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# RECENT ADVANCEMENTS IN THE USE OF PREFABRICATED VERTICAL DRAINS IN SOFT SOILS

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## ABSTRACT

A system of prefabricated vertical drains with surcharge load to accelerate consolidation by shortening the drainage path is one of the most popular methods of soft ground improvement. An analytical solution is proposed based on radial (lateral) soil permeability while considering variations in vacuum pressure. The predicted smear zone and effects of drain unsaturation are compared with laboratory data obtained from large-scale radial consolidation tests. When a higher load is required to meet the desired rate of settlement and the cost of raising a surcharge embankment is also significant, the application of vacuum pressure with a reduced surcharge load can be used. In this method, the vacuum creates a suction head that increases the effective stress. Analytical and numerical analyses were conducted for several case histories using the equivalent plane strain solution for Darcian and non-Darcian flows. The effectiveness of vertical drains on cyclic loading was also investigated based on a laboratory study. This paper shows that vertical drains can dissipate the built up excess pore pressure under repeated loading, and that short drains can be sufficient in certain cases rather than driving the drains to cover the entire depth of soft clay deposits. The research findings verify that the effects of soil disturbance and vacuum pressure can affect soil consolidation considerably, which means that these aspects need to be modelled correctly in any numerical approaches.

## 1. INTRODUCTION

Preloading is usually an economic and successful ground improvement technique that can be used to stabilise soft clays. It involves loading the ground surface to induce a greater proportion of ultimate settlement than the soil foundation is expected to experience after construction (Richart, 1957; Indraratna and Redana, 2000; Indraratna et al., 2005a). In order to control the development of excess pore water pressure, a surcharge embankment is usually constructed as a multi-stage exercise with rest periods between the loading stages (Jamiolkowski et al., 1983). Since most compressible soils are characterised by very low permeability, depending on their thickness, consolidation may take a long time and require a very high surcharge load (Indraratna et al., 1994), which may not always be appropriate for tight construction schedules. Installation of vertical drains can significantly reduce the preloading period by reducing the drainage path radially, because, the consolidation time is inversely proportional to the square of the length of the drainage path (Hansbo, 1981; Indraratna and Redana, 1998; Indraratna and Redana, 2000). Due to the rapid initial consolidation, vertical drains will increase the stiffness and bearing capacity of soft foundation clays (Bo et al., 2003). Geosynthetic vertical drains are usually composed of a plastic core that is protected by a fabric filter with a longitudinal channel. The filter (sleeve) is made of synthetic or natural fibrous material with a high resistance to clogging. Vertical drains are most appropriate for moderate to highly compressible soils which are normally consolidated or lightly over consolidated. Pre-consolidation allows coastal structures such as transport systems, embankments and tall buildings to be more stable under large static and cyclic loads.

In this paper, the effects of the compressibility indices, variation of soil permeability, and the magnitude of preloading are examined. The equivalent (transformed) permeability coefficients for plane strain condition are incorporated in finite element codes ABAQUS, using the modified Cam-clay theory. A case history, including the site of the New Bangkok International Airport (Thailand), is discussed and analyzed and the numerical predictions are compared with the available field data.

## 2. CHARACTERISTICS OF VERTICAL DRAIN SYSTEMS

### 2.1. PURPOSE AND APPLICATION

Various types of vertical drains, including sand drains, sand compaction piles, prefabricated vertical drains (PVDs) and gravel piles have commonly been used in the past. Apart from the increasing cost of sand quarrying and strict environmental regulations in many countries, plus conventional sand drains that can be adversely affected by lateral soil movement, flexible PVD systems enjoying relatively rapid installation, have often replaced the original sand drains and gravel piles. The most common band shaped (wick) drains have cross sectional dimensions of 100mm × 4mm. For design purposes the rectangular (width- $a$ , thickness- $b$ ) section must be converted to an equivalent diameter,  $d_w$ , because the conventional theory of radial consolidation assumes circular drains (Figure 1).

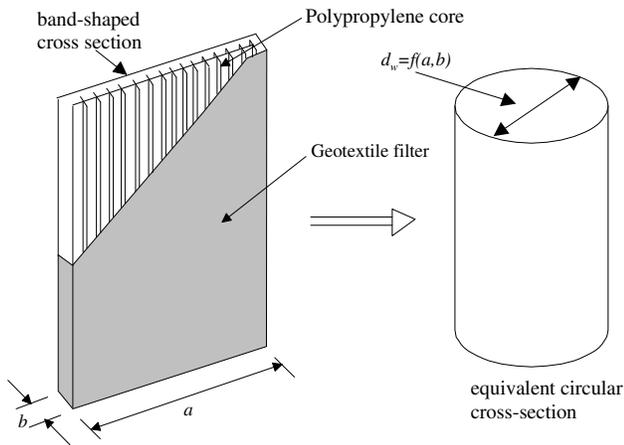


Figure 1: Conceptual illustration of band-shaped PVD and equivalent diameter of drain well (Indraratna et al., 2005f)

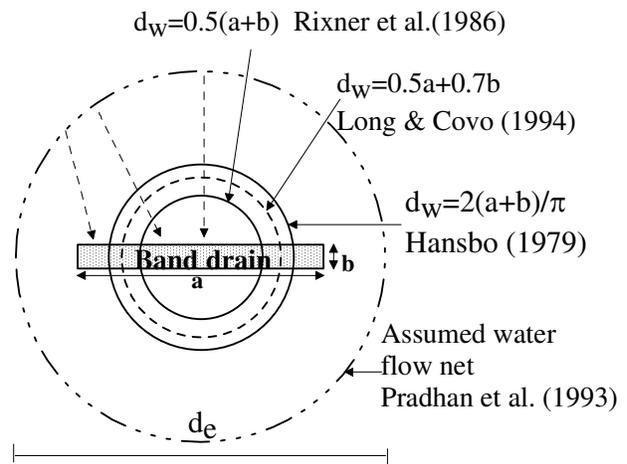


Figure 2: Assessment of equivalent diameter of band shaped vertical drains (Indraratna et al., 2005f)

Figures 1 and 2 define the appropriate dimensions of the band shaped PVD and the equivalent transformation to a circular cross section.

The following typical equation based on a circular perimeter is used to determine the equivalent drain diameter:

$$d_w = 2(a+b)/\pi \quad (\text{Hansbo, 1979}) \quad (1)$$

Atkinson and Eldred (1981) proposed that a reduction factor of  $\pi/4$  could be applied to Eq. 1 to take account of the corner effect, where the flow lines rapidly converge. From finite element studies, Rixner et al. (1986) proposed that:

$$d_w = (a+b)/2 \quad (2)$$

Pradhan et al. (1993) suggested that the equivalent diameter of band-shaped drains could be estimated by considering the net flow around a soil cylinder of diameter  $d_e$  (Figure 2). The mean square distances of their net flow is calculated as:

$$s^{-2} = \frac{1}{4}d_e^2 + \frac{1}{12}a^2 - \frac{2a}{\pi^2}d_e \quad (3)$$

$$\text{Then, } d_w = d_e - 2\sqrt{\left(\frac{-2}{s}\right)} + b \quad (4)$$

More recently, Long and Covo (1994) found that the equivalent diameter ( $d_w$ ) could be computed using an electrical analogue field plotter:

$$d_w = 0.5a + 0.7b \quad (5)$$

The discharge capacity is one of the most important parameters that control the performance of PVDs. The discharge capacity depends primarily on the following factors (Figure 3): (i) the area of the drain; (ii) the effect of lateral earth pressure; (iii) possible folding, bending and crimping of the drain, and (iv) infiltration of fine particles into the drain filter (Holtz et al., 1991).

