[13]This change in Simon's research focus is discussed in Simon (1982, p.401) and Simon (1991b, p.189).

"One interesting and important direction of research in computational complexity lies in showing how the complexity of problems might be decreased by weakening the requirements for solution - by requiring solutions only to approximate the optimum, or by replacing an optimality criterion by a satisficing criterion."

"The theory of heuristic search is concerned with devising or identifying search procedures that will permit systems of limited computational capacity to make complex decisions and solve difficult problems. When a task environment has patterned structure, so that solutions are not scattered randomly throughout it, but are located in ways related to the structure, then an intelligent system capable of detecting the pattern can exploit it in order to search for solutions in a highly selective way."

3.2 Bounded Rationality: Empirical and Theoretical Developments

The rich ideas of Simon not withstanding, it took another decade, namely in the late 1980s, before bounded rationality received significant attention within economics (Klaes and Sent, 2005, p.45). Even so, not all researchers whether they are empiricists (which includes experimentalists) or theoreticians used the term 'bounded rationality' in the same manner. Within the empriricists/experimentalists camp, which originated from the field of psychology, the early work on heuristics and biases by Tversky and Kahneman (1974) made no reference to Simon work. This could be due to the distinction that they make between reasoning and intuitive thoughts, the latter being more important to judgements and choices (Kahneman, 2003, p.1450). Simon does not seem to have made this distinction even though he did acknowledge the importance of the unconscious: " ... we cannot rule out the possibility that the unconscious is a better decision-maker than the conscious" (Simon, 1955, p.104). In Kahneman's Nobel lecture, he describes his work with Tversky as an exploration of the psychology of intuitive beliefs and choices and an examination of their bounded rationality (Kahneman, 2003, p.1449). However, in their seminal works, explicit references to bounded rationality were only made in discussing framing effects (Tversky and Kahneman, 1981, p.458). Much of the body of research carried out under the banner of "Behavioral Economics" or "Economics and Psychology" were devoted to testing departures from elements of substantive rationality such as complete preferences, expected utility, Bayesian updating and exponential

discounting, amongst others (see Camerer (1998) and Rabin (1998)). This can also be inferred in the following remarks in Rabin (1998, fn.1) which could be interpreted as a reference to Simon's approach:

> "Another topic I have omitted is "non-psychological" models of bounded rationality. Researchers have formulated models of bounded rationality (based on intuition, computer science, or artificial intelligence) meant to capture cognitive limits of economic actors, but which do not invoke research on the specific patterns of errors that human beings make."

On the theoretical front, a number of different approaches to modeling bounded rationality have been adopted since the 1980s. To make sense of the literature, it is perhaps useful to try to classify the diverse models and methods that have been used to theorize some of the implications of bounded rationality.

One approach originates from the efforts by mathematicians and computer scientists to understand the foundation of mathematics as well as the nature and limits of computation. The origins of computability theory can be traced back to two monumental works in the 1930s. In 1931, Kurt Gödel shattered the Hilbertian program of attempting to derive all mathematics from a complete and consistent set of axioms. Gödel showed that there are true statements within such systems that are not provable. For the theory of computation, this result implies that "there are some functions on the integers that cannot be represented by an algorithm - that is, cannot be computed" (Russell and Norvig, 1995, p.11). This was to be followed by Alan Turing's proof that there are some functions that even powerful computing devices (such as the Universal Turing Machine) cannot compute.

The body of literature known as 'computable economics' and 'algorithmic economics' associated with the works of K. Vela Velupillai represent, perhaps, the most sustained application of computability theory to economics.[14] Surprisingly, there has been very little discussions on bounded rationality within a computability framework. Exceptions include Velupillai (2000) and Vellupilai (2010). Both works advanced several important points. The modeling of bounded rationality algorithmically (e.g. via the use of Turing machines)

---

[14]These terms were coined by K.Vela Velupillai. Early contributors who have applied computability theory to decision making problems include Michael Rabin, Alain Lewis and Preston McAfee (see Velupillai, 2000, p.17). Other contributors in the 1980s include Luca Anderlini and Kislaya Prasad.

implies that choice and decision-making are intrinsically dynamic processes. Equally important, bounded rationality is not a constrained version (or a special case) of rationality in the so-called 'Olympian models' (in our parlance, models where agents are fully-rational). Rather, bounded rationality should be perhaps considered as the general case and full-rationality a special case.

Another line of theoretical approach to bounded rationality that is related to computational theory but is different from the computability approach discussed above, involves the use of finite state automata to model limits to strategies that players can employ in games.[15] Early pioneers include Neyman (1985) and Rubinstein (1986). In these works, each player is assumed to employ an automaton (often a Moore machine) to play the game. Here, 'bounded rationality' can be interpreted in terms of limits to the number of states of the automaton. Such limits can either be exogenously determined (as in Neyman) or endogenously determined (as in Rubinstein). In the latter case, the number of states in the automaton is determined by a trade-off between the cost of maintaining such states and payoffs from a repeated game.

