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Abstract

New amicable orthogonal designs AODs(8; 1; 1; 1; 2; 2; 2), AODs(8; 1; 1; 4; 1; 2; 2), AODs(8; 1; 2;2; 2; 2; 4), AODs(8; 1; 2; 2; 1; 2; 4), AODs(8; 1; 1; 2; 1; 2; 4), AODs(8; 1; 2; 4; 2; 2; 2), AODs(8; 1; 1; 4; 1; 1; 2; 2), AODs(8; 2; 2; 2; 2; 2; 2; 2; 2) and AODs(8; 1; 1; 1; 2; 1; 2; 2; 2) are found by applying a new theorem or by an exhaustive search. Also some previously undecided cases of amicable pairs are demonstrated to be non-existent after a complete search of the equivalence classes for orthogonal designs.

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Amicable Orthogonal Designs of Order 8 for Complex Space-Time Block Codes

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Abstract

New amicable orthogonal designs $AODs(8;1,1,1;2,2,2)$, $AODs(8;1,1,4;1,2,2)$, $AODs(8;1,2,2;2,2,4)$, $AODs(8;1,2,2;1,2,4)$, $AODs(8;1,1,2;1,2,4)$, $AODs(8;1,2,4;2,2,2)$, $AODs(8;1,1,4;1,1,2,2)$, $AODs(8;2,2,2,2;2,2,2,2)$ and $AODs(8;1,1,1,2;1,2,2,2)$ are found by applying a new theorem or by an exhaustive search. Also some previously undecided cases of amicable pairs are demonstrated to be non-existent after a complete search of the equivalence classes for orthogonal designs.

2000 *Mathematics subject classification (Amer. Math. Soc.)*: 05B30, 15A36.

1 Motivation

Complex orthogonal space-time block codes(STBCs) based on Amicable Orthogonal Designs(AODs) [1] are known for relatively simple receiver structure and minimum processing delay when dealing with complex signal constellations. The simplest complex STBCs is an Alamouti code [8] for two transmit antennas, which is based on an amicable orthogonal design of order 2. The complex STBCs for more than two transmit antennas cannot achieve rate one [7], but they can still provide full diversity. To date, only the complex STBCs based on full(i.e., no zeros) designs of order two and four have been proposed [5][8]. The known complex STBCs for higher number of transmit antennas, e.g.

eight, have zeros in the code matrices that results in the waste of several time slots when no useful information is being transmitted from a given antenna. Moreover, those zeros make implementation of the transmitter rather difficult at high symbol rates, as it requires the transmitter to be switched on and off frequently. It has been shown that by utilizing some amicable pairs, especially the new $AODs(8;2,2,2,2;2,2,2,2)$ which is the only full design with 4 variables of order 8, the new complex codes have much better error performance than the conventional ones [9][10].

2 Introduction and Basic Definitions

Definition 2.1 Let $x_1, x_2, \cdots, x_t$ be commuting indeterminates. An orthogonal design X of order n and type $(s_1, s_2, \cdots, s_t)$ denoted $OD(n; s_1, \cdots, s_t)$, where s_i are positive integers, is a matrix of order n with entries from $\{0, \pm x_1, \cdots, \pm x_t\}$, such that

$$XX^T = (\sum_{i=1}^{t} s_i x_i^2) I_n,$$

where X^T denotes the transpose of X and I_n is the identity matrix of order n.

Alternatively, each row of X has s_i entries of the type $\pm x_i$ and the rows are pairwise-orthogonal under the Euclidean inner product [1]. The above description of X applies to the columns of X as well.

Definition 2.2 Let X be an $OD(n; u_1, \cdots, u_s)$ on the variables $\{x_1, \cdots, x_s\}$ and Y an $OD(n; v_1, \cdots, v_t)$ on the variables $\{y_1, \cdots, y_t\}$. It is said that X and Y are *amicable orthogonal designs* $AODs(n; u_1, \cdots, u_s; v_1, \cdots, v_t)$ if $XY^T = YX^T$.

In the complex multiple antenna communication systems, each transmitted complex symbol contains both real and imaginary part. Hence, the concept of complex orthogonal designs used for STBCs is different from the definition introduced in Geramita-Geramita [2].

