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# The cyclical and trend behaviour of Australian investment and savings

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## **Abstract**

A spectral analysis of the Australian time series for the investment and savings ratios on monthly data over the period from September 1959 to December 2005 reveals that the major cyclical components of the savings and investment ratios cohere strongly. This suggests there is a medium to long term relationship between investment and savings. Further, the investment and saving ratios cohere strongly with the business cycle suggesting a procyclical pattern of investment and saving behaviour on Australian data. A subsequent long memory analysis reveals that the saving and investment ratios, the investment ratio and real GDP and the savings ratio and real GDP are fractionally cointegrated. The policy implications are explained.

## **Keywords**

savings, cyclical, investment, australian, behaviour, trend

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**THE CYCLICAL AND TREND BEHAVIOUR OF AUSTRALIAN  
INVESTMENT AND SAVINGS**

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# THE CYCLICAL AND TREND BEHAVIOUR OF AUSTRALIAN INVESTMENT AND SAVINGS

**Abstract:** A spectral analysis of the Australian time series for the investment and savings ratios on quarterly data over the period from September 1959 to December 2005 reveals that the major cyclical components of the savings and investment ratios cohere strongly. This suggests there is a medium to long term relationship between investment and savings. Further, the investment and saving ratios cohere strongly with the business cycle suggesting a procyclical pattern of investment and saving behaviour on Australian data. A subsequent long memory analysis reveals that the saving and investment ratios, the investment ratio and real GDP and the savings ratio and real GDP are fractionally cointegrated. The policy implications are explained.

**Keywords:** Spectral analysis, cycles, trends, fractional cointegration, long memory, investment, savings

**JEL Codes:** E21, E22, E32

## **1. The Significance of the Saving Investment Relationship**

The aim in this study is to analyse the cyclical and trend behaviour of the Australian gross investment and gross savings ratios and identify the economic implications of their covariation. A prominent feature of many time series is their cyclical variation. The preferred option for this individual study is spectral analysis described briefly in Section 3 below. This technique can provide a frequency based description of the time series and indicate characteristics such as long memory, the presence of high frequency variation and cyclical behaviour. This technique is better suited to the analysis of cycles in an individual series than other techniques because it allows researchers to model the non linearity directly and to identify more than one major cyclic (harmonic) component of each series. A major component is one which explains a large proportion of the total variance of an individual series. Once the major components of  $I/Y$ ,  $S/Y$  and real GDP are identified it is appropriate to extend this analysis to answer questions about the coherence of investment and savings ratios and the correspondence of each of these with real GDP using cross spectral analysis. The use of spectral analysis will inform Australian policy makers about the correlation between particular components of the individual  $I/Y$  and  $S/Y$  and real GDP series, but it will not provide precise information about the presence of a long run relationship between savings, investment and real GDP and for this purpose, an Autoregressive Fractionally Integrated Moving Average (ARFIMA) process is applied to determine if  $I/Y$ ,  $S/Y$  and real GDP are fractionally cointegrated. The ARFIMA model accommodates a long memory process in which current period values of a variable are interdependent with values of the same variable in periods long past. Further policy related economic issues can be addressed from the outcomes of this ARFIMA study: the presence of a long run relationship between these

variables and the relevance of the Feldstein-Horioka (1980) condition using Australian data.

A literature generally related to the issues examined in this study is the synchronisation of international business cycles. The time series domain techniques currently applied to this problem are two in number: Markov switching regimes such as those applied to the synchronisation issue by Smith and Summers (2005), Phillips (1991) and Krolzig (1997) and second linear VAR analysis. The Markov switching analysis developed by Hamilton (1989) is employed by Harding and Pagan (2002) plus Hess and Iwata (1997) to examine the performance of Markov switching techniques against an AR(1) model in analysing the synchronisation of international business cycles. These authors find that Markov switching models perform less effectively than simple AR(1) models in explaining synchronisation. Thus, we prefer to use spectral analysis to address the objectives of this paper.

The motivation for this study stems from the centrality of the relationship between these economic aggregates for the achievement of both internal and external balance. In relation to the first of these, undergraduate students of economics are taught that a general macroeconomic equilibrium is consistent with the equality of leakages and injections such as gross saving and investment. This is quite surprising to beginning students once they discover that those who save are not generally synonymous with those who invest because the motives of savers and investors differ. This prompts us to analyse the behaviour of the individual time series for the investment ratio ( $I/Y$ ) and savings ratio ( $S/Y$ ) separately to determine if savings and investor motives are evident in the behaviour of each through time. Further, the interrelationship between

I/Y and S/Y is a widely studied issue, in particular, Feldstein and Horioka (1980) indicate that the nature of the correlation between these cast some light on the degree of financial capital mobility confronting the open economy. If there is zero correlation between I/Y and S/Y, then financial interdependence is complete. At the other extreme, if there is perfect linear correlation between I/Y and S/Y, the economy in question is characterised by a state of financial autarky. No recent examination of the Feldstein-Horioka condition using Australian data exists. It is also of importance to know if the two ratios have a long run equilibrium and if each ratio has a long term relationship with real GDP. Once this is known a cross spectral analysis will answer questions about the coherence of investment and savings ratios and the correspondence of each of these with real GDP.

In summary the four goals of this study are as follows: to determine if the individual ratios display patterns consistent with agents motives; to assess the strength of correlation between I/Y and S/Y; to test for any pro or countercyclical patterns against the variation of real GDP as a benchmark and to establish the presence or otherwise of a long run equilibrium among these three variables and to test for the existence of the Feldstein-Horioka condition using Australia data.