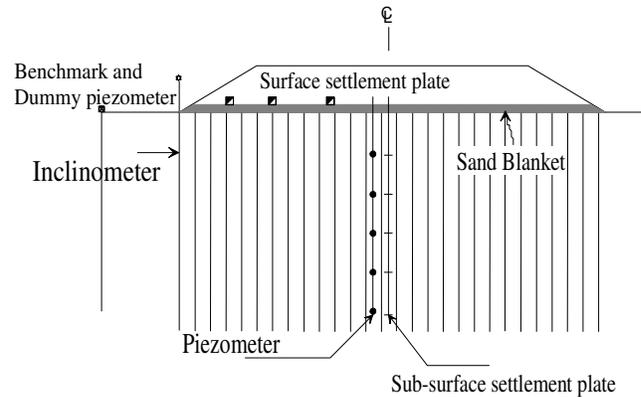
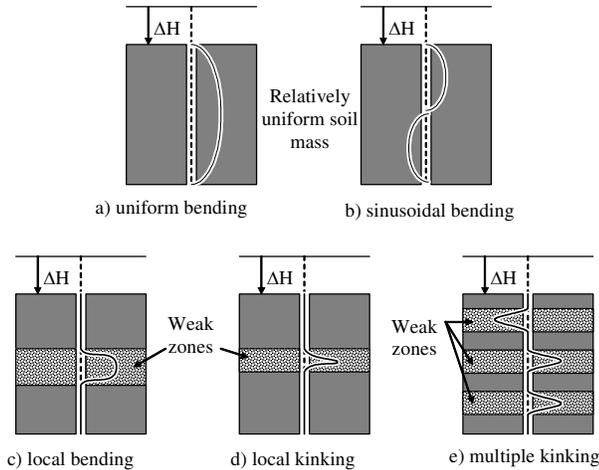


Figure 3: PVD Deformation modes (after Holtz et al., 1991)      Figure 4: System of PVDs with sand blanket and surcharge preloading (Indraratna et al., 2005d)

In practice, static and dynamic methods can be used to install PVDs. The static procedure is preferred for driving the mandrel into the ground, whereas the dynamic method can be used to penetrate weathered crust (e.g. drop hammer impact or vibrating hammer). A typical PVD installation in the field is shown in Figure 4. The degree of disturbance during installation depends on the types of soil, the size and shape of the mandrels, and the soil macro-fabric. The sand blanket system is used to expel water away from the drains and provide a sound working mat for erecting the drain installation rigs. Before installing PVDs, surface soil and vegetation must be removed so the site can be graded and a sand blanket placed and compacted.

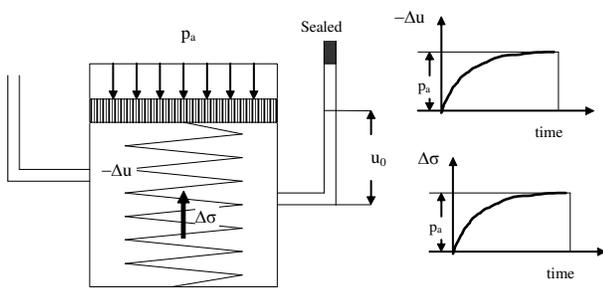
Field instrumentation for monitoring and evaluating embankments is vital to examine and control any geotechnical problems. Instrumentation can be separated into two categories (Bo et al., 2003) depending on the construction stages; the first is used to prevent sudden failures during construction (e.g. settlement plates, inclinometers and piezometers), whereas the second is related to the rate of settlement and excess pore pressure during loading (e.g. multi-level settlement gauges and piezometers).

## 2.2. PVD WITH VACUUM PRELOADING

The vacuum preloading method was originally introduced in Sweden by Kjellman (1952) for cardboard wick drains. Since then it has been used to accelerate consolidation and improve soft ground such as the Philadelphia International Airport, USA and Tianjin port, China (Holtan, 1965 and Yan and Chu, 2003). When a larger amount of surcharge load is required to meet the expected settlement, a combined vacuum and fill surcharge can be used to reduce the cost of quarry. In a soft clay area where a high surcharge embankment cannot be constructed quickly without compromising stability, it is preferable to apply a vacuum. This PVD system has recently been used to distribute vacuum pressure to deep subsoil layers, thereby increasing the consolidation rate of reclaimed land or dredged deposits (e.g. Indraratna et al., 2005d; Chu et al., 2000). The mechanism for vacuum preloading can be described by the spring analogy (Figure 5) proposed by Chu and Yan (2005).

With vacuum-assisted preloading, installing some horizontal drains transversely and longitudinally is compulsory after placing the sand blanket. These drains can then be connected to the edge of a peripheral Bentonite slurry trench, which is normally sealed by an impervious membrane (Figure 6), after which the trenches can be filled with water to improve sealing between the membrane and Bentonite slurry. The vacuum pumps are connected to the discharge system extending from the trenches and the suction head generated by the pump accelerates the dissipation of excess pore water pressure in the soil radially towards the drains and vertically towards the surface. The effective stress increases through the vacuum load while the total stress remains constant (Figure 7). Since then, vacuum pressure systems without a membrane have also been introduced where the suction head is directly applied to the PVD (Seah, 2006).

When the effective stress related to suction pressure increases equi-axially, the corresponding lateral movement is compressive, and therefore, the risk of shear failure can be minimized even with a higher rate of embankment construction (Qian et al., 1992). The extent of surcharge fill can be decreased to achieve the same amount of settlement, depending on the efficiency of the vacuum system in the field (i.e., air leaks). With vacuum pressure, an inevitable unsaturated condition at the soil-drain interface caused by swift mandrel withdrawal may be improved, resulting in an increased rate of consolidation (Indraratna et al. 2004).



$$u_0 = p_a$$

$$\Delta\sigma = p_a - (u_0 - \Delta u) = \Delta u$$

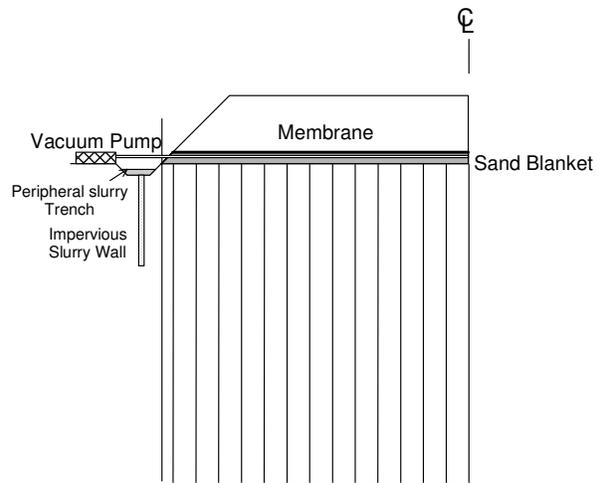


Figure 5: Spring analogy of vacuum consolidation process (adopted from Chu and Yan, 2005)

Figure 6: Vacuum-assisted preloading system (Indraratna et al., 2005d)

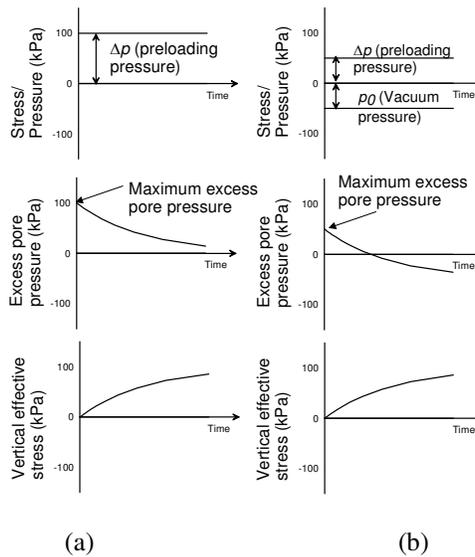


Figure 7: Consolidation process: (a) conventional loading (b) idealised vacuum preloading (Indraratna et al., 2005d)

### 2.3. FIELD OBSERVATION OF RETARDED PORE PRESSURE DISSIPATION

In spite of the advantages of PVDs, excess pore water pressure does not dissipate as expected; this is often attributed to filter clogging, extreme reduction of the lateral permeability of soil, damage to piezometer tips etc. Numerical analysis suggests that very high lateral strains and corresponding re-distribution of stresses (e.g. substantial heave at the embankment toe) can also be a factor (c.f. Mandrel-Cryer effect). Some examples are shown in Figures. 8 and 9.