Definition 2.3 A *complex orthogonal design for STBC* (CODSTBC, throughout this paper referred as COD for short) Z of order n is an $n \times n$ matrix on the complex indeterminates $s_1, \cdots, s_t$, with entries chosen from $0, \pm s_1, \cdots, \pm s_t$, their conjugates $\pm s_1^*, \cdots, \pm s_t^*$, or their product with $i = \sqrt{-1}$ such that

$$Z^H Z = (\sum_{i=1}^{t} |s_i|^2) I_n, \tag{1}$$

where Z^H denotes the conjugate transpose of Z and I_n is the identity matrix of order n.

Given an orthogonal design over s variables, we can get new designs of the same order but different types by setting variables equal to one another or zero. The concept of "Equating and Killing variables" was first stated as Lemma 4.4 in [1].

Definition 2.4 Let X be an $OD(n; u_1, \cdots, u_s)$ on the variables $\{x_1, \cdots, x_s\}$, then there exists $OD(n; u_1, \cdots, u_i + u_j, \cdots, u_s)$ and $OD(n; u_1, \cdots, u_{i-1}, u_{i+1}, \cdots, u_s)$ on $s-1$ variables.

The description of Equating and Killing variables applies as well to amicable orthogonal designs.

Example 2.1 Given an $AODs(8; 1, 1, 2, 2; 1, 1, 2, 2)$, if we set $u_3^{'}$=u_3+u_4 for one of the amicable pair, then we get a new $AODs(8; 1, 1, 4; 1, 1, 2, 2)$ with same order but different type. We can also get new designs $AODs(8; 1, 1, 2; 1, 1, 2, 2)$, $AODs(8; 1, 2, 2; 1, 1, 2, 2)$ by killing variables.

The existence and non-existence results for AODs are established by using the theory of quadratic forms. For future reference, a detailed description about the linkage between quadratic forms and amicable orthogonal designs can be found in Street's Thesis [3].

3 Theorems and Examples

In this section, two previously known theorems will be quoted for constructing AODs. The first contribution is due to Wolfe [4] and the second one can be found in Street's thesis [3]. An example of Street's theorem is given followed by a new construction theorem.

Theorem 3.1 (Wolfe, 1975). *If there exist a pair of AODs* $(n; a_1, ..., a_s; b_1, ..., b_t)$ *and a pair of AODs* $(m; c_1, ..., c_u; d_1, ..., d_v)$*, then there exists a pair of AODs* $(mn; b_1c_1, ..., b_1c_{u-1}, a_1c_u, ..., a_sc_u; b_1d_1, ..., b_1d_v, b_2c_u, ..., b_tc_u)$.

The observation of Wolfe's theorem leads to the following corollary.

Corollary 3.1 *If there exist a pair of amicable orthogonal designs* $AODs(n; a_1, ..., a_s; b_1, ..., b_t)$*, then there exists a pair of amicable orthogonal designs of type*
a)$AODs(2n; a_1, ..., a_s, a_s; b_1, ..., b_t, a_s)$,
b)$AODs(2n; a_1, a_1, 2a_2..., 2a_s; 2b_1, ..., 2b_t)$,
c)$AODs(2n; a_1, a_1, a_2..., a_s; b_1, ..., b_t)$.

Proof. Let $X = \sum_{i=1}^{s} A_i x_i$ and $Y = \sum_{j=1}^{t} B_j y_j$ are the amicable designs in order n.
a) Let one pair be $AODs(2; 1, 1; 1, 1)$ in Theorem 2.1.
b) Let weighing matrices $M = \begin{bmatrix} 0 & 1 \\ - & 0 \end{bmatrix}$, $N = \begin{bmatrix} 1 & 1 \\ 1 & - \end{bmatrix}$ and construct the matrices

$$P = (A_1 \otimes I_2)p_1 + (A_1 \otimes M)p_2 + \sum_{i=2}^{s}(A_i \otimes N)p_{i+1} \tag{2}$$

and

$$Q = \sum_{j=1}^{t} (B_j \otimes N) q_j \tag{3}$$

c) Same as b), only set $N = \begin{bmatrix} 1 & 0 \\ 0 & - \end{bmatrix}$.

Here $\otimes$ denotes the Kronecker product and the p_i's and q_i's are distinct commuting indeterminates. It's obvious that all the coefficient matrices P_i's and Q_i's satisfy the conditions in (2) because the weighing matrices M and N have the following properties: $M = -M^t, N = N^t$, and $MN^t = NM^t$. □

Theorem 3.2 (Street, 1981). *Suppose (A, B) and (C, D) are both AODs $(n; a_1, ..., a_s; b_1, ..., b_t)$. Suppose further that there exists a weighing matrix $W(n, k)$ such that*

$$AW^T = WC^T, BW^T = -WD^T$$

Then there exist AODs $(2n; k, a_1, ..., a_s; k, b_1, ..., b_t)$.