Another automata-based approach which is closer to 'applied' computational theory or computational complexity theory is that of Gilboa (1988) and Ben-Porath (1990). In contrast the works of Neyman and Rubinstein, which looks at complexity of implementation, these works consider the complexity of computational involved in selecting strategies. Here, the analysis of the complexity of computation involves assessing the amount of resources (such as time and memory) required to solve computational problems. This is analyzed in terms of whether a polynomial time algorithm exists to solve a given computation problem such as Nash equilibrium. Overall, the results obtained suggest that Nash equilibria can be hard to compute (i.e. requires non-polynomial time) except for restricted cases (e.g. anonymous opponents and graphical structures).[16] What of bounded rationality within a computational complexity context? Roughgarden (2010, p.231) argues that:

> "For equilibrium computation problems, polynomial-time solvability correlates well with efficient learnability and has an ap-

[15]See Aumann (1997) for a good summary and Chatterjee and Sabourian (2008) for a more extensive survey.

[16]For a recent summary of the state of art research in this area, see Daskalakis *et al.* (2009) and Kalai (2009). The fact that both tend to be mutually exclusively applied in economic theory reflects the difficulties encountered in reconciling both within a single modeling exercise, as Roughgarden (2010, p.210) has argued.

pealing interpretation in terms of boundedly rational participants. While the exact definition of hard varies with the nature of the problem, all such hardness results suggest that no fast and general algorithm will be discovered, motivating the study of heuristics, distributional analyses, domain-specific special cases, and alternative, more tractable equilibrium concepts."

The above remarks suggest that questions pertaining to the hardness of solving a problem, ways of reducing this hardness and the nature of the problem to be solved are inextricably linked. This insight is not entirely new and can be found in some of Herbert Simon's earliest published works. It provides a call for a reconsideration of how bounded rationality should be framed.

4   Bounded Rationality and the Complexity of Environment

4.1   Some Early Views From Simon

Herbert Simon's seminal 1955 paper that contained his early ideas of bounded rationality was followed by an equally interesting paper that was published a year later. In the paper, Simon (1956, p.120) articulated the importance of considering the environment within which bounded rational agents operate:

> "A great deal can be learned about rational decision making by taking into account, at the outset, the limitations upon the capacities and complexity of the organism, and by taking into account of the fact that the environments to which it must adapt possesses properties that permit further simplification of its choice mechanisms."

This line of thinking continued to preoccupy Simon and his collaborator, Allen Newell, in their subsequent work. For example, an entire chapter (three) in Newell and Simon (1972) was devoted to the 'task environment' defined as "an environment coupled with a goal, problem, or task". The centrality of the interdependence between bounded rationality and the task environment is clearly articulated in the book using a scissors metaphor reminiscence of Marshall's use of the same metaphor for demand and supply:

> "Just as a scissors cannot cut without two blades, a theory of thinking and problem solving cannot predict behavior unless it encompasses both an analysis of the structure of task environments and an analysis of the limits of rational adaptation to task requirements." (Newell and Simon, 1972, p.55)

In their theory, the locus of the links and interactions between the external environment and bounded rationality is the 'problem space' which is described as "the space in which his problem solving activities takes place" (Newell and Simon, 1972, p.59) This problem space is not exogenously given but is something that is derived internally (within the agent's mind) via the construction of an internal representation of the task environment (ibid, p.59).

Another interesting point that Newell and Simon make is the possibility of shifting the boundary between the problem solver (as an information processing system or IPS) and the environment (ibid, p.81) - which is accompanied by the need for another parameter, namely the intelligence of the problem solver (ibid, p.82). A reading of Newell and Simon (1972, pp.81-82) suggests that intelligence is related to the predictive abilities of a problem solver, which in turn can only be defined in terms of the type (or classes) of environment in which such abilities are valid. In an intriguing discussion of an extreme case of shifting such boundaries, Newell and Simon suggests that:

> "We must exercise caution, however, in shifting the boundary between problem solver and environment. If we move particular operators and classify them with the task environment, there is a danger that a problem solver will disappear entirely, and that there will be no room at all for a theory of him." (ibid, p.81)

This is indeed what the literature on situated or embodied cognition seems to imply (Anderson, 2003). The other extreme would be of course, to shift the boundary entirely away from the environment which perhaps have an equally disturbing implication that there is actually no problem to be solved except for that which exists in the mind of the problem solver! Another interesting point that Newell and Simon (1972, pp.93-94) make is the possibility of employing methods to reduce the problem space that needs to be explored to find a solution. After employing such methods, the agent will be left with a irreducible problem space that has to be examined in its entirety.