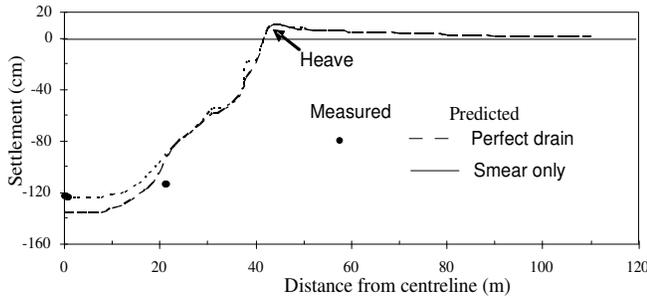


Figure 8: Surface settlement profile after 400 days (Indraratna & Redana, 2000; Indraratna & Chu, 2005)

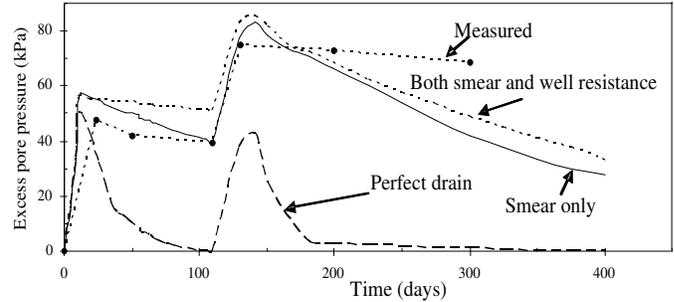


Figure 9: Excess pore water pressure variation at piezometer location, P6 (after Indraratna & Redana, 2000)

## 3. THEORY OF RADIAL CONSOLIDATION

### 3.1. AXI-SYMMETRIC UNIT CELL ANALYSIS

#### 3.1.1. Linear Darcian flow:

Conventional radial consolidation theory has commonly been used to predict the behaviour of vertical drains in soft clay. Its mathematical formulation is based on the small strain theory, where for a given stress range, a constant volume compressibility ( $m_v$ ) and a constant coefficient of lateral permeability ( $k_h$ ) are assumed (Barron, 1948; Hansbo, 1981). However, the value of  $m_v$  varies along the consolidation curve over a wide range of applied pressure ( $\Delta p$ ). In the same manner  $k_h$  also changes with the void ratio ( $e$ ). Indraratna et al. (2005c) have replaced  $m_v$  with the compressibility indices ( $C_c$  and  $C_r$ ), which define the slopes of the  $e$ - $\log \sigma'$  relationship. Moreover, the variation of horizontal permeability coefficient ( $k_h$ ) with the void ratio ( $e$ ) during consolidation is represented by the  $e$ - $\log k_h$  relationship which has a slope of  $C_k$ .

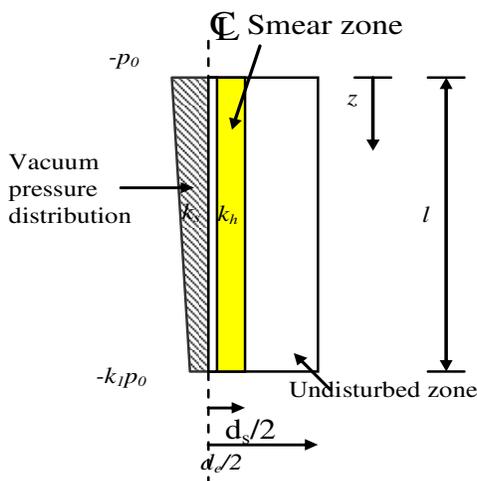


Figure 10: Cylindrical unit cell with linear vacuum pressure distribution (modified after Indraratna et al., 2005d)

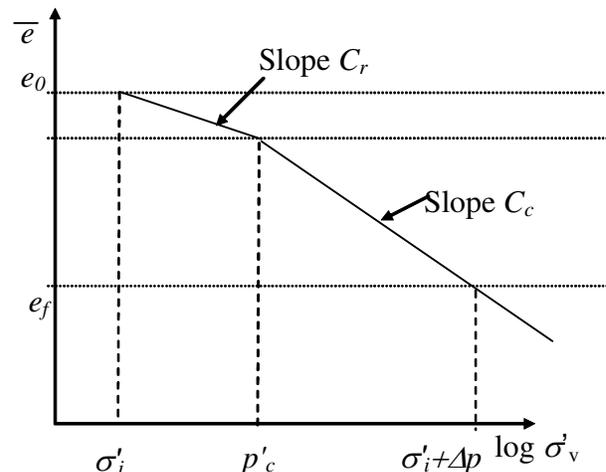


Figure 11: Soil compression curve (after Indraratna et al., 2005c)

The main assumptions are given below (Indraratna et al., 2005c):

(1) According to Indraratna et al. (2004), the distribution of vacuum pressure along the boundary of the drain is considered to vary linearly from  $-p_0$  at top to  $-k_1 p_0$  at the bottom, where  $k_1$  is a ratio between the vacuum at the top and bottom of the drain (Figure 10).

(2) The relationship between the average void ratio and logarithm of average effective stress in the normally consolidated range (Figure 11) can be expressed by  $\bar{e} = e_0 - C_c \log(\sigma' / \sigma'_i)$ . If the current vertical effective stress ( $\sigma'$ ) is less than  $p'_c$ , then for this over-consolidated range, the re-compression index ( $C_r$ ) is used rather than  $C_c$ .

(3) For radial drainage, the horizontal permeability of soil decreases with the average void ratio (Figure 12). The relationship between these two parameters is given by Tavenas et al. (1983):  $\bar{e} = e_0 + C_k \log(k_h / k_{hi})$ . The permeability index ( $C_k$ ) is generally considered to be independent of stress history ( $p'_c$ ), as explained by Nagaraj et al. (1994).

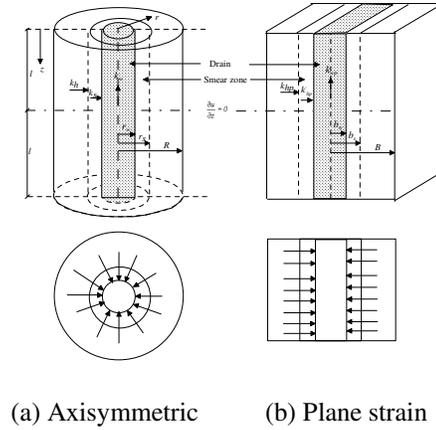
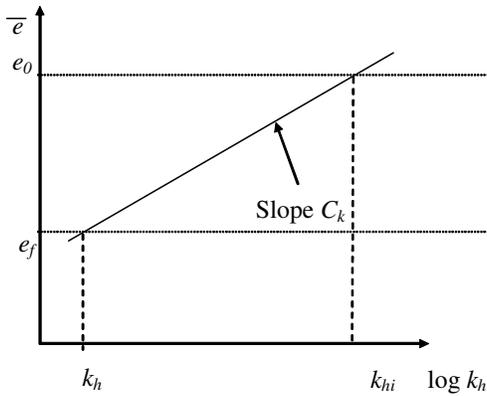


Figure 12: Semi-log permeability-void ratio (after Indraratna et al., 2005c)

Figure 13: Conversion of an axisymmetric unit cell into plane strain condition (after Indraratna and Redana, 2000)

The dissipation rate of average excess pore pressure ratio ( $R_u = \bar{u}_r / \Delta p$ ) at any time factor ( $T_h$ ) can be expressed as:

$$R_u = \left( 1 + \frac{p_0 (1+k_1)}{\Delta p} \right) \exp\left( \frac{-8T_h^*}{\mu} \right) - \frac{p_0 (1+k_1)}{\Delta p} \quad (6)$$

In the above expression,

$$T_h^* = P_{av} T_h \quad (7)$$

$$P_{av} = 0.5 \left[ 1 + \left( 1 + \Delta p / \sigma'_i + p_0 (1+k_1) / 2\sigma'_i \right)^{1-C_c/C_k} \right] \quad (8)$$

$$T_h = c_h t / d_e^2 \quad (9)$$

$$\mu = \ln \frac{n}{s} + \frac{k_h}{k'_h} \ln s - 0.75 \quad (10)$$

where  $\mu$  = a group of parameters representing the geometry of the vertical drain system and smear effect,  $n = d_s/d_w$ ,  $s = d_s/d_w$ ,  $d_e$  = equivalent diameter of cylinder of soil around drain,  $d_s$  = equivalent diameter of smear zone and  $d_w$  = equivalent diameter of drain well,  $k_h$  = average horizontal permeability in the undisturbed zone (m/s), and  $k'_h$  = average horizontal permeability in the smear zone (m/s).  $\Delta p$  = preloading pressure,  $T_h$  is the dimensionless time factor for consolidation due to radial drainage.