Example 3.1 Given $W(4, 4)$, $AODs(4; 1, 1, 1; 1, 1, 1)$, we use Theorem 3.2 to construct $AODs(8; 1, 1, 1, 4; 1, 1, 1, 4)$. We have

$$A = \begin{bmatrix} x_1 & 0 & x_3 & x_2 \\ 0 & x_1 & -x_2 & x_3 \\ -x_3 & x_2 & x_1 & 0 \\ -x_2 & -x_3 & 0 & x_1 \end{bmatrix}; B = \begin{bmatrix} y_1 & 0 & y_2 & y_3 \\ 0 & y_1 & y_3 & -y_2 \\ y_2 & y_3 & -y_1 & 0 \\ y_3 & -y_2 & 0 & -y_1 \end{bmatrix}, C = \begin{bmatrix} x_1 & x_2 & x_3 & 0 \\ -x_2 & x_1 & 0 & x_3 \\ -x_3 & 0 & x_1 & -x_2 \\ 0 & -x_3 & x_2 & x_1 \end{bmatrix};$$

$$D = \begin{bmatrix} -y_3 & -y_2 & -y_1 & 0 \\ -y_2 & y_3 & 0 & -y_1 \\ -y_1 & 0 & y_3 & y_2 \\ 0 & -y_1 & y_2 & -y_3 \end{bmatrix} \text{ and } W = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & - & 1 & - \\ 1 & 1 & - & - \\ 1 & - & - & 1 \end{bmatrix}.$$

The above matrices satisfy the conditions of Theorem 3.2 and give $AODs(8; 1, 1, 1, 4; 1, 1, 1, 4)$. Such an amicable pair can be used to construct $COD(8; 1, 1, 1, 4; 1, 1, 1, 4)$ for 8 transmitter antennas communication system. The pair of AODs is given below (we use $\bar{x_1}$ for $-x_1$, $\bar{x_2}$ for $-x_2$ and so on).

$$X: \begin{bmatrix} x_1 & 0 & x_3 & x_2 & x_4 & x_4 & x_4 & x_4 \\ 0 & x_1 & \bar{x_2} & x_3 & x_4 & \bar{x_4} & x_4 & \bar{x_4} \\ \bar{x_3} & x_2 & x_1 & 0 & x_4 & x_4 & \bar{x_4} & \bar{x_4} \\ \bar{x_2} & \bar{x_3} & 0 & x_1 & x_4 & \bar{x_4} & \bar{x_4} & x_4 \\ \bar{x_4} & \bar{x_4} & \bar{x_4} & \bar{x_4} & x_1 & x_2 & x_3 & 0 \\ \bar{x_4} & x_4 & \bar{x_4} & x_4 & \bar{x_2} & x_1 & 0 & x_3 \\ \bar{x_4} & \bar{x_4} & x_4 & x_4 & \bar{x_3} & 0 & x_1 & \bar{x_2} \\ \bar{x_4} & x_4 & x_4 & \bar{x_4} & 0 & \bar{x_3} & x_2 & x_1 \end{bmatrix}; Y: \begin{bmatrix} y_1 & 0 & y_2 & y_3 & y_4 & y_4 & y_4 & y_4 \\ 0 & y_1 & y_3 & \bar{y_2} & y_4 & \bar{y_4} & y_4 & \bar{y_4} \\ y_2 & y_3 & \bar{y_1} & 0 & y_4 & y_4 & \bar{y_4} & \bar{y_4} \\ y_3 & \bar{y_2} & 0 & \bar{y_1} & y_4 & \bar{y_4} & \bar{y_4} & y_4 \\ y_4 & y_4 & y_4 & y_4 & \bar{y_3} & \bar{y_2} & \bar{y_1} & 0 \\ y_4 & \bar{y_4} & y_4 & \bar{y_4} & \bar{y_2} & y_3 & 0 & \bar{y_1} \\ y_4 & y_4 & \bar{y_4} & \bar{y_4} & \bar{y_1} & 0 & y_3 & y_2 \\ y_4 & \bar{y_4} & \bar{y_4} & y_4 & 0 & \bar{y_1} & y_2 & \bar{y_3} \end{bmatrix}$$