The historical record of the investment and saving ratio is examined in the following section of the paper. The methods and the properties of the data set particularly in relation to covariance stationarity are described in Section 3 of the paper. The results of the cyclical analysis are discussed in Section 4 while Section 5 involves an explanation of the results from the long run trend analysis. Finally, the significance of all results is explained in a closing section.

## 2. The Historical Record

The entire officially recorded history of the Australian savings and investment experience is captured on Figure 1 where the graphs of the investment  $\left(\frac{I}{Y}\right)$  and saving ratios  $\left(\frac{S}{Y}\right)$  are shown for a quarterly time series dating back to the first publication of savings and investment in 1959(3) and extending to the December quarter of 2005.

[Figure 1, about here]

A ratio analysis is preferred to the levels of savings and investment throughout this study for two reasons: first, the ratios  $I/Y$  and  $S/Y$  are pure numbers free of scale and second, data expressed in the form of ratios is better suited to the analysis of some of the questions posited above, in particular, the ratio of investment and savings to nominal GDP is required to test for the presence of the Feldstein-Horioka result using Australian data.

The salient features of Figure 1, are first that the correspondence between savings and investment behaviour in Australia is stronger in the first half of the period dating from September 1959 to June 1974. The two ratios are locked together in the range 25 to 35 percent for much of this time suggesting that negative current account balances are modest compared with the second half of Australia's investment-savings history. A substantial change in the pattern of the relationship occurs in mid 1975 when a gap between the two ratios develops. This change in the relationship between  $I/Y$  and  $S/Y$  is associated with the timing of the first oil price shock in 1974-75 an event which economic agents in many oil dependent economies regard as a critical time in their financial history. Consequently the second half of this era (1975-05), is



marked by an excess of the investment ratio over the savings ratio an outcome which applies right through to the end of 2005. This widening of the I/Y, S/Y gap indicates a worsening current account balance and an associated accumulation of foreign indebtedness of Australian institutions.

A second characteristic of the I/Y and S/Y time series is their apparent cyclical behaviour. From Figure 1, downturns in both ratios are synchronised in the sense that peaks and troughs in each individual series appear to be coincidental in their timing. Further, the amplitude of saving cycles in each series increases post 1975 and appears to be accentuated by the effect of recessions. From Figure 1, substantial downturns are evident in 1959, 1961 (the credit squeeze), 1965, 1969, 1974 (first oil price shock), 1978 (second oil price shock), 1982, 1986, 1991 (recession), 1997 (Asian crisis) and 2001 (September 11). Downswings in I/Y and S/Y appear to coincide with well known cycles in the behaviour of the economy's rate of growth. This is an issue to be resolved formally in this study.

A further interpretation of Figure 1 is that Australians have become less thrifty over time. Prior to the first oil price shock (1974-75) the gross saving ratio hovers in the range of 31%-35%, but post 1975 S/Y has fallen and fluctuates in the 15%-20% range. The S/Y series hits a low of 15% in the middle of the 90/93 recession. The investment ratio has also fallen although on certain occasions (1982 and 1989) when it bounces back to the thirty percent of GDP range. These observations highlight a fundamental weakness of the Australian economy. The decline of the gross savings ratio has created a situation in which saving from domestic sources has not been able to keep pace with a stronger demand for investment goods in an economy which

continues to grow strongly. The result is burgeoning foreign indebtedness as borrowers turn to foreign savings sources. The issue of how thrift is encouraged remains at centre-stage in an Australian policy context.

### **3 Data and Methodology**

There are two analytical tools used to address the economic issues described in the first section of this paper: the first, is the spectral/cross spectral analysis of the cyclical behaviour of the individual time series for  $I/Y$ ,  $S/Y$  and real GDP and the second is the autoregressive fractionally cointegrated (ARFIMA) analysis of the series. The spectral/cross spectral analysis provides insights into those issues associated with the apparent cyclical characteristics of each series as follows: the identification of the major cycles in each individual series; the extent of coherence between the major cyclical components of each series and the question about the procyclical behaviour of the  $I/Y$  and  $S/Y$  series. The ARFIMA process involves an analysis of these further issues: the existence or otherwise of a long run relationship between these series and a test to determine the degree of capital mobility on Australian data.

The measurement of the investment and savings ratios are described in the following manner. Gross investment is constructed by adding to gross capital expenditure in the Australian National Accounts the value of investment in inventories. Further, Gross Savings are found by adding the aggregate measures National Savings and depreciation (capital used up in production). The denominator of these two ratios is the current price value of Australia's gross domestic product seasonally adjusted. Real GDP is the chain volume measure recorded in the Australian National accounts.

All data are quarterly and seasonally adjusted the series run from 1959:3 to 2005:4. The Australia Bureau of Statistics is the source of all data.