Since the relationship between effective stress and strain is non-linear the average degree of consolidation can be described on either excess pore pressure (stress) ( $U_p$ ) or strain ( $U_s$ ).  $U_p$  indicates the rate of dissipation of excess pore pressure

whereas  $U_p$  shows the development rate of surface settlement. Normally,  $U_p \neq U_s$  except when the relationship between effective stress and strain is linear, which is in accordance with Terzaghi's one-dimensional theory, and therefore the average degree of consolidation based on excess pore pressure can be obtained as follows:

$$U_p = 1 - R_u \quad (11)$$

The average degree of consolidation based on settlement (strain) is defined by:

$$U_s = \frac{\rho}{\rho_\infty} \quad (12)$$

The associated settlements ( $\rho$ ) are then evaluated by the following equations:

$$\rho = \frac{HC_r}{1 + e_0} \log\left(\frac{\sigma'}{\sigma'_i}\right), \quad \sigma'_i \leq \sigma' \leq p'_c \quad (13a)$$

$$\rho = \frac{H}{1 + e_0} \left[ C_r \log\left(\frac{p'_c}{\sigma'_i}\right) + C_c \log\left(\frac{\sigma'}{p'_c}\right) \right], \quad p'_c \leq \sigma' \leq \sigma'_i + \Delta p \quad (13b)$$

$$\rho = \frac{HC_c}{1 + e_0} \log\left(\frac{\sigma'}{\sigma'_i}\right) \text{ for normally consolidated clay} \quad (13c)$$

It is noted that  $\rho_\infty$  can be obtained by substituting  $\sigma' = \sigma'_i + \Delta p$  into the above equations. In the above equations,  $\rho$  = settlement at a given time,  $\rho_\infty$  = total primary consolidation settlement,  $\sigma'_i$  = effective in-situ stress,  $\sigma'$  = effective stress,  $C_c$  = compression index,  $C_r$  = recompression index and  $H$  = compressible soil thickness.

Depending on the location of the initial and final effective stresses with respect to the normally consolidated and over-consolidated domains, the following is a summary of the relevant computational steps.

- (i) If both the initial and final effective stresses are in the normally consolidated range, Equations (6) and (11) are used to calculate  $U_p$ , whereas Equations (12) and (13c) are used to compute  $U_s$ .
- (ii) If both the initial and final effective stresses are in the over-consolidated range, Equations (6) and (11) are used to calculate  $U_p$ , and Equations (12) and (13a) are used to determine  $U_s$ .
- (iii) If the initial effective stress falls on the over-consolidated domain and the final effective stress is on the normally consolidated domain, then Equations (6) and (11) are used to calculate  $U_p$ , Equations (12) (13a) and (13b) are used to calculate  $U_s$ .

When the value of  $C_r/C_c$  approaches unity and  $p_0$  becomes zero, the author's solution converges to the conventional solution proposed by Hansbo (1981):

$$R_u = \exp(-8T_h/\mu) \quad (14)$$

### 3.1.2. Non-Darcian flow:

Hansbo (1997) stated that at small hydraulic gradients, the conventional linear Darcy's law may be replaced by a non-Darcian flow condition defined by an exponential relationship. Based on non-Darcian flow, Hansbo (1997) modified the classical axisymmetric solutions. The pore water flow velocity  $v$  caused by a hydraulic gradient  $I$ , might deviate from the original Darcy's law  $v = k i$ , where under a certain gradient  $i_o$  below which no flow occurs. Then the rate of flow is given by  $v = k(i - i_o)$ , hence the following relations have been proposed:

$$v = \kappa i^n \quad \text{for } i \leq i_1 \quad (15)$$

$$v = k(i - i_o) \quad \text{for } i \geq i_1 \quad (16)$$

$$\text{where, } i_1 = \frac{i_o n}{(n-1)} \text{ and } \kappa = (n^{-1} i_1^{1-n}) \kappa \quad (17)$$

In order to study the non-Darcian effects Hansbo (1979, 1997) proposed an alternative consolidation equation where the time required to attain a certain average degree of consolidation, including the smear effect, is given by:

$$t = \frac{\alpha D^2}{\lambda} \left( \frac{D \gamma_w}{u_o} \right)^{n-1} \left( \frac{1}{(1-\bar{U}_h)^{n-1}} - 1 \right) \quad (18a)$$

where, the coefficient of consolidation  $\lambda = \frac{\kappa_h M}{\gamma_w}$ ,  $M=1/m_v$  is the oedometer modulus,  $D$  is the diameter of the drain influence zone,  $d_s$  is the diameter of smear zone,  $n=D/d_w$  where  $d_w$  is the drain diameter,  $u_o$  is the initial average excess pore water pressure, and  $\alpha$  is  $\frac{n^{2n} \beta^n}{4(n-1)^{n+1}}$  in which,

$$\beta = \frac{1}{3n-1} - \frac{n-1}{n(3n-1)(5n-1)} - \frac{(n-1)^2}{2n^2(5n-1)(7n-1)} + \frac{1}{2n} \left[ \left( \frac{\kappa_h}{\kappa_s} - 1 \right) \left( \frac{D}{d_s} \right)^{-(1-(1/n))} - \frac{\kappa_h}{\kappa_s} \left( \frac{D}{d_w} \right)^{-(1-(1/n))} \right] \quad (18b)$$

When  $n \rightarrow 1$ , Eq. (18) gives the same result as the average degree of consolidation represented by Eq. (9), provided that well resistance is neglected as well as assuming  $\lambda=c_h$  and  $\kappa_h/\kappa_s=k_h/k_s$ .

### 3.2. EQUIVALENT PLANE STRAIN APPROACH FOR MULTI-DRAIN ANALYSIS

For multi-drain simulation, the plane strain finite element analysis can be readily adapted to most field situations (Hansbo, 1981; Indraratna and Redana, 1997; Indraratna and Redana, 2000). Nevertheless, realistic field predictions require the axisymmetric properties to be converted to an equivalent 2D plane strain condition, especially with regard to the permeability coefficients and drain geometry (Indraratna and Redana, 1997). The plane strain analysis can also accommodate vacuum preloading in conjunction with vertical drains (e.g. Gabr and Szabo, 1997). Mohamedelhassan and Shang (2002) discussed the application of vacuum pressure and its benefits, but without any vertical drains. Subsequently, Indraratna et al. (2005b) proposed the equivalent plane strain approach to simulate the vacuum pressure of a vertical drain system.

#### 3.2.1. Darcian Flow:

Indraratna and Redana (1997, 1998, and 2000) and Indraratna et al. (2005b) converted the vertical drain system shown in Figure 13 into an equivalent parallel drain wall by adjusting the coefficient of permeability of the soil and by assuming a plane strain cell width of  $2B$ . The half width of the drain  $b_w$  and half width of the smear zone  $b_s$  may be kept the same as their axisymmetric radii  $r_w$  and  $r_s$ , respectively, which suggests  $b_w = r_w$  and  $b_s = r_s$ .

Indraratna et al. (2005b) proposed the average degree of consolidation in plane strain condition by:

$$\frac{\bar{u}}{u_o} = \left( 1 + \frac{p_{0p}}{u_o} \frac{(1+k_1)}{2} \right) \exp \left( -\frac{8T_{hp}}{\mu_p} \right) - \frac{p_{0p}}{u_o} \frac{(1+k_1)}{2} \quad (19a)$$

and,

$$\mu_p = \left[ \alpha + (\beta) \frac{k_{hp}}{k'_{hp}} \right] \quad (19b)$$

where,  $\bar{u}_0$  = initial excess pore pressure,  $\bar{u}$  = pore pressure at time  $t$  (average values) and  $T_{hp}$  = time factor in plane strain,  $k_{hp}$  and  $k'_{hp}$  are the undisturbed horizontal and the corresponding smear zone equivalent permeability, respectively. The geometric parameters  $\alpha$  and  $\beta$  are given by:

$$\alpha = \frac{2}{3} - \frac{2b_s}{B} \left( 1 - \frac{b_s}{B} + \frac{b_s^2}{3B^2} \right) \quad (20a)$$

$$\beta = \frac{1}{B^2} (b_s - b_w)^2 + \frac{b_s}{3B^3} (3b_w^2 - b_s^2) \quad (20b)$$

At a given effective stress level and at each time step, the average degree of consolidation for both axisymmetric ( $\bar{U}_p$ ) and equivalent plane strain ( $\bar{U}_{p,pl}$ ) conditions are made equal, hence:

$$\bar{U}_p = \bar{U}_{p,pl} \quad (21)$$

Combining Equations (14) and (17) with equation (14) of original theory by Hansbo, 1981, the time factor ratio can be represented by the following equation:

$$\frac{T_{hp}}{T_h} = \frac{k_{hp}}{k_h} \frac{R^2}{B^2} = \frac{\mu_p}{\mu} \quad (22)$$

Making the magnitudes of  $R$  and  $B$  to be the same, Indraratna and Redana (2000) presented a relationship between  $k_{hp}$  and  $k'_{hp}$ . The influence of smear effect can be modelled by the ratio of the smear zone permeability to the undisturbed permeability, as follows:

$$\frac{k'_{hp}}{k_{hp}} = \frac{\beta}{\frac{k_{hp}}{k_h} \left[ \ln\left(\frac{n}{s}\right) + \left(\frac{k_h}{k'_h}\right) \ln(s) - 0.75 \right] - \alpha} \quad (23)$$

If smear and well resistance effects are ignored in the above expression, then the simplified ratio of plane strain to axisymmetric permeability is readily obtained, as also proposed earlier by Hird et al. (1992), as follows:

$$\frac{k_{hp}}{k_h} = \frac{0.67}{[\ln(n) - 0.75]} \quad (24)$$

For vacuum preloading, the equivalent vacuum pressure in the plane strain and axi-symmetric condition is the same.