4.2 Some Recent Developments

The vision on the interdependence between bounded rationality and the environment within which decisions are made continue to find resonance in contemporary views:

> "Models of bounded rationality describe how a judgement or decision is reached (that is, the heuristic processes or proximal mechanisms) rather than merely the outcome of the decision, and they describe the class of environments in which these heuristics will succeed or fail." (Gigerenzer and Selten, 2001, p.4)

The emphasis on context/environment suggests that there has to be a matching between the decision/judgement processes that are used and the structure/complexity of the environment (problems) to which they are applied. The term 'ecological rationality' has been used by Vernon Smith to denote this heuristic-environment matching. This can contrasted with the term "constructivist rationality" in which social institutions are created 'top-down' by what Smith (2003, p.467) describes as "conscious deductive processes of human reason".

What are the sources of ecological rationality and how do they come about? Vernon Smith (2003, p.469) suggests that such order "emerges out of cultural and biological evolutionary processes". The diverse views on the subject matter seem to suggest that our current state of knowledge in this area is far from complete and definitive. For example, there are at least five different views/theories on this subject, namely sociobiology, human behavioral ecology, evolutionary psychology, memetics and gene-culture co-evolution (Laland and Brown, 2002).

Following a co-evolutionary line of argument and perhaps inspired by similar views advanced by Simon (1956) and Newell and Simon (1972), it can be perhaps be further argued that the bounded rational nature of the human species is inextricably linked to the complexity of the environment. This is put forward succinctly by Gigerenzer (2001, p.5):

> "Simple heuristics can succeed by exploiting the structure of information in an environment. In other words, the environment can do part of the work for the heuristic."

A similar view is advanced by Rubinstein (1998, p.3) on the relationship between formal social institutions and decision-making:

> "Many social institutions, like standard contracts and legal procedures, exist, or are structured as they are, in order to simplify decision making"

Furthermore, the bounded rational nature of humans also leads to the formation of formal and non-formal social structures and relationships (e.g. markets, non-markets, organizations, institutions, norms, conventions) that not only affect the choice and efficacy of 'approximating procedures' (to use Simon's terminology) but are themselves outcomes of such procedures:

> "Rules emerge as a spontaneous order - they are found - not deliberately designed by one calculating mind. Initially constructivist institutions undergo evolutionary change adapting beyond the circumstances that gave them birth. What emerges is a form of "social mind" that solves complex organization problems without conscious cognition. This "social mind" is born of the interaction among all individuals through the rules of institutions that have to date survived cultural selection processes." (Smith (2003), p.500)

Thus, what emerges is a social structure that acts as a collective problem-solving mechanism that sometimes complements and substitutes the judgement and decision processes at the individual level. Here, we recognize the ability of bounded-rational agents to partake in some rational constructions that can further adapt and evolve, sometimes in unpredictable directions (this is a view articulated by Vernon Smith). This also extends to more tacit social constructions/relations such as norms and conventions. For example, Conlisk (1996, p.677) suggests that:

> "Norms might be the cause of bounds on individualistic rationality. Or norms might be the effect of bounded rationality ... docility to social norms improves economic fitness by inducing people to augment their limited rationality with the collective wisdom of their social group."

Such ideas also seem to be related to what Arrow (1986) considers to be Adam Smith's profound insight captured in the invisible hand metaphor:

"Actually, the classical view had much to say about the role of knowledge, but in a very specific way. It emphasized how a complete price system would require individuals to know very little about the economy other than their own private domain of production and consumption. The profoundest observation of Smith was that the system works behind the backs of the participants; the directing "hand" is "invisible." Implicitly, the acquisition of knowledge was taken to be costly." (p.S391)

The invisible hand metaphor has been the subject of market experiments for some time. An early example is Smith (1962), who provided experimental evidence that price adjustments were not consistent with that associated with a Walrasian tântonnement and that a decentralized trading system could bring about economic equilibrium. Perhaps, even more startling, Gode and Sunder (1993) showed that allocative efficiency can be achieved in markets populated by traders with zero intelligence. Aside from these works, there have been other attempts by economists to discuss and model decentralized (micro-level) interactions, often incorporating some form of departures from full-rationality. Kirman (2003, p.22) emphasizes the importance of decentralized market interactions involving individuals with "limited reasoning and calculating capacities". Both Kirman (1997) and Axtell (2007) argue that a fruitful way forward is to model decentralized interactions within networks. In a later paper, Kirman *et al.* (2007) modeled the self-organization of social networks via the assumption of bounded rational agents in a 'spatially myopic sense' (i.e. capable of interacting with only a subset of neighbors).