Let $\{X_i\}$ and $\{Y_i\}$ be coefficient matrices and $\{s_i = s_i^R + is_i^I\}_{i=1}^4$ be a set of complex symbols. then the corresponding complex orthogonal space-time block code, by using equation $Z = \sum_{i=1}^4 X_i s_i^R + i\sum_{i=1}^4 Y_i s_i^I$, is

$$\begin{bmatrix} s_1 & 0 & s_3^R + is_2^I & s_2^R + is_3^I & \frac{s_4}{2} & \frac{s_4}{2} & \frac{s_4}{2} & \frac{s_4}{2} \\ 0 & s_1 & -s_2^R + is_3^I & s_3^R - is_2^I & \frac{s_4}{2} & -\frac{s_4}{2} & \frac{s_4}{2} & -\frac{s_4}{2} \\ -s_3^R + is_2^I & s_2^R + is_3^I & s_1^* & 0 & \frac{s_4}{2} & \frac{s_4}{2} & -\frac{s_4}{2} & -\frac{s_4}{2} \\ -s_2^R + is_3^I & -s_3^R - is_2^I & 0 & s_1^* & \frac{s_4}{2} & -\frac{s_4}{2} & -\frac{s_4}{2} & \frac{s_4}{2} \\ -\frac{s_4^*}{2} & -\frac{s_4^*}{2} & -\frac{s_4^*}{2} & -\frac{s_4^*}{2} & s_1^R - is_3^I & s_2^* & s_3^R - is_1^I & 0 \\ -\frac{s_4^*}{2} & \frac{s_4^*}{2} & -\frac{s_4^*}{2} & \frac{s_4^*}{2} & -s_2 & s_1^R + is_3^I & 0 & s_3^R - is_1^I \\ -\frac{s_4^*}{2} & -\frac{s_4^*}{2} & \frac{s_4^*}{2} & \frac{s_4^*}{2} & -s_3^R - is_1^I & 0 & s_1^R + is_3^I & -s_2^* \\ -\frac{s_4^*}{2} & \frac{s_4^*}{2} & \frac{s_4^*}{2} & -\frac{s_4^*}{2} & 0 & -s_3^R - is_1^I & s_2 & s_1^R - is_3^I \end{bmatrix}$$

We now give a new theorem and all six new amicable designs derived from this theorem can be found in Table 4.3.

Theorem 3.3 *If there exist a pair of amicable orthogonal designs $AODs(n; a_1, ..., a_s; b_1, ..., b_t)$, then there exists a pair of amicable orthogonal designs $AODs(2n; e_1a_1, e_2a_2..., e_sa_s; k_1b_1, k_2b_2, ..., k_tb_t)$ for every $e_i, k_j \in \{1, 2\}$.*

Proof. Let $X = \sum_{i=1}^s A_i x_i$ and $Y = \sum_{j=1}^t B_j y_j$ are the amicable designs in order n.
Let $C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $C_2 = \begin{bmatrix} 1 & 1 \\ 1 & - \end{bmatrix}$, they have the following properties: $C_1C_1^t = I_2, C_2C_2^t = 2I_2$, and $C_1C_2^t = C_2C_1^t$. Construct the matrices

$$P = \sum_{i=1}^{s} (A_i \otimes C_{e_i}) p_i \tag{4}$$

and

$$Q = \sum_{j=1}^{t} (B_j \otimes C_{k_j}) q_j \tag{5}$$

□

Some new amicable orthogonal designs including $AODs(8; 2, 2, 2, 2; 2, 2, 2, 2)$ and $AODs(8; 1, 2, 2, 2; 1, 1, 1, 2)$ of order 8 are found as part of a complete search of the equivalence classes for orthogonal designs. A detailed description about the search method can be found in [11].

Example 3.2 For the $AODs(8; 2, 2, 2, 2; 2, 2, 2, 2)$ given in [11], the corresponding $COD(8; 2, 2, 2, 2; 2, 2, 2, 2)$ is a full(i.e., no zeros) complex orthogonal design which results in no wasted time slots and efficient implementation.