### 3.2.2. Non-Darcian Flow:

Sathanathan and Indraratna (2005) determined a solution for an equivalent plane strain under non-Darcian flow where the converted permeability relationship is given by:

$$\kappa_{hp} = 2\kappa_h \left( \frac{n-1}{2n^2} \frac{\beta_p}{\beta} \right)^n \quad (25)$$

Ignoring the smear effect in Eq. (25), the equivalent plane strain permeability in the undisturbed zone is now obtained as:

$$\frac{\kappa_{hp}}{\kappa_h} = \frac{\lambda_{hp}}{\lambda} = 2 \left( \frac{f_p \left( n, \frac{b_w}{B} \right)}{2f \left( n, \frac{r_w}{R} \right)} \right)^n \quad (26a)$$

## 4. FACTORS INFLUENCING THE EFFICIENCY OF A VERTICAL DRAIN

### 4.1. SMEAR ZONE DETERMINATION

The term 'smear zone' or soil disturbance can occur when installing a vertical drain with a steel mandrel. Soil permeability around the drain can be decreased significantly, which retards the rate of consolidation. This decrease in permeability can be expressed by the ratio of average permeability in the smear zone and the in-situ horizontal permeability ( $k_h/k_s$ ). This ratio depends on the sensitivity and macro fabric of the soil. The area of the smear zone ( $d_s$ ) can be influenced by the size and shape of the mandrel, method of installation, and type of soil. These factors can be determined using large-scale

consolidation testing (Indraratna and Redana, 1997; Sharma and Xiao, 2000). The extent of the smear zone can be determined using variations in permeability and water content (Sathanantha and Indraratna, 2006).

Based on laboratory tests conducted on a large-scale consolidometer at the University of Wollongong, the extent of the smear zone can be quantified either by variations in permeability or water content along the radial distance (Indraratna and Redana, 1997; Sathanantha and Indraratna, 2006). Figure 14 shows the variation of the ratio of the horizontal to vertical permeability ( $k_h/k_v$ ) at different consolidation pressures along the radial distance, obtained from large-scale laboratory consolidation. Variations in the water content with radial distance are shown in Figure 15. As expected, the water content decreases towards the drain but is greater towards the bottom of the cell at all radial locations. Walker and Indraratna (2007) showed that the smear zone can overlap and create more soil disturbance when the minimum radius of influence is 0.6 times the radius of the smear zone. The soil properties for the above case are given in Table 1.

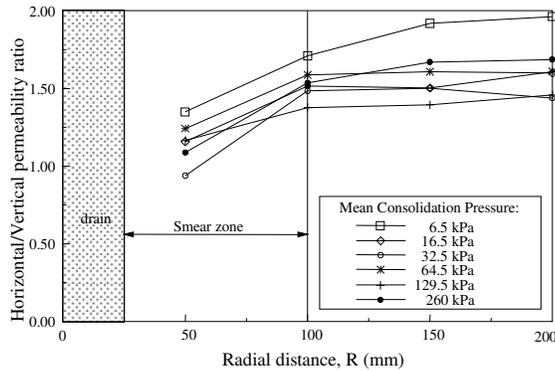


Figure 14: Ratio of  $k_h/k_v$  along the radial distance from the central drain (after Indraratna and Redana, 1995)

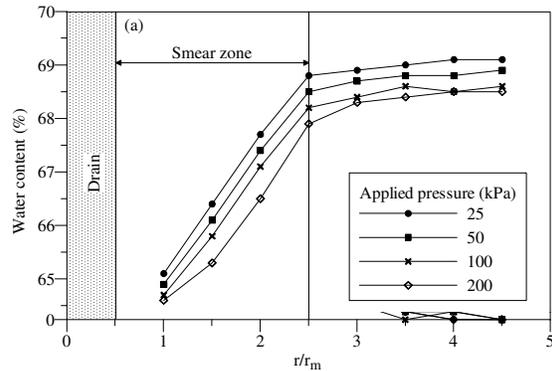


Figure 15: (a) water content radial distance at (after Sathanantha and Indraratna, 2006)

Table 1: Modified Cam-clay parameters used in consolidometer analysis (Indraratna et al., 2004)

Soil properties	Magnitudes
$\kappa$	0.05
$\lambda$	0.15
Critical state line slope, $M$	1.1
Critical state void ratio, $e_{cs}$	1.55
Poisson's ratio, $\nu$	0.25
Permeability in smear zone, $k'_{hp}$ (m/s)	$3.6 \times 10^{-11}$
Permeability in undisturbed zone, $k_{hp}$ (m/s)	$9.1 \times 10^{-11}$

## 4.2. THE EFFECT OF DRAIN UNSATURATION DURING INSTALLATION

Unsaturated soil adjacent to the drain can occur due to mandrel withdrawal (air gap) and dry conditions of PVDs. The apparent retardation of pore pressure dissipation and consolidation can be found at the initial stage of loading (Indraratna et al., 2004). Figure 16 shows using a numerical simulation, how the top of the drain takes a longer time to be saturated compared to the bottom of the drain. This example assumes that the PVD was 50% saturated at the start. Even for a short drain such as 1m, the time lag for complete drain saturation can be significant. The soil properties for this simulation were assumed to be the same as in Section 4.1.

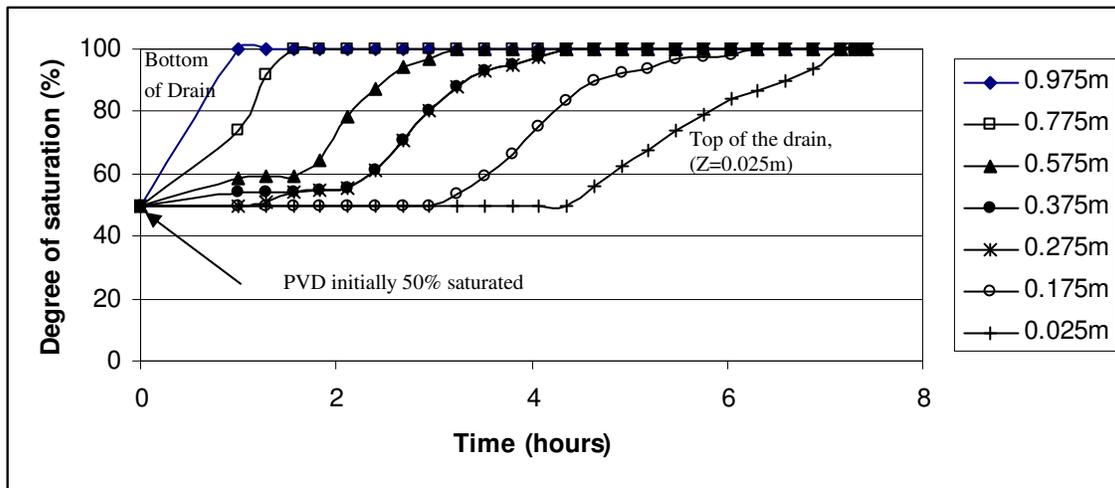


Figure 16: Degree of drain saturation with time (after Indraratna et al. 2004)

## 5. APPLICATION TO CASE HISTORIES

### 5.1. SECOND BANGKOK INTERNATIONAL AIRPORT

#### 5.1.1. Site Characteristics and Embankment Details

The Second Bangkok International Airport (SBIA) is located on low-lying soft clay which means that ground improvement techniques are essential before construction can begin. Three trial embankments were built, one of them (TV2) was built with PVDs and vacuum application (Asian Institute of Technology, 1995). Figure 17 shows the vertical cross section and position of the field instruments. PVDs of 12m in length were installed using a steel mandrel. The sand blanket (0.8m thick) was placed and compacted together with the drainage system consisting of perforated horizontal pipes wrapped in geotextiles. An air and water tight membrane was placed on top of the drainage system, and the edges of the liner were immersed at the bottom of a bentonite trench to ensure minimum risk of any air leaks (Figure 17). The predicted variation in the depth of the smear zone for each layer of soil is shown in Figure 18.