An issue that arise in virtually all these works is the the emergence of patterns arising from local interactions albeit without sufficient attention paid to the computational aspects whether from a computability or a computational complexity point of view. On this issue, it is perhaps useful to note recent attempts to deal with some of the computational theoretical problems associated with general equilibrium theory. Whilst Axtell (2005) argued that the Walrasian general equilibrium is an implausible conception given the difficulty (NP hard) to compute it. In an even more devastating critique of the theory, Velupillai (2006) argued that the standard CGE model is not computable. In other words, since the equilibrium in a GE model cannot even be computed (solved algorithmically without any resource constraints) - there is not much point in even discussing the computational complexity of the problem (or computation under resource constraints). This is a distinction between what is solvable in principle (computable) and what is solvable in practice (tractable). A non-computable problem cannot be solved

at all whilst a computable problem may not be solvable in practice i.e. intractable.(See Davis *et al.* (1994, p.444)).

Clearly, the topic on the relationship between bounded rationality and the environment requires further research especially from more empirical evidence (anchored within a information processing and computational framework) as well as theoretical explorations along the lines of computability and computational complexity. The full richness and relevance of Simon and Newell's vision on the subject can be seen from other disciplines. A particularly fruitful area of research involves the study of decision-making in social insects. Interesting results include ant colonies that exhibits optimal decision-making (e.g. Edwards and Pratt, 2009) and that can even solve NP-hard problems such as the Travelling Salesman Problem (Dorigo and Gambardella, 1997). Even more intriguing are perhaps studies on colony-level cognition where internal representation of cognition are found in individual insects and their interactions with each other (Marshall and Franks, 2009). Within social insect colonies, mutually interacting populations must reach an activation threshold before a decision takes place. Similar to the role played by Simon's satificing concept, the decision threshold "can be varied to to achieve either quick but inaccurate, or accurate but slow decisions" (Marshall and Franks, 2009, p.R395). Empirical observations in this area have been accompanied by computational complexity-based modeling (of the Crutchfield statistical complexity variety) e.g. Delgado and Sole (1997).

There has been little interests in drawing lessons from such studies for economics, perhaps because many economists would consider ants are way too different ('less intelligent' or complex?) than humans. Kirman (1993) is a rare exception. In his paper, Alan Kirman emphasizes on how asymmetric outcomes (e.g. herding) can 'emerge' from stochastic interactions between symmetric (identical) and simple agents (ants). Another interesting feature of Kirman's model is the analysis of equilibrium as a process where the only meaningful characterization would be in terms of the "equilibrium distribution of the process". One interesting question that comes to mind is whether the shifting of the boundaries between the problem solver and environment might entail a trade-off between centralization and decentralization in a parallel information processing system. How then would a shift change computational complexity of the colony? A question of a similar spirit can also be ask about the computability of such a system. Finally, such discussions also compel us to ponder another question - whether it might be useful to consider how 'simplicity' as in bounded rationality can emerge and co-evolve with a changing complex environment.

5 The Emergence of Simplicity Amidst Complexity

5.1 The Complexity of Organisms and Environment

Complexity and complex system are issues that have preoccupied scholars for a long time including Herbert Simon. After publishing two seminal papers that articulated the idea and importance of bounded rationality and how they relate to the environment (Simon (1955) and Simon (1956)), Herbert Simon went on to publish a paper titled "The Architecture of Complexity" in 1962. In a sense, Simon's interests in complexity and complex systems is a natural extension of his earlier works published since the mid-1950s. In Simon (1955, p.101), the "levels of computational complexity" was an issue raised in relation to bounded rationality (in terms of the employment of 'schemes of approximation'). Furthermore, not only is an organism complex (Simon, 1956, p.129), the choice process associated with bounded rationality can differ in terms of different degree of rationality (ibid, p.133). In addition, the 'complexity' of the environment also matters. In Simon's example involving the case of an organism seeking food, the 'complexity' of the environment matters. This was discussed in terms of the richness of randomly distributed food points and the density of paths (leading to them) (ibid, p.131). In addition, Simon also noted that learning could enhance an organism's survival if food points are not randomly distributed such that clues to their location exists along paths leading to them (ibid, p.135).

To discuss the issue of the complexity (or simplicity) of an organism and its environment, a more precise way to defining complexity is needed. One such approach would be to use Kolgomorov Complexity (or algorithmic information content) to measure complexity. The Kolgomorov Complexity K of an object is measured by the smallest program that can be used to compute it. A random object of length n would have a maximum K number equal to n. In contrast, an object comprising a string of n ones is relatively low (with K approximately equal to $\log n$). The next challenge is to decide what to measure - the physical/biological or behavioral aspects of an organism. It is a difficult question to address as it leads to deep questions such as those relating to the roles of biological and cultural evolution. Both are related as an organism's capacity to learn, produce and transmit knowledge is likely to be biologically constrained to some extent.