### *3-1 Spectral Analysis*

Spectral/ cross spectral methods are applicable only to time series which are covariance stationary which holds if the mean of the series is constant through time and if the autocovariance function is determined by periodic time intervals (s-t), but not by historical time. These requirements are not satisfied by the individual series (I/Y) and (S/Y) and real GDP. In order to achieve this desired covariance stationary property in the three series we filter the data in each series by applying the Hodrick-Prescott (HP - 1997) filter. The application of this filter is subject to the reservation that HP filtering could produce a spurious peak in the individual spectrum of the GDP series. Baxter and King (1999) compare the properties of the HP filter and their own preferred band-pass filter which give essentially the same outcomes for quarterly series on US GNP. This reinforces our argument that the HP filter provides an acceptable measure of the Australian business cycle using quarterly data. Baxter and King also identify the observable properties of the HP filter: first, the HP filter does not involve a phase shift; second, it detrends an individual series; third it places zero weight at the zero frequency and it approximates Baxter and Kings band pass filter. Guay and St-Amant (1997) assess the capacity of both the HP and Baxter and King filters to extract business cycle components from the data. Guay and St-Amant find that both filters perform adequately when the spectrum of the original series peaks at business cycle frequencies. The behaviour of the spectrum for Australia's real GDP as shown on Figure 2 is relevant here. This spectrum shows a major cyclical component of 17.067 quarters duration. There is some corroborating evidence to

support the notion that the Australian business cycle is of 17 quarters duration. Cotis and Coppel (2005) find that the average duration of downswings of the Australian business cycle is 6.3 quarters, while upswings last for 10.7 quarters, so from peak to peak the average duration of the Australian business cycle is exactly 17 quarters which is almost precisely the duration of the cycle uncovered in this study. Thus in the light of the desirable properties of the HP filter, its concordance with the results of Baxter and Kings filter on quarterly data and its relevance to business cycle peaks, prompts a preference for the HP methodology.

Spectral analysis is applied to establish the cyclical behaviour of both ratios through time. This frequency domain methodology has many applications and is described originally by Fishman (1969), Rayner (1971) and Koopmans (1974) and more recently by Hamilton (1994). In an economic context, the relationship between time and frequency domain techniques is examined by Engle (1976) in the following terms: spectral analysis decomposes a stationary, stochastic series into a set of uncorrelated cycles, each associated with a frequency ( $\lambda$ ) or period (the inverse of the frequency) which is the time required for the series to complete a whole cycle. The variation of an individual series ( $x$ ) is represented by the spectrum, which is the Fourier transform of the autocovariance function. The spectrum  $[g_x(\lambda)]$  decomposes the time series into a sequence of sine and cosine waves of differing frequencies with just the right number of amplitudes to compose the whole series. It has the following definition:

$$g_x(\lambda) = (2\pi)^{-1} \sum_{s=-\infty}^{\infty} e^{-i\lambda s} \gamma(s) \quad (2)$$

where  $\lambda = \text{frequency}$ ,  $i = (-1)^{1/2}$ ,  $\gamma(s) = \text{autocovariance function at lag } (s)$  of the series in  $x$ ,  $e^{-i\lambda} = \cos \lambda - i \sin \lambda$ . The formal relationship between the spectrum and the autocovariance function is defined in (2) where the integral  $g_x(\lambda)$  is the area under the spectrum and is equal to the variance  $\gamma(0)$ . Thus the spectrum decomposes the variance into the components contributed by each frequency and in each series, cycles of differing length can be identified, In the spectral analysis of each series the objective is to identify the predominant cycles. The importance of each frequency component is assessed in terms of its power or contribution to the total variance of the series. The power density function described by Rayner [1971, p.24] estimates the percentage contribution of each cycle (frequency component) to the total variance of the series as follows:

$$P_D = \left[ \frac{A^2(k)}{\sum_k A^2(k)} \right] \cdot 100 \quad (3)$$

where  $A^2(k)$  is the estimated variance of the  $k$ th frequency component and  $\sum_k [A^2(k)]$  is the estimated variance of the whole series. The major cycle in each series is the one with the greatest power density. The variance of each frequency component is half the square of its amplitude  $A^2(k)$  and the estimates of these at each frequency are provided by estimates of the spectrum for each series. This is estimated by smoothing or averaging the periodogram, which is the square of the absolute value of the Fourier transform of the autocovariance of the series at each frequency divided by the total number of observations. Estimation is completed by applying the Fast Fourier Transform (FFT). Spectral estimation provides reliable estimates if the series in question is covariance stationary, requiring the autocovariance function and the

mean of the process to be independent of time. These requirements are often violated by economic time series especially those which are heavily trended.

### 3.2 Cross Spectral Analysis

The correlation of the two series is assessed in a pairwise comparison and it is assessed by applying the technique known as cross spectral analysis. This method analyses the joint variation of pairs of variables in the frequency domain and is equivalent to a series of individual regressions between sinusoids in two different series at the same frequency ( $\lambda$ ). The cross spectrum is the bivariate equivalent of the spectrum in the single variable case and is defined in terms of its imaginary and real components – the quadrature spectrum  $q(\lambda)$  and co-spectrum  $c(\lambda)$ . The cross spectrum describing the joint variation of the two series  $x$  and  $y$  at frequency ( $\lambda$ ) is defined as follows:

$$g_{xy}(\lambda) = c(\lambda) - iq(\lambda) \quad (4)$$

The cross spectrum is not estimated directly and its properties are summarized in two statistics associated with the quadrature and co-spectra. These statistics provide all the information required for the purpose of the study. The coherence is analogous to the correlation coefficient in the time domain and measures the strength of association of two interdependent series at particular frequencies and is formally defined as follows:

$$coh(\lambda) = \left\{ \left[ c^2(\lambda) + q^2(\lambda) \right] / \left[ g_x(\lambda) g_y(\lambda) \right] \right\}^{1/2}$$

$$0 \leq coh(\lambda) \leq 1 \quad (5)$$

The coherence indicates the strength of association of the common harmonic components in the two series and is employed to indicate the correlation of I/Y and

S/Y series. If the coherence of the I/Y and S/Y series are significant at the five percent level, according to the tests specified by Fishman [(1969), p. 138], then I/Y and S/Y are deemed to be correlated. If the coherence is not significant according to these tests, correlation does not occur.