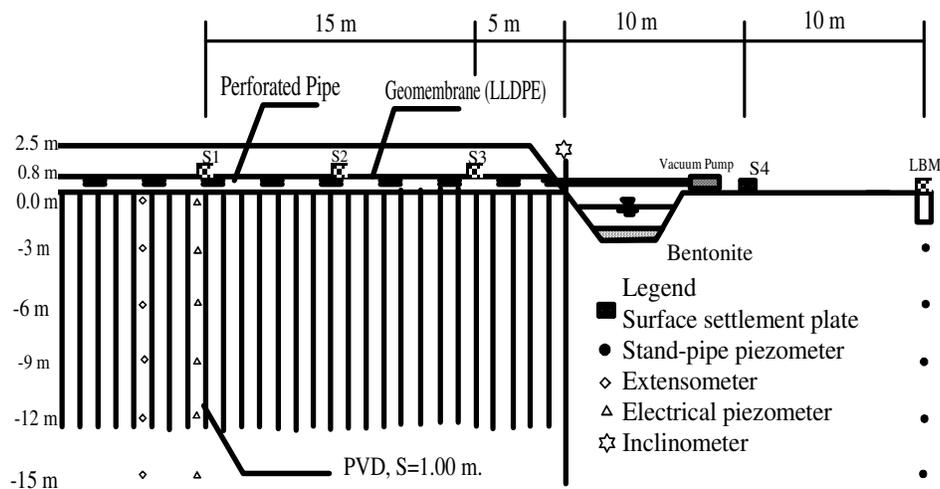


Figure 17: Cross section of embankment TV2 and location of monitoring system (Indraratna et al., 2005d)

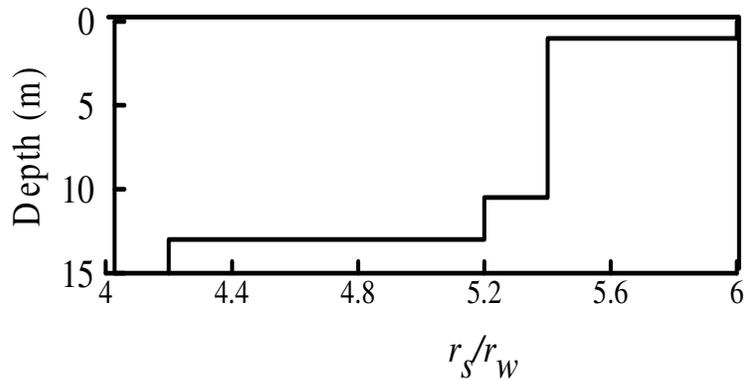


Figure 18: Variation of smear zone with depth by the Cavity Expansion theory (Indraratna et al., 2005d)

A suction pressure up to -80kPa was applied using a commercial vacuum pump system. A four-stage surcharge load up to 2.5m high was applied (the unit weight of surcharge fill is  $18 \text{ kN/m}^3$ ), as illustrated in Figure 19. During this vacuum application the suction head dropped as shown in Figure 20. This reduction was attributed to air leaks through the surface membrane or loss of suction head beneath a certain depth for long PVD. The settlement, excess pore water pressure, and lateral movement were recorded for 5 months.

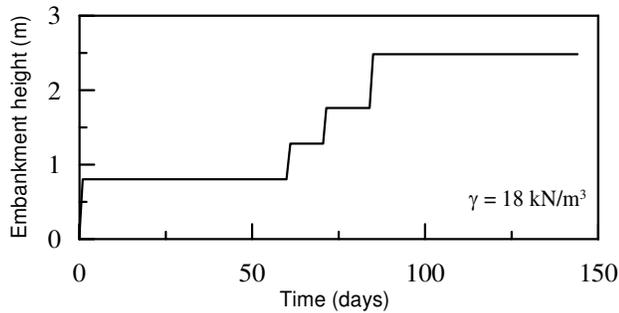


Figure 19: Multi-stage loading (Indraratna et al., 2005d)

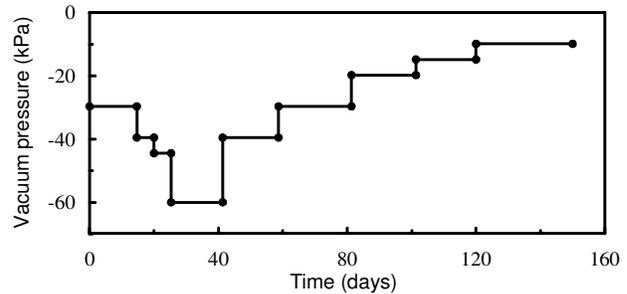


Figure 20: Time dependent vacuum pressure (Indraratna et al., 2005d)

### 5.1.2. Multi-drain Analysis Using FEM Incorporating Proposed Equivalent Plain Strain Model

Consolidation was studied using the finite element software ABAQUS. The *equivalent* plane strain model (Equations 14-15) and the modified Cam-clay theory (Roscoe and Burland, 1968) were used in the analysis (Indraratna et al., 2005d). The ratios of  $k_f/k_s$  and  $d_s/d_w$  obtained in the laboratory were approximately 1.5-2.0 and 3.0-4.0, respectively. However, in practice these ratios may vary from 1.5 to 6.0 depending on the type of drain, size and shape of mandrel, and installation procedures (Indraratna and Redana, 2000). The constant values of  $k_f/k_s$  and  $d_s/d_w$  for this case study were in the order of 2 and 6, respectively (Indraratna et al., 2004). For the plane strain FEM simulation, the equivalent permeability inside and outside the smear zone was determined using Equations (14) and (15), while the role of well resistance hence the implications of discharge capacity ( $q_w$ ) was neglected (Indraratna and Redana, 2000). As modern PVDs have  $q_w$  values greater than  $150 \text{ m}^3/\text{year}$ , well resistance is insignificant compared to the smear effects.

The finite element mesh contained 8-node bi-quadratic displacement and bi-linear pore pressure elements (Figure 21). It was sufficient to model just the right hand side of the embankment due to symmetry. The incremental surcharge loading was simulated at the upper boundary.

The following 4 distinct models were numerically examined under the 2D (plane strain) multi-drain analysis (Indraratna et al., 2005d):

Model A: Conventional analysis (i.e., no vacuum application);

- Model B: Vacuum pressure varies according to field measurement and decreases linearly to zero at the bottom of the drain ( $k_1=0$ );
- Model C: No vacuum loss (i.e. -60kPa vacuum pressure was kept constant after 40 days); vacuum pressure diminishes to zero along the drain length ( $k_1=0$ ); and
- Model D: Constant time-dependent vacuum pressure throughout the soil layer ( $k_1=1$ ).

The settlement predictions for Model B agree with the field data (Figure 22). A comparison between all the different vacuum pressure conditions shows that vacuum application combined with a PVD system can accelerate the consolidation process significantly. Most of the primary consolidation can be achieved in around 3-4 months by applying a vacuum pressure, whereas conventional surcharge (same equivalent pressure) needs more time to attain primary consolidation (after about 5 months). A greater settlement can be obtained by minimising any loss of vacuum (Model C).

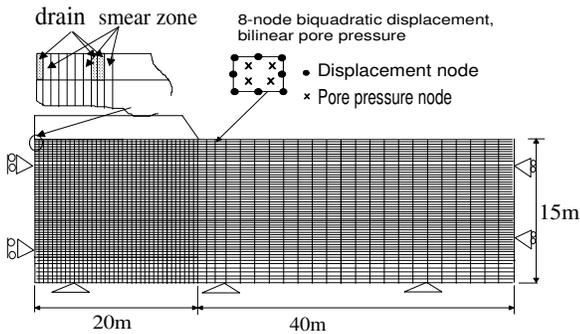


Figure 20: Finite element mesh for plane strain analysis (modified after Indraratna et al., 2005d)

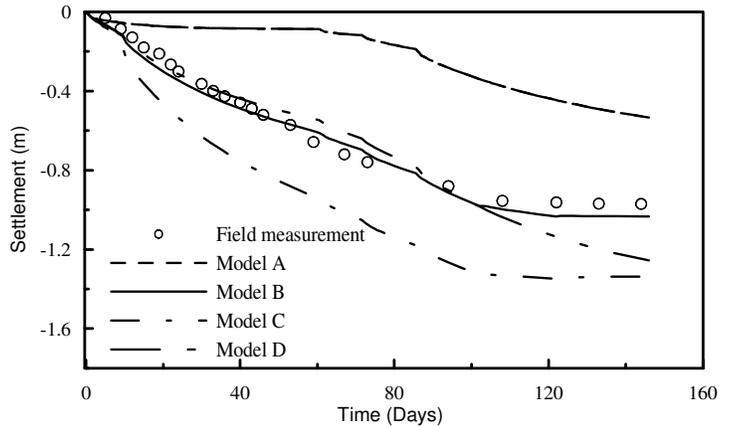


Figure 21: Surface settlements (modified after Indraratna et al., 2005d)