5.2 Bounded Rationality and the Environment

In the model used in Simon (1956), the organism is assumed to have a fixed aspiration level (in terms of maintaining average food intake rather than maximizing) and an ability to see a finite number of moves ahead. When food points are randomly distributed, these two characteristics of the organism is sufficient to ensure a higher survival probability for the organism than when it behaves randomly. This example suggests that when the environment is maximally Kolgomorov complex (randomly distributed food points), the organism is likely to be found to be using decision procedures that are less than maximal Kolgomorov complex.[17]

What if food points are not randomly distributed and, in addition, clues on the distribution of food points can be detected by the organism? In such a case, Simon argued that the adoption of a systematic exploration strategy (i.e. heuristics) is associated with a higher survival probability than a completely random behavior. This suggests that it pays (in terms of survival probability) for organisms to be less than maximal Kolgomorov complex when the environment is also less than maximal Kolgomorov complex (i.e. food points are not randomly distributed). An important issue to consider is whether some degree of randomness must be present in to ensure survivability especially in a changing environment.

The above discussions suggest that not only is bounded rationality (as in less than maximal Kolgomorov complex) may be more prevalent, the form in which it takes (satisficing, heuristics or both) depends on the nature of the environment i.e. heuristics involving learning to exploit some systematic feature of the environment. The prevalence of bounded rationality could also be interpreted to mean that the type of 'Olympian rationality' commonly assumed in many mainstream economics models is indeed a special case (as Vellupilai (2010) has argued) i.e. precisely because the spectrum encompassing the collection of choice procedures are obviously less than that associated with maximal Kolgomorov complexity. It is a special case where the organism has no constraints in terms of its ability to look forward (full knowledge of how food points are distributed) and/or is a maximizer (searching the entire problem space).

---

[17]In the paper, the food points appear to be distributed over a graph-theoretic (regular) tree (in Simon's words, "branching system of paths", ibid, p.131). There is a need to differentiate between the Kolgomorov complexity measure K of a regular graph tree (which is low) and the Kolgomorov complexity of the randomly distributed food points (which is high). The latter suggests that the Kolgomorov complexity of the environment facing the organism is high.

Would such interpretations differ in the case of organisms undertaking parallel information processing such as social insects? Going back to the earlier example, the employment of random foraging strategies (high K) under parallel information processing conditions (and in an environment where food points are randomly distributed) could be enough to ensure survivability of the colony. Does this mean that less complex choice procedures under parallel information processing work as well as complex choice procedures under serial information processing?[18] This may be a wrong comparison (question) as random foraging behavior within social insect colonies may not necessarily be less complex. Why? The collective computations undertaken by social insects, even though based on simple choice procedures (e.g. randomizing) at the individual level, may be interpreted (as many have done) as an emergent property. Thus, if it can be shown that the Kolgomorov complexity measure of the resulting 'collective choice procedures' (K') is actually lower. Can this be true? Can a collective choice procedure, comprising of individual choice procedure with high Ks, have a lower K'? It is possible that the Kolgomorov complexity of parallel information processing systems may be lower even without any emergent properties. For example, the average lower bound Kolgomorov complexity of a sequential sorting of n elements is in the order of $\frac{1}{2}\log n$ whereas that of a parallel stacks is $\sqrt{n}$ (Vitanyi, 2007). However, could emergent properties in such systems (provided they exist) drive the Kolgomorov complexity even lower?

5.3   Emergence and Hierarchies

There is currently no definition of the term 'emergence' that is universally accepted in the academic research community across different disciples. Some refer to emergence as a "*property* of a system not reducible to, nor readily predictable from the properties of individual system components" (Halley and Winkler, 2008, p.10). Some have emphasized emergence as a "*process* that leads to the appearance of a structure not directly described by the defining constraints and instantaneous forces that control a system" (Crutchfield, 1994, p.12). Others consider emergence as involving the *detection* of "some new feature that makes the overall description of the system simpler than it was before" (Deguet *et al.*, 2006, p.24).