A second statistic associated with cross spectral analysis is the phase angle in radians or the time difference in quarters between peaks in each series. The phase angle  $p(\lambda)$  is computed from the quad and co-spectra as follows:

$$p(\lambda) = \tan^{-1}[-q(\lambda)/c(\lambda)] \quad (6)$$

$p(\lambda)$  assesses the fraction of a cycle by which one series leads the other. The phase in radians can be expressed as a time difference in quarters by dividing the phase by its frequency:  $p(\lambda)/\lambda$ .

## **4 The Cyclical Behaviour of Investment and Savings**

### **4.1 Individual Spectra**

The individual spectra for the savings ratio (S/Y), investment ratio (I/Y), and real GDP (Y) are estimated as the Fast Fourier transform of the autocovariance function using the RATS 6.0 package. The results of the individual spectral studies for I/Y, S/Y and real GDP are shown on Table 1.

**Table 1: Maximum Power Spectra: Australian Real GDP, Investment and Savings Ratios: 1959(3) to 2005(4)**

Series	Real GDP	I/Y	S/Y
Entry ( <i>j</i> )	31	32	30
Frequency (radians)	0.368	0.380	0.356
Duration (Quarters)	17.067	16.516	17.650
Estimate	2.897	2.801	2.126
% Variance	39.54	37.40	44.64
t statistic of sample mean	9.131	11.443	9.823
skewness	2.132	1.719	2.151
kurtosis	3.596	2.587	3.754
J-B	0.772	0.500	0.794

The columns of Table 1 show the frequency point (*j*), at which the maximum individual power spectra occurs in the first row of each column. The second row shows the frequency in radians at which the maximum spectra is observed. The percentage contribution of the maximum estimated power spectra to the variance of each series is calculated by applying the ratio in equation (5) to the estimated spectrum. The total variance of each series is the area under the power spectra on Figures 2, 3 and 4 and the percentage explained at the peak spectra is shown in the fifth row of Table 1. The diagnostics in the last few rows of Table 1 relate to the significance of the mean of the power spectra in each series; the t-ratio in each case suggests that the estimated mean of the estimated power spectra is significant in each case. In addition to the properties of the data set, there is some evidence of positive skewness in the data. The Jarque Bera statistics for the individual series indicate that the null of normality can be accepted at the 0.05 percent significance level.



The results of the spectral estimates are interesting. The most important component of each individual series occurs at data entry points  $j = 31, 32$  and  $30$  for real GDP, I/Y and S/Y respectively. These points and frequencies relate to a harmonic component of around 4 to 4.25 years duration (periodicity). In the case of Australia's real GDP this harmonic has been associated with the business cycle. It is also of interest to note that the peak spectral estimate also occurs on a frequency component of similar duration in the case of both I/Y and S/Y. For I/Y, this is data entry  $j=32$  and for S/Y  $j=30$ . This peak component is evident on Figure 2, 3 and 4 below at approximately the same frequency. The respective peak frequencies are 0.368 for real GDP, 0.380 for I/Y and 0.356 for S/Y. This band of frequencies explains 39.54 percent, 37.4 percent and 44.64 percent of the total variance of real GDP, I/Y and S/Y respectively.

The individual series behave differently when harmonic components other than the peak ones are considered. In particular, there is clearly a longer swing in real GDP occurring at a lower frequency than the business cycle on Figure 2.

[Figure 2, about here]

This occurs at entry  $j=14$  at a low frequency of 0.159 radians consistent with a periodicity of 39.386 quarters a much longer swing of almost 10 years in duration. By way of contrast, the graph of the investment ratio (Figure 3) reveals a second peak at frequency  $\pi/4$  with a periodicity of 10 quarters or two and a half years, shorter than the main components of 4 years.

[Figure 3, about here]

The savings ratio also contains the same longer swinging component evident in the real GDP series. At the frequency of 0.159 and periodicity of 39.385 quarter, and approximately 10 years duration. This may reflect the long term motives of savers in comparison with investors where the second important component is one which completes a full cycle in 2.5 years.

[Figure 4, about here]

In summary, the major component of each of the three series is one which completes a full cycle in 4 to 4.25 years. As mentioned above, Cotis and Coppel (2005) indicate that this is the Australian business cycle. A longer swinging 10 year cycle is evident in the Australian real GDP series although it does not contribute as much to its total variance GDP in comparison with the four year business cycle. Similarly, the second most important contributor to the variance of Australia's S/Y series is this same 10 year swing observed in the real GDP series, while the I/Y series also displays a 2 year cycle shorter in duration than the business cycle. These differing attributes of the I/Y and S/Y series may reflect the different motives of investors and savers: some investors are motivated by short term gains while some savers are motivated by longer term considerations.

#### *4.2 Cross Spectral Relationships*

An interesting aspect of the results from the univariate spectral analysis is that the peak frequencies in each individual case appear to occur on cycles of similar periodicity: the predominant cycle in each series has a duration of 4 to 4.25 years. This correspondence of cycles in the individual series prompts further bivariate cross spectral analysis with the purpose of determining if the predominant cycles

in each series coheres. The results of this cross spectral analysis are shown on Table 2.