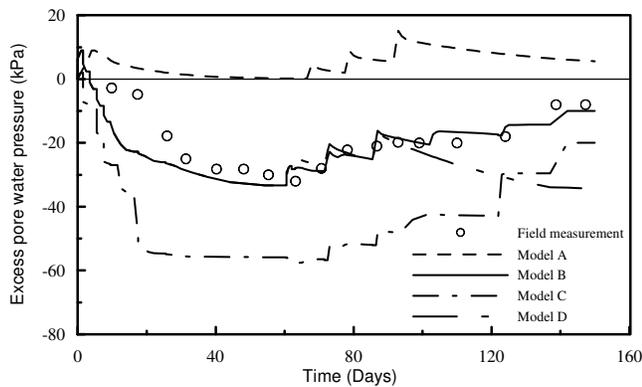


Figure 23: Variation of excess pore water pressure at 3m below the surface and 0.5m away from centerline (modified after Indraratna et al., 2005d)

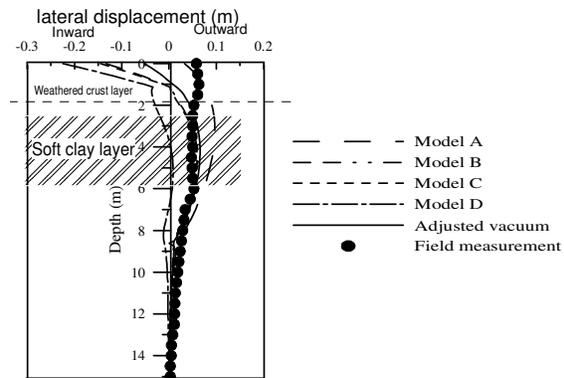


Figure 24: Calculated and measured lateral displacements distribution with depth (modified after Indraratna et al., 2005d)

The field observations are closest to Model B, which implies that the author's assumption of a linearly decreasing time-dependent suction head along the length of the drain is justified (Figure 23). Negative pore pressure generated by the application of vacuum pressure (increased effective stress) enables a higher rate of embankment construction compared to the conventional construction.

The lateral displacements measured and predicted at the end of construction are shown in Figure 24. As described by Indraratna et al. (2005d), the lateral displacements do not agree with all vacuum pressure models. The predictions from Models B and C, in the middle of the very soft clay layer (4-5m deep), are closest to the field measurements. However, the field observations near the surface do not agree with the ‘inward’ lateral movements predicted by Models B and C. The discrepancy between the finite element models and the measured data is more evident at the topmost crust (0-2 m).

As discussed by Indraratna et al. (1997), if a surcharge embankment without PVDs is raised quickly, it can fail in 13 days without effective pore pressure dissipation (Figure 25), but the same clay formation stabilised with PVDs shows insignificant lateral displacement after 13 days. Even after 7 years, the normalised lateral displacements will be less than that without PVDs. In Figure 25, the normalised lateral displacement is the absolute lateral displacement divided by the maximum height of the embankment.

## 5.2. SKA-EDEBY EMBANKMENT, STOCKHOLM, SWEDEN

The practical application of a non-Darcian plane strain solution is demonstrated through a well documented pilot study (Ska-Edeby, 25 km west of Stockholm, Sweden). Site details, including construction history and soil parameters are given elsewhere by Hansbo, 1997; 2005. Here, Area II with an equivalent loading of 32 kPa is selected. Sand drains with a radius of 90 mm are installed in a triangular pattern 1.5 m apart.

In Figure 26, the estimated degree of consolidation based on the Darcian axi-symmetric, non-Darcian axi-symmetric (Hansbo, 1997) and non-Darcian plane strain solutions (Sathananthan and Indraratna, 2005) are plotted at the centerline of the embankment with the available field data. The predicted values based on non-Darcian flow agree better with the field data in relation to the Darcian (conventional) analysis, but the difference is usually small. This proves that conventional Darcy flow conditions are sufficient for all practical purposes.

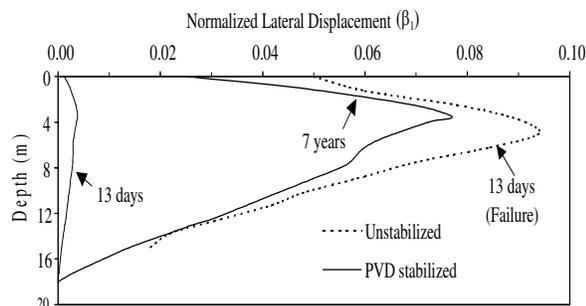


Figure 25: Curtailing lateral displacements due to PVDs (after Indraratna et al., 1997)

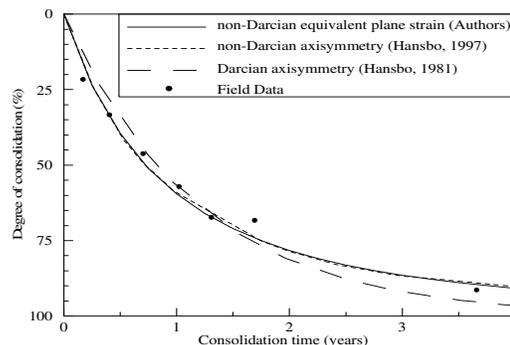


Figure 26: Degree of consolidation at the embankment centerline with time for Area II, Ska-Edeby field study (after Hansbo, 2005; Sathananthan & Indraratna, 2005)

## 6. PERFORMANCE OF SHORT VERTICAL DRAINS SUBJECTED TO CYCLIC LOADS

Very low permeable plastic clays can sustain high excess pore water pressure during static and (repeated) cyclic loading such as in the case of rail track environments. In poorly drained situations an increase in pore pressure will decrease the bearing capacity of the formation. Even if the rail tracks are well built structurally, undrained formation failures can adversely influence train speeds resulting in the inevitable operational delays. In such conditions, clay slurring (c.f. liquefaction) may occur under high cyclic loads which clogs the ballast layer and causes poor track drainage.

The stability of rail tracks and highways built on soft saturated clays is often controlled by the magnitude of lateral movements, even if consolidation provides a gain in shear strength and load bearing capacity. If excessive initial settlement of deep estuarine deposits cannot be tolerated in terms of maintenance (e.g. in newly built railway tracks where ballast packing may be required), the rate of settlement may still be controlled by using relatively short PVDs, and optimising the drain spacing and pattern of installation. In this way a reduction in lateral strain and a gain in shear strength of the soil beneath the track significantly improve its stability.

## 6.1. LABORATORY TESTING WITH CYCLIC LOADS

A large-scale triaxial test (300 mm diameter and 600 mm high) was used to examine the effect of cyclic load on the radial drainage and consolidation by PVDs (Figure 27). Excess pore water pressure was measured via miniature pore pressure transducers which were saturated and fitted through the base of the cell to their sample locations.

Remoulded estuarine clay was lightly compacted to a unit weight of about  $17 \text{ kN/m}^3$  with  $k_0$  conditions in the range of 0.6-0.7. The natural water content exceeded 75% and Plasticity Index was above 35%. It is not uncommon to find the undrained shear strength of the softest estuarine deposits to be less than 8 kPa. The tests were conducted at frequencies of 5-10 Hz, typically simulating 60-100 km/h train speeds of say of 25-30 tonnes/axle train loads. Figure 28 presents the excess pore pressure recorded, indicating that the maximum excess pore water pressure near the PVD during a cyclic load (T4) are considerably less than that in the proximity of the cell boundary (T3). As expected, the excess pore pressure near the outer cell boundary (e.g. T1 and T3) dissipated at a slower rate than T4 and T2 closer to the PVD. This demonstrates without doubt that PVDs effectively decrease the maximum excess pore pressure even under cyclic loading.