Instead of trying to reconcile these definition, it might be perhaps more useful to discuss theoretically how emergence might occur. To begin with,

---

[18]Perhaps even better given that social insects have been around and may be around longer than humans!

emergence is an outcome of local interactions between large number of components of the system. Such local or micro-level interactions lead to the formation of spatio-temporal patterns or properties at the global or macro-level. Obviously, not all local-level interactions lead to emergence. In system with emergent properties, local interactions have been described as 'non-trivial' in the sense that they result in a reduction in the degrees of freedom (Prokopenko *et al.*, 2008, p.11). This is similar to Simon (1962, p.476) description of 'strong interactions' which not only reduces the capacity the components to interact further but leads to the formation of sub-systems that interact weakly amongst themselves. Thus, systems with emergent properties are hierarchic systems (in the sense of being "composed of interrelated sub-systems") with the property of 'near-decomposability' (where the system can be described at the aggregate or macro-level in terms of the weak interactions between the sub-systems (ibid, p.468 and 478). Furthermore, "only the aggregate properties of their parts enter into the description of the interactions of those part" (Simon, 1962, p.478). This would also imply that the Kolgomorov complexity of the system at the macro-level is actually lower (as argued earlier). It should also be noted that Simon has attempted to operationalize the strength of interactions in terms of frequency interactions, an idea which has some resonance in institutional economics (Williamson, 2000).

What are implications of such views on bounded rationality? First, bounded rationality relates to micro-level interactions in complex social systems. Since there is diversity in the degree of complex social systems across time and space, the complexity of any (social) system may be found in the nature of bounded rationality of organisms/agents in that system. This in turn affects the type of interactions in the system - whether they are non-trivial or not - with consequence on the formation or non-formation of hierarchies. For example, the response threshold to stimuli (read bounded rationality, satisficing) of each individual will affect the emergence of collective phenomena in social insects. A even more challenging question is what drives bounded rationality. From an evolutionary point of view, the prevailing form of bounded rationality would have been subject to selection within a given environment. Thus the degree of 'simplicity' that is associated with bounded rationality would be inextricably linked to the environment. This has led some scholars to identify "genomic complexity with the the amount of information a sequence stores about its environment" (Adami *et al.*, 2000, p.4463).

6   Concluding Remarks

The predominant view of rationality in economics today is one of 'rationality as consistency' or 'rationality as maximizing'. Whilst departures from such notions of Olympian rationality (to paraphrase Professor Velupillai) do get some attention in the research community in the form of bounded rationality, the resulting body of research is one that mostly either treats bounded rationality as constrained form of rationality (therefore a special case) or is devoid of the rich computational implications suggested in Herbert Simon's original works. Furthermore, a key insight of Simon's work is missing - that of the inter-dependence between the problem solver and his/her environment.

On this, several important insights can be derived from Simon's work as well as other recent contributions from outside economics. The boundaries between bounded rationality and its environment can shift. The form and degree of complexity of bounded rationality is dependent on the structure (and hence the complexity of) the environment. Collective decision-making processes involving multiple local interactions can result in the emergence of hierarchies. This reduces the Kolgomorov complexity of the system. Thus, the 'simplicity' that we associate with bounded rationality in the form of simplifying mechanisms such as satisficing is in fact crucial for the emergence of complex systems.

A minority of economists have already begun incorporating some of these insights in their work e.g. North (2005). The road ahead for those interested to pursue their research along such lines will not be an easy one due the number of disciplines outside economics that are involved, most of which are unfamiliar to the average economists. However, the returns from learning from these other disciplines are likely to be very large indeed.

References

Adami, C., Ofria, C., and Collier, T. (2000). Evolution of Biological Complexity. *Proceedings of the National Academy of Sciences of the United States of America*, **97**(9), 4463–4468.

Anderson, M. (2003). Embodied cognition. *Artificial Intelligence*, **149**, 91–130.

Arrow, K. (1986). Rationality of Self and Others in an Economic System. *Journal of Business*, **59**(4), S385–S399.

Aumann, R. J. (1997). Rationality and Bounded Rationality. *Games and Economic Behavior*, **21**, 2–14.

Axtell, R. (2005). Complexity of Exchange. *Economic Journal*, **115**, F193–F210.

Axtell, R. (2007). What Economic Agents Do: How Cognition and Interaction Lead to Emergence and Complexity. *Review of Austrian Economics*, **20**, 105–122.

Becker, G. (1962). Irrational Behavior and Economic Theory. *Journal of Political Economy*, **70**(1), 1–13.

Ben-Porath, E. (1990). The Complexity of Computing a Best Response Automaton in Repeated Games with Mixed Strategies. *Games and Economic Behavior*, **2**(1), 1–12.

Blaug, M. (1985). *Economic Theory in Retrospect*. Cambridge University Press, Cambridge, Third edition.

Blaug, M. (1992). *The Methodology of Economics*. Cambridge University Press, Cambridge.

Blume, L. and Easley, D. (2008). Rationality. In S. Durlauf and L. Blume, editors, *The New Palgrave Dictionary of Economics, Second Edition*, Second Edition, London. Palgrave Macmillan.