**Table 2: Cross Spectral Analysis of S/Y and I/Y and GDP**

I/Y and S/Y				
<i>j</i>	Coherence	F <sup>(1)</sup>	Phase (Radians)	Time Difference
30	0.879	627.2	-0.221	-0.620
31	0.900	895.3	-0.219	-0.595
32	0.876	610.4	-0.237	-0.624
Min Coh	0.221			
Max Coh	0.949			
Mean	0.548		-0.006	
T ratio	34.25		-0.186	
Skewness	-0.344		-0.142	
Kurtosis	-0.707		0.512	
J-B	0.592		6.261	
I/Y and Real GDP				
<i>j</i>	Coherence	F <sup>(1)</sup>	Phase (Radians)	Time Difference
30	0.907	857.9	0.034	0.095
31	0.904	827.6	0.030	0.081
32	0.889	697.2	0.022	0.058
Min Coh	0.014			
Max Coh	0.941			
Mean	0.619		-0.005	
T ratio	41.267		-0.236	
Skewness	-0.558		0.537	
Kurtosis	-0.558		3.725	
J-B	0.579		0.070	
S/Y and Real GDP				
<i>j</i>	Coherence	F <sup>(1)</sup>	Phase (Radians)	Time Difference
30	0.887	685.5	-0.254	-0.714
31	0.903	817.2	-0.254	-0.690
32	0.988	869.1	-0.262	-0.705
Min Coh	0.101			
Max Coh	0.988			
Mean	0.594		-0.071	
T ratio	34.941		-2.730	
Skewness	-0.374		0.173	
Kurtosis	-0.808		1.725	
J-B	0.628		0.073	

(1) Koopmans (1974 p.284) test statistic for the coherence on Table 2 will exceed the  $\alpha = 0.05$  critical value for Fisher's F distribution.  $F_{2,2(n-1)} = (n-1)c\hat{oh}^2 / (1-c\hat{oh})$  has the critical value 19.5 at 2, 2(n-1) degrees of freedom. The null of  $c\hat{oh} = 0$  is rejected in each case.

The estimated coherences shown on Table 2 are all significant at the 5 percent level and there is a strong correlation of the peaks in the individual series at the same frequency. At  $j = 30$ , a frequency component of 17.5 quarter duration in each of the two series is strongly correlated in all three bivariate studies. This is indicated by the following estimated coherence statistics: 0.879 in the study of the two ratios I/Y and S/Y, 0.907 in the analysis of I/Y with real GDP and 0.887 for the study of S/Y with real GDP. Similar arguments apply to entry  $j=31$  (17 quarters duration). The coherence between this component in this bivariate analysis is 0.900 (I/Y with S/Y), 0.904 (I/Y with Real GDP) and 0.903 (S/Y with real GDP). Finally, the coherence between the  $j=32$  (16.52) quarters) component of each series is also strong. The estimated coherences applying to this component are 0.876, 0.889 and 0.898 respectively for the I/Y-S/Y, I/Y-Real GDP, S/Y-Real GDP analyses.

The diagnostics at the bottom half of Table 2 indicate the significance of the mean coherence in three bivariate studies: I/Y, I/Y – real GDP and S/Y- real GDP. The value of the coherence at each of the three data entries  $j=30, 31, 32$  is significantly different from zero using the test specified by Koopman's (1974, p.284) which is distributed as Fisher's F distribution with 2 and  $2(n-1)$  degrees of freedom. The JB statistic in each of the three cross spectral studies is less than the 5% critical value indicating that the hypothesis of normality is not rejected.

This cross spectral analysis indicates the presence of a medium term relationship between the I/Y and S/Y ratios of approximately four to four and one quarters years duration. This frequency component was also found to be the predominant harmonic component in the individual series and is also strongly correlated in the bivariate case.

The correlation of the 4.25 years swing in the saving ratio and the four year swing in the  $I/Y$  series may be explained by their strength of association with the predominant 4.5 year swing in real GDP. In addition we can add that the Australian real GDP and savings ratio series exhibit a procyclical characteristic cohering strongly with the predominant 4 year cycle in real GDP. The phase angle in radians when expressed as a time difference in quarters indicates that the lag and lead relationships between peaks in the  $I/Y$  and  $S/Y$  series at the same component are in general within quarter relationships. For example, from Table 2, the negative signs in each estimated time lag indicate that the investment ratio leads the savings ratio by 0.620 quarters at  $j=30$ , by 0.595 quarters at  $j=31$  and by 0.624 at  $j=32$  quarters. A similar argument applies to lags and leads between real GDP and  $I/Y$  and the analysis of real GDP and  $S/Y$ . In each case lags and leads are less than one quarters duration.

The outstanding finding from this cross spectral study is that the savings and investment ratios cohere strongly and are also strongly correlated with the major swing in real GDP. The policy implications which follow from this frequency domain analysis are discussed in the concluding section but prior consideration is given to a long memory analysis. Spectral analysis provides insights into the structure of individual time series by identifying those components of an individual series which explain substantial proportions of the variance of that series. It also settles issues about the correlation and concordance of the same components in the individual series. However, spectral methods alone cannot settle issues about the existence of a long run relationship between these variables. To analyse the properties of a long run equilibrium between  $I/Y$  and  $S/Y$  on Australian data, we turn to the ARFIMA methodology.

## 5 The GPH Test for Fractional Cointegration

Many economic time series exhibit a high degree of persistence. Sowell (1990), Diebold and Rudebusch (1991) among others show that traditional unit root tests have low power against a fractional alternative because they are restricted to integer order I(1) or I(0). Permitting the integration order of a series to take on any fraction is called fractional integration (long memory models). Long-memory, or long-term dependence implies that a series is dependent on its values in the distant past. Such series are characterized by distinct but nonperiodic cyclical patterns such as those evident in the preceding spectral analysis. Hence the Geweke and Porter-Hudak (1983) test based on spectral regression estimates is the fractional differencing parameter  $d$  which in turn is based on the slope of the spectral density function around the angular frequency  $= 0$  is used for this purpose.