Figure 27: (a) Large-scale triaxial rig, (b) soil specimen

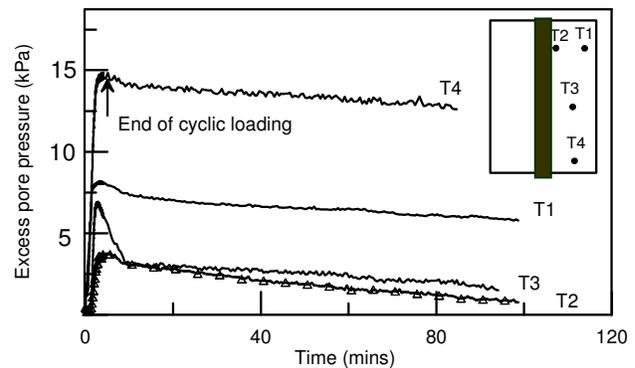


Figure 28: Dissipation of excess pore pressure at various locations from the PVD

## 6.2. NUMERICAL ANALYSIS

A significant proportion of the applied load beneath railway tracks is usually propagated within the first 5-6 meters of the soft formation. Therefore, usually it may not be necessary to improve the entire depth of a soft clay layer; i.e. relatively short PVDs without prolonged preloading may still be adequate in design. For instance, a pattern of short PVDs (5-8m) can still provide a “stiffened” section of the soft clay up to several metres deep, supporting the rail track within the critical influence zone of vertical stress distribution beneath the track.

In railway engineering the impact load factor may be changed according to the field conditions simulated on the track (Esveld, 2001). In the illustrative example described below, a static load of say 80kPa with an impact factor of 1.3 to represent dynamic conditions is assumed to model standard gauge, 25 tonne axle trains (i.e.104 kPa total load). Figure 29 shows a typical cross-section beneath the rail track, where a shallow soft clay deposit is underlain by a layer of slightly stiffer clay. Assuming that PVDs are only installed to stabilise the shallow layer of the softest clay, a FEM, 2D equivalent plane strain model (Indraratna and Redana, 2000) using triangular elements with 6 displacement nodes and 3 pore pressure nodes is considered in the analysis.

The assumed soil properties are summarised in Table 2, where the compacted soil crust on the top, including the sub-ballast fill and ballast layers are modelled by linear Mohr-Coulomb theory. The two layers of normally consolidated clays are represented using the modified Cam-clay theory (Roscoe and Burland, 1968).

The dissipation of excess pore water pressure via the short PVDs is clearly shown in Fig. 30. More than 60% dissipation of excess pore pressure can be obtained within the first 4-5 months, while the initial settlement induced by the PVDs is less

than 0.5m after 90 days. This settlement can still be acceptable over a routine maintenance period by packing more ballast. A more significant reduction of lateral displacement in the PVD stabilised soil underlying the compacted crust is shown in Figure 31. While long term lateral displacement at shallow depths (@ 3m) could be as large as 250-300mm, the PVDs decrease the lateral displacement by about 25%. This numerical example with equivalent dynamic loads reveals the positive role of short PVDs installed beneath a simulated rail track.

Table 2: Assumed parameters for soft soil foundation and ballasted track (300mm of ballast thickness and 1m thick compacted fill and crusted layer)

Depth of layer (m)	Model	c (kPa)	$\phi$ (degree)	$\lambda/(1+e_0)$	$\kappa/(1+e_0)$
+0.3 (ballast)	M-C	5	45		
0 -1 (crust)	M-C	29	29		
1 - 9 (very soft clay)	MCC	10	25	0.15	0.03
9 -35 (stiffer soft clay)	MCC	15	20	0.12	0.02

Note: M-C= Mohr-Coulomb, MCC= Modified Cam-Clay

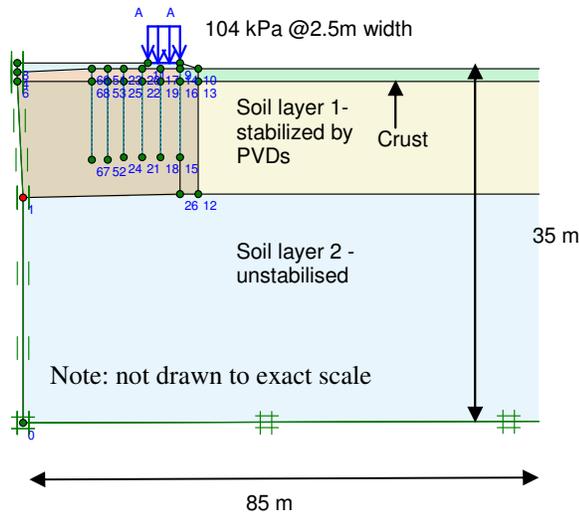


Figure 29: Vertical cross section of track and formation

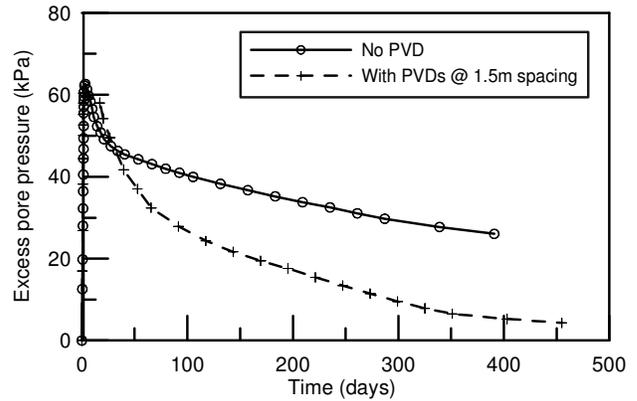


Figure 30: Excess pore pressure dissipation at 2 m depth at centre line of rail track

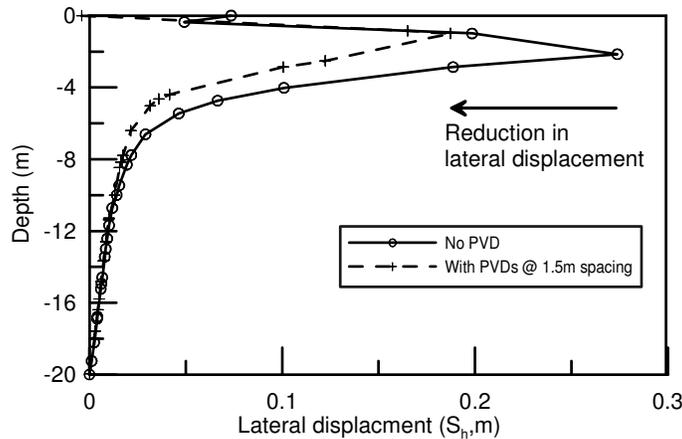


Figure 31: Ultimate lateral displacement profiles near the embankment toe

## 7. CONCLUSIONS

Prefabricated vertical drains have been used to accelerate the rate of primary consolidation. An analytical model for soft clay subjected to radial drainage and incorporating the compressibility indices ( $C_c$  and  $C_r$ ) and vacuum pressure has been introduced. The lateral displacement and dissipation of pore pressure associated with radial consolidation are not easy to predict numerically. This may be due to the difficulty of evaluating the soil parameters around PVDs and correct drain properties, as well as the aspects of the soil-drain interface. Therefore, one needs to use suitable laboratory techniques to determine these parameters, preferably large scale testing apparatus, where it was found that the smear zone diameter was 2-3 times the diameter of the mandrel, and the soil permeability in the smear zone was 1.5-2.0 times higher than in the undisturbed zone.

Plane strain analysis is sufficient for large construction projects given the computational efficiency for thousands of metres of PVDs that are installed. A conversion from axisymmetric to an equivalent plane strain condition used in the numerical analysis gives a good agreement with measured data. The finite element code ABAQUS was used to analyse the behaviour of multi-drains and compared with the field data. An equivalent plane strain technique was applied for selected case histories, demonstrating its capability for field predictions. It indicated that the efficiency of vacuum preloading depends on the magnitude and distribution of vacuum pressure as well as the degree of drain unsaturation at the soil-drain interface.

An accurate prediction of lateral displacement at shallow depths needs a correct assessment of soil properties (i.e. over consolidated surface crust). This near surface stiff compacted layer resists lateral movement of the soil (in the same way as a geogrid) even after a vacuum pressure is applied. The modified Cam-clay model is not suitable for modelling the behaviour of the compacted layer. This surface crust can be modelled as an elastic layer rather than a 'soft' elasto-plastic medium. An analysis of the case history proves that the vacuum application via PVD significantly decreases lateral displacement and increases the effective stresses, with the favourable result that the potential shear failure during rapid embankment construction can be avoided.

A system of vacuum-assisted consolidation with PVDs is a useful and practical approach for accelerating radial consolidation, because, it reduces the need for a high surcharge load, provided that air leaks can be prevented. Accurate modelling of vacuum preloading requires both laboratory and field studies to quantify the nature of vacuum distribution within a given formation and drain system. These ground improvement techniques, including PVDs, can be applied prior to rail track construction in coastal areas containing soft soils. Relatively short prefabricated vertical drains can be used under rail tracks to improve track stability by dissipating excess pore pressure and to curtail lateral displacement within the critical shallow depth beneath tracks.

## 8. ACKNOWLEDGEMENT

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