Camerer, C. (1998). Bounded Rationality in Individual Rationality Decision Making. *Experimental Economics*, **1**(2), 163–183.

Chatterjee, K. and Sabourian, H. (2008). Game Theory and Strategic Complexity. Technical report, mimeo.

Conlisk, J. (1996). Why Bounded Rationality? *Journal of Economic Literature*, **34**(2), 669–700.

Crutchfield, J. (1994). The Calculi of Emergence: Computation, Dynamics and Induction. *Physica D*, **75**, 11–54.

Daskalakis, C., Goldberg, P. W., and Papadimitriou, C. H. (2009). The Complexity of Computing a Nash Equilibrium. *Communications of the ACM*, **52**(2), 89–97.

Davis, M., Sigal, R., and Weyuker, E. (1994). *Computability, Complexity and Languages*. Academic Press, San Diego.

Deguet, J., Demazeau, Y., and Magnin, L. (2006). Elements of the Emergence Issue: A Survey of Emergence Definitions. *Complexus*, **3**, 24–31.

Dekel, E. and Gul, F. (1997). Rationality and Knowledge in Game Theory. In D. Kreps and K. Wallis, editors, *Advances in Economics and Econometrics: Theory and Applications*, Cambridge. Cambridge University Press.

Delgado, J. and Sole, R. (1997). Collective-Induced Computation. *Physical Review E*, **1997**(55), 2338–2344.

Dorigo, M. and Gambardella, L. M. (1997). Ant Colonies for the Travelling Salesman Problem. *BioSystems*, **1997**(43), 73–81.

Edwards, S. and Pratt, S. (2009). Rationality in Collective Decision-Making by Ant Colonies. *Proceedings of the Royal Society*, pages 1–7.

Gigerenzer, G. (2001). Decision Making: Nonrational Theories. *International Encyclopedia of the Social and Behavioral Sciences*, **5**, 3304–3309.

Gigerenzer, G. and Selten, R., editors (2001). *Bounded Rationality: The Adaptive Toolbox*. MIT Press, Cambridge MA.

Gilboa, I. (1988). The Complexity of Computing Best-Response Automata in Repeated Games. *Journal of Economic Theory*, **45**, 342–352.

Giocoli, N. (2003). *Modeling Rational Agents*. Edward Elgar, Cheltenham.

Gode, D. and Sunder, S. (1993). Allocative Efficiency of Markets with Zero-Intelligence Traders: Market as a Partial Substitute for Individual Rationality. *Journal of Political Economy*, **101**(1), 119–137.

Halley, J. and Winkler, D. (2008). Classification of Emergence and its Relation to Self-Organization. *Complexity*, **13**(5), 10–15.

Hayek, F. (1991). On being an economist. In W. W. Bartley and S. Kresge, editors, *The Trend of Economic Thinking; Essays on Political Economists and Economic History, Volume III*, London. Routledge.

Ingrao, B. and Israel, G. (1990). *The Invisible Hand: Economic Equilibrium in the History of Science.* MIT Press, Cambridge MA.

Kahneman, D. (2003). Maps of Bounded Rationality: Psychology for Behavioral Economics. *American Economic Review*, **93**(5), 1449–1475.

Kalai, E. (2009). The Complexity of Computing Nash Equilibrium. *Commun. ACM*, **52**(2), 88–88.

Kirman, A. (1993). Ants, Rationality and Recruitment. *Quarterly Journal of Economics*, **108**(1), 137–156.

Kirman, A. (1997). The Economy as an Evolving Network. *Journal of Evolutionary Economics*, **7**, 339–353.

Kirman, A. (2003). The Structure of Economic Interaction: Individual and Collective Rationality. mimeo.

Kirman, A., Markose, S., Giansante, S., and Pin, P. (2007). Marginal Contribution, Reciprocity and Equity in Segregated Groups: Bounded Rationality and Self-Organization in Social Networks. *Journal of Economic Dynamics and Control*, **31**, 2085–2107.

Klaes, M. and Sent, E.-M. (2005). A Conceptual History of the Emergence of Bounded Rationality. *History of Political Economy*, **37**(1), 27–59.

Laland, K. and Brown, G., editors (2002). *Sense and Nonsense: Ecolutionary Perspectives on Human Behavior.* Oxford Press, Oxford.

Marshall, J. and Franks, N. (2009). Colony-Level Cognition. *Current Biology*, **2009**(19), R395–R396.

Myerson, R. (1991). *Game Theory.* Harvard University Press, Cambridge, MA.

Newell, A. and Simon, H. (1972). *Human Problem Solving.* Prentice-Hall, Englewood Cliffs, NJ.