A time series,  $y$ , is said to follow an autoregressive fractionally integrated moving average (ARFIMA process of order  $(p, d, q)$ ) with mean  $\mu$  if:

$$\phi(L)(1-L)^d(y_t - \mu) = \theta(L)\varepsilon_t \quad \varepsilon_t \sim i.i.d(0, \sigma_u^2) \quad (7)$$

where  $\phi(L)$  is an autoregressive coefficient of order  $p$  and  $\theta(L)$  is a moving average coefficient of order  $q$  and  $\varepsilon_t$  is a white noise process.  $(1-L)^d$  is the fractional differencing operator defined as follows:

$$(1-L)^d = \frac{\sum_{k=0}^{\infty} \Gamma(k-d)L^k}{\Gamma(-d)\Gamma(k+1)} \quad (8)$$

with  $\Gamma(\cdot)$  denoting the generalised factorial function. The parameter  $d$  is permitted to assume any real value. If  $d$  is not an integer, the series can be said to be fractionally integrated. In the time domain, the series can be expected to exhibit an

hyperbolically decaying ACF. In the frequency domain, the process  $y$  is both stationary and invertible if all roots of  $\phi(L)$  and  $\theta(L)$  are outside the unit circle and  $0 > d > 1/2$ . The series is non stationary and possesses an infinite variance for  $d \geq 1/2$ . See Granger and Joyeux (1980) and Hosking (1981).

Geweke and Porter-Hudak (GPH) have developed a non parametric test for estimating the fractional differencing parameter,  $d$ . The GPH test is carried out on the first differences of the series to ensure that stationarity and invertibility are achieved. Geweke and Porter-Hudak show that the differencing parameter,  $d$ , which is also called a long memory parameter, can be estimated consistently from the least squares regression:

$$\ln(I(\omega_j)) = \theta + \lambda \ln(4 \sin^2(\omega_j / 2)) + v_j; \quad j = 1, \dots, J \quad (9)$$

where  $\theta$  is a constant,  $\omega_j = 2\pi j / T$  ( $j = 1, \dots, T-1$ ) denotes the Fourier frequencies of the sample,  $J = f(T^\mu)$ .  $J$  is an increasing function of  $T$  where  $T$  are the number of observations and  $0 < \mu < 1$ .  $I(\omega_j)$  is the periodogram of the time series at frequency  $\omega_j$ . For a sample series with  $T$  observations, the periodogram at harmonic frequency  $\omega_j$ ,  $I(\omega_j)$  is computed in the following manner:

$$\hat{I}(\omega_j) = \frac{1}{2\pi} \sum_{k=-T+1}^{T-1} \hat{\gamma}_j e^{-i\omega_j k}, i^2 = -1, \text{or equivalently,}$$

$$\hat{I}(\omega_j) = \frac{1}{2\pi} (\hat{\gamma}_0 + 2 \sum_{k=1}^{T-1} \gamma_k \cos(\omega_j k)), \quad (10)$$

where  $\gamma_j$  is order  $j$  auto-covariance of sample series,

$$\hat{\gamma}_j = \begin{cases} T^{-1} \sum_{t=j+1}^T (y_t - \bar{y})(y_{t-j} - \bar{y}), & j = 0, 1, 2, \dots, T-1 \\ \hat{\gamma}_{-j}, & j = -1, -2, \dots, -T+1 \end{cases}. \quad (11)$$

The existence of a fractional order of integration can be tested by examining the statistical significance of the differencing parameter,  $d$ . The estimated  $d$  values can be interpreted as follows: the process is mean reverting if  $d < 1$  and exhibits long memory if  $0 < d < 1$ . If  $d \geq 0.5$  the process is non-stationary but mean reverting and exhibits long memory; if  $0 < d < 0.5$  the process is stationary and exhibits long memory. The process is stationary and has short memory if  $-1/2 < d < 0$ .<sup>1</sup> Table 3 reports the  $\tilde{d}$  estimates for  $J = T^{0.45}, T^{0.5}, T^{0.55}$ . Different values of  $\mu$  are used in order to check the sensitivity of the results to changes in  $\mu$ . Fractional cointegration requires testing for fractional integration in the error correction term of the cointegrating regression.

The null hypothesis of  $d = 1 (d = 0)$  can be tested against the alternative of  $d \neq 1 (d \neq 0)$ . The results are broadly consistent with the different choices of  $\mu$ . The savings ratio and investment ratios are  $0 < d < 1$  suggesting that they follow a fractionally integrated process and therefore have long memory. Both series are  $d \geq 0.5$  for all values of  $\mu$  suggesting that the two series are non stationary but mean reverting and exhibit long memory. The null hypothesis of  $d = 1$  cannot be rejected for the real GDP series  $J = T^{0.5}, T^{0.55}$  suggesting that the series is non stationary.

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<sup>1</sup> See Granger and Joyeux (1980) and Hosking (1981) for a detailed discussion.