Neyman, A. (1985). Bounded Complexity Justifies Cooperation in Finitely Repeated Prisoners' Dilemma. *Economic Letters*, **19**(1), 227–229.

North, D. (2005). *Understanding the Process of Economic Change.* Princeton University Press, Princeton.

Prokopenko, M., Boschetti, F., and Ryan, A. (2008). An Information-Theoretic Primer on Complexity, Self-Organization, and Emergence. *Complexity*, **15**(1), 11–28.

Rabin, M. (1998). Psychology and Economics. *Journal of Economic Literature*, **36**(1), 11–46.

Roughgarden, T. (2010). Computing Equilibria: A Computational Complexity Perspective. *Economic Theory*, **42**, 193236.

Rubinstein, A. (1986). Finite Automata Play the Repeated Prisoners' Dilemma. *Journal of Economic Theory*, **39**, 83–96.

Rubinstein, A. (1998). *Modeling Bounded Rationality.* MIT Press, Cambridge, MA.

Russell, S. and Norvig, P. (1995). *Artificial Intelligence: A Modern Approach.* Prentice Hall, Englewood Cliffs, NJ.

Samuelson, L. (1996). Bounded Rationality and Game Theory. *Quarterly Review of Economics and finance*, **1996**(36), 17–35.

Schrödinger, E. (1967). *What is Life?* Cambridge University Press, Cambridge.

Sen, A. (1977). Rational Fools: A Critique of the Behavioral Foundations of Economic Theory. *Philosophy and Public Affairs*, **6**(4), 317–344.

Simon, H. (1955). A Behavioral Model of Rational Choice. *Quarterly Journal of Economics*, **69**, 99–118.

Simon, H. (1956). Rational Choice and the Structure of the Environment. *Psychological Review*, **63**(2), 129–138.

Simon, H. (1957). *Models of Man.* Wiley, New York.

Simon, H. (1962). The Architecture of Complexity. *Proceedings of the American Philosophical Society*, **106**(6), 467–482.

Simon, H. (1976). From Substantive to Procedural Rationality. In S. Latsis, editor, *Method and Appraisal in Economics*, Cambridge. Cambridge University Press.

Simon, H. (1978). Rationality as Process and as Product of Thought. *American Economic Review*, **68**(2), 1–16.

Simon, H. (1982). *Models of Bounded Rationality*. MIT Press, Cambridge, MA.

Simon, H. (1991a). Game Theory and the Concept of Rationality. Mimeo, Paper No.493 in the Herbert Simon Collection.

Simon, H. (1991b). *Models of My Life*. Basic Books, New York.

Simon, H. (1997). *An Empirically Based Microeconomics*. Cambridge University Press, Cambridge.

Simon, H. (2000). Bounded Rationality in Social Science: Today and Tomorrow. *Mind & Society*, **1**(1), 25–39.

Smith, V. (2003). Constructivist and Ecological Rationality in Economics. *American Economic Review*, **93**(3), 465–508.

Smith, V. L. (1962). An Experimental Study of Competitive Market Behavior. *The Journal of Political Economy*, **70**(2), pp. 111–137.

Tversky, A. and Kahneman, D. (1974). Judgement Under Uncertainty: Heuristics and Biases. *Science*, **185**(4157), 1124–1131.

Tversky, A. and Kahneman, D. (1981). The Framing of Decisions and the Psychology of Choice. *Science*, **211**(4481), 453–458.

Varian, H. (1992). *Microeconomic Analysis*. Norton, New York, third edition.

Vellupilai, K. (2009). Uncomputability and Undecidability in Economic Theory. *Applied Mathematics and Computation*, **215**(4), 1404 – 1416.

Vellupilai, K. (2010). Foundations of Boundedly Rational Choice and Satisficing Decisions. *Advances in Decision Sciences*, **2010**, 1–16.

Velupillai, K. (2000). *Computable Economics*. Oxford University Press, Oxford.

Velupillai, K. V. (2006). Algorithmic Foundations of Computable General Equilibrium Theory. *Applied Mathematics and Computation*, **179**(1), 360 – 369.

Vitanyi, P. (2007). Analysis of sorting algorithms by kolmogorov complexity (a survey). In I. Csiszar, G. Katona, and G. Tardos, editors, *Entropy, Search, Complexity, Bolyai Society Mathematical Studies, 16*, New York. Springer-Verlag.

Waldrop, M. (1992). *Complexity: the Emerging Science at the Edge of Order and Chaos.* Simon & Schuster, New York.

Walsh, V. (1996). *Rationality, Allocation and Reproduction.* Clarendon Press, Oxford.

Williamson, O. E. (2000). The New Institutional Economics: Taking Stock, Looking Ahead. *Journal of Economic Literature*, **38**(3), 595–613.