**Table 3: GPH Test Results for Fractional Integration and Fractional****Cointegration**

	d=.45	d=.5	d=.55
Results for Fractional Integration			
I/Y	0.745 (0.469)	0.543 (0.363)	0.7379 (0.312)
S/Y	0.854 (0.319)	0.8386 (0.251)	0.9604 (0.293)
Real GDP	0.965 (0.031)	1.00 (0.031)	1.006 (0.025)
Results for Fractional Cointegration			
$\varepsilon_1$	0.281 (0.267)	0.229 (0.222)	0.375 (0.171)
$\varepsilon_2$	0.552 (0.430)	0.410 (0.314)	0.675 (0.288)
$\varepsilon_3$	0.630 (0.373)	0.647 (0.305)	0.663 (0.227)

Note:  $\tilde{d}=0.45$ ,  $\tilde{d}=0.5$  and  $\tilde{d}=0.55$  give the  $\tilde{d}$  estimates corresponding to the GPH spectral regression of sample size  $J = T^{0.45}$ ,  $J = T^{0.5}$ ,  $J = T^{0.55}$ . The standard errors are reported in parenthesis. The term in parenthesis is the OLS standard error.

The hypothesis of fractional cointegration requires a test of the error correction term ( $\varepsilon$ ) obtained from the cointegrating regression of the individual series. For the regression of I/Y on S/Y, the fractional cointegration tests  $0 < d < .5$  for the error correction term ( $\varepsilon_1$ ) implying that the series are stationary and exhibit long memory.

The fact that the two series are fractionally cointegrated implies that a long run relation exists between the two series. According to the Feldstein Horioka condition, if perfect capital mobility exists then there should be no correlation between the investment and savings ratios of a country. For the error correction terms from the cointegrating regressions of I/Y on RGDP ( $\varepsilon_2$ ) and S/Y on RGDP ( $\varepsilon_3$ ) respectively,  $0 < d < 1$ . Therefore although the individual real GDP series wanders, deviations from the cointegrating relation are mean reverting implying that a shock to the system will eventually subside and an equilibrium relation will exist in the long

run between real GDP and the investment ratio and real GDP and the savings ratio. The findings of this study therefore do not support the Feldstein-Horioka condition.

## **6 Conclusion**

The stylised facts about saving and investment in Australia reveal a close correspondence between private sector gross investment and gross saving each expressed as a ratio of Australia's GDP from 1959 to 1974. The first signs of a weakening of this correspondence is evident in the aftermath of the first oil price shock in 1974-75. On the other hand, the demand for investment goods holds up throughout as the gross investment ratio remains in the range of 25 to 35 percent of GDP. However, the savings ratio trends down from this same range to 20 to 25 percent before settling around a constant 20 percent of GDP post 1992, a year in which S/Y troughed at 14 percent of GDP. Policy initiatives since then do not seem to have pushed the savings ratio back to its glory days of 25 to 35 percent of GDP and S/Y remains fixed at around 20% of GDP.

An examination of the historical record indicates a basic fundamental weakness of the Australian economy. Domestic saving has not been sufficient to match the demand for investment goods since 1975. Investors have turned to foreign sources of savings and Australia's foreign debt has escalated as a consequence. Both the I/Y and S/Y series exhibits much greater volatility following the first oil price shock as the amplitude of cycles in each series seems to increase, particularly, over the period 1975 to 1992 when both series decline sharply in the 1990-92 recession. The amplitude of cycles in S/Y and I/Y lessen following the 1992 recession.

These cyclical characteristics of the data prompted a further, formal study of the cyclical behaviour of I/Y and S/Y and confirm that both series are cyclical in nature. The most important cyclical component in each series is a swing of 4 years periodicity. A second noteworthy feature of the I/Y series is the presence of a cyclical component of 2.5 years somewhat shorter than the 4 year business cycle perhaps reflecting the shorter term motives of some investors. The S/Y series also displays a longer run swing of 10 years duration reflecting the long run motivation of some savers. Finally, the major component of the real GDP series is a cyclical component of 4 years consistent with the commonly accepted duration of the business cycle. This business cycle component explains 37 percent of the total variation of the real GDP series. Further, a longer swing of 10 years duration is also evident in the real GDP series. This component explains one quarter of the variation of the real GDP series.

The study of the individual I/Y and S/Y series is augmented by bivariate comparisons of cycles in the individual series and in this way we find the strongest coherence between the I/Y and S/Y series occurring on the cyclical component of 4 years duration. So from this outcome it can be argued that the saving and investment ratios are strongly correlated in this medium term context. Further, the coherence between the variation of the I/Y series and real GDP is at a maximum on a cyclical component of 4.25 about the same duration as the business cycle. Finally, the saving ratio and real GDP exhibit their strongest coherence on a cycle of 17 quarters again close to the duration of the business cycle. Thus, the I/Y and S/Y ratios appear to behave in a procyclical manner.

This finding is significant for policy coordination. If investment and saving ratios are coherent with the business cycle then policies motivated by the presence of internal imbalances will have a simultaneous impact on saving and investment and consequently on external balance. Thus the procyclical characteristic of saving and investment greatly enhances the prospective benefits of harmonising policy strategies to achieve both internal and external balance. The policy harmonisation task must be more complicated if saving and investment are not procyclical and are subject to cyclical episodes of differing duration. The task of predicting the impacts on saving and investment and external balance will be more complex in this disharmonious setting and some assignment of policy instrument may be called for once more. However, if saving and investment are procyclical the effects of individual policy settings on both internal and external balance may be more easily predicted given that the timing of such impacts on individual aggregates is synchronised.

The presence of a long run relationship between the I/Y and S/Y ratios is confirmed in a fractional cointegration analysis. In relation to capital mobility, if the Feldstein Horioka (FH) condition holds and there is no correspondence between I/Y and S/Y then perfect capital mobility is presumed. However, the error correction term obtained from the cointegrating regression for the individual series suggests the presence of a long run relationship between the investment and savings ratio suggesting that the FH condition does not hold on Australian data. It is possible that the various forms of capital controls existing in Australia over the period 1959 to 2005 have impeded progress towards freeing up capital flows in spite of the maintenance of a comparatively liberal exchange rate mechanism from 1984 to 2005. Alternatively, the failure of the FH condition may simply indicate a home bias such as

the one discussed by Feldstein (1983) so that there is a propensity to retain domestic savings in the home country.

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Figure 1

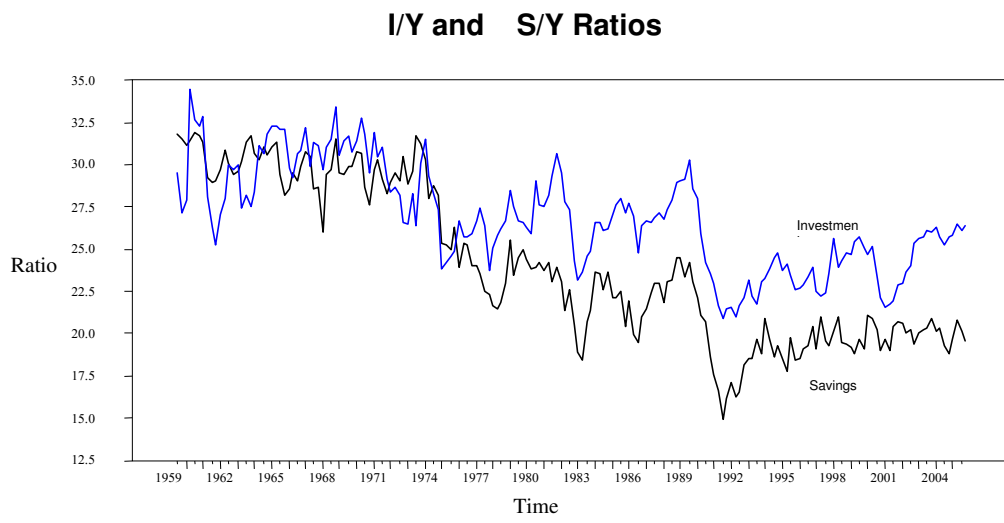




Figure 2

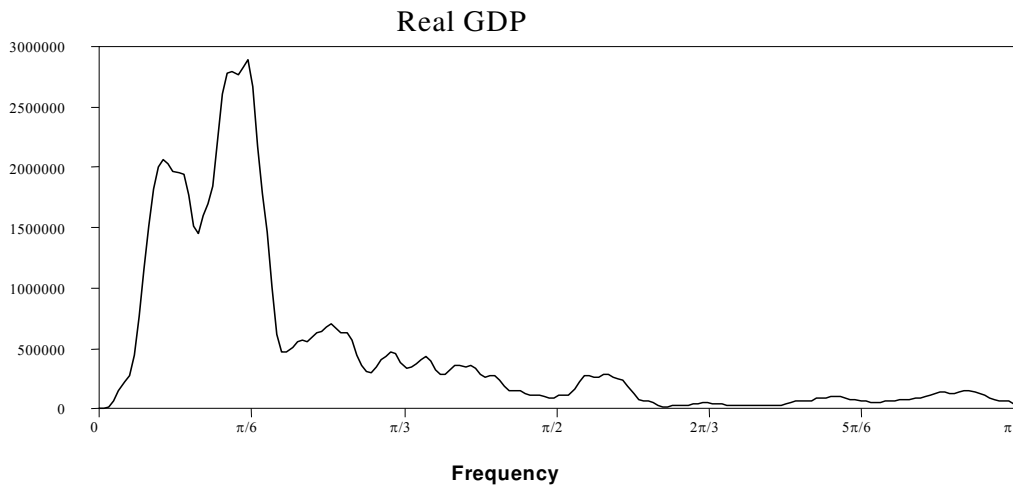


Figure 3

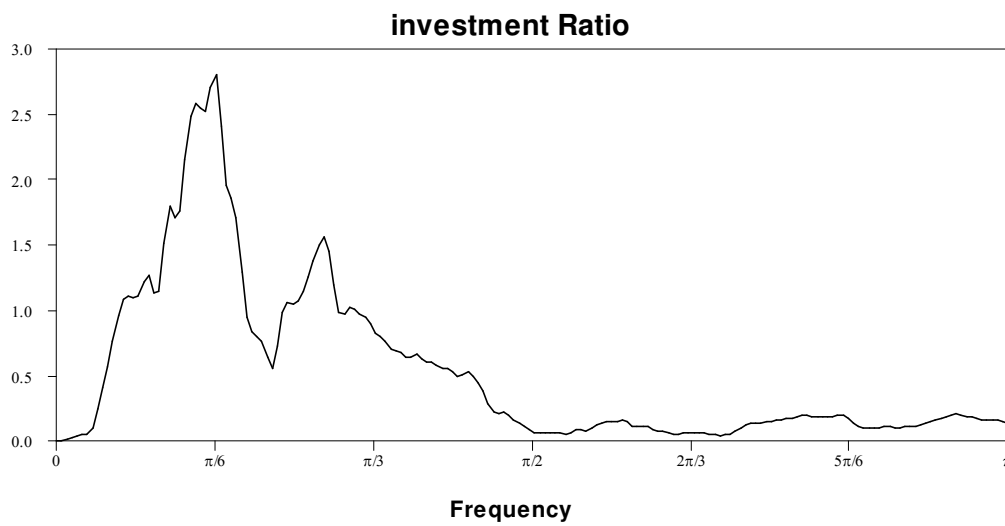


Figure 4