$$\begin{bmatrix}
\frac{s_1}{\sqrt{2}} & \frac{s_1}{\sqrt{2}} & \frac{s_2}{\sqrt{2}} & \frac{s_2}{\sqrt{2}} & \frac{s_3}{\sqrt{2}} & \frac{s_4}{\sqrt{2}} & \frac{s_3}{\sqrt{2}} & \frac{s_4}{\sqrt{2}} \\
\frac{s_1}{\sqrt{2}} & -\frac{s_1}{\sqrt{2}} & \frac{s_2}{\sqrt{2}} & -\frac{s_2}{\sqrt{2}} & \frac{s_4^*}{\sqrt{2}} & -\frac{s_3^*}{\sqrt{2}} & \frac{s_4^*}{\sqrt{2}} & -\frac{s_3^*}{\sqrt{2}} \\
\frac{s_2^*}{\sqrt{2}} & \frac{s_2^*}{\sqrt{2}} & -\frac{s_1^*}{\sqrt{2}} & -\frac{s_1^*}{\sqrt{2}} & \frac{s_3}{\sqrt{2}} & \frac{s_4}{\sqrt{2}} & -\frac{s_3}{\sqrt{2}} & -\frac{s_4}{\sqrt{2}} \\
\frac{s_2^*}{\sqrt{2}} & -\frac{s_2^*}{\sqrt{2}} & -\frac{s_1^*}{\sqrt{2}} & \frac{s_1^*}{\sqrt{2}} & \frac{s_4^*}{\sqrt{2}} & -\frac{s_3^*}{\sqrt{2}} & -\frac{s_4^*}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} \\
\frac{-s_4^R+is_3^I}{\sqrt{2}} & \frac{-s_3^R+is_4^I}{\sqrt{2}} & \frac{-s_4^R+is_3^I}{\sqrt{2}} & \frac{-s_3^R+is_4^I}{\sqrt{2}} & \frac{s_2^R-is_1^I}{\sqrt{2}} & \frac{s_2^R-is_1^I}{\sqrt{2}} & \frac{s_1^R-is_2^I}{\sqrt{2}} & \frac{s_1^R-is_2^I}{\sqrt{2}} \\
\frac{-s_3^R-is_4^I}{\sqrt{2}} & \frac{s_4^R+is_3^I}{\sqrt{2}} & \frac{-s_3^R-is_4^I}{\sqrt{2}} & \frac{s_4^R+is_3^I}{\sqrt{2}} & \frac{s_2^R-is_1^I}{\sqrt{2}} & \frac{-s_2^R+is_1^I}{\sqrt{2}} & \frac{s_1^R-is_2^I}{\sqrt{2}} & \frac{-s_1^R+is_2^I}{\sqrt{2}} \\
\frac{-s_4^R+is_3^I}{\sqrt{2}} & \frac{-s_3^R+is_4^I}{\sqrt{2}} & \frac{s_4^R-is_3^I}{\sqrt{2}} & \frac{s_3^R-is_4^I}{\sqrt{2}} & \frac{s_1^R+is_2^I}{\sqrt{2}} & \frac{s_1^R+is_2^I}{\sqrt{2}} & \frac{-s_2^R-is_1^I}{\sqrt{2}} & \frac{-s_2^R-is_1^I}{\sqrt{2}} \\
\frac{-s_3^R-is_4^I}{\sqrt{2}} & \frac{s_4^R+is_3^I}{\sqrt{2}} & \frac{s_3^R+is_4^I}{\sqrt{2}} & \frac{-s_4^R-is_3^I}{\sqrt{2}} & \frac{s_1^R+is_2^I}{\sqrt{2}} & \frac{-s_1^R-is_2^I}{\sqrt{2}} & \frac{-s_2^R-is_1^I}{\sqrt{2}} & \frac{s_2^R+is_1^I}{\sqrt{2}}
\end{bmatrix}$$

4 Amicable Pairs with Order Eight

Given an $AODs(8; u_1, \cdots, u_s; v_1, \cdots, v_t)$, then the maximum achievable value for $s+t$ is 8 according to Corollary 5.32 in [1]. We summarize all existence and non-existence results of AODs which are of 4-4-variables type, 3-4-variables type or 3-3-variables type in Table 4.1, Table 4.2 and Table 4.3 respectively. In these tables

$\star$ denotes such an AODs is newly constructed (remains undecided in Street's paper [6]),

$\times$ denotes such an AODs does not exist,

$\otimes$ denotes such an AODs is newly found non-existent (remains undecided in Street's paper [6]),

U denotes such an AODs remains unknown,

$\diamond$ denotes such an AODs can be constructed using Equating and Killing variables concept,

C denotes such an AODs can be constructed using Corollary 3.1,

T1 denotes such an AODs can be constructed using Theorem 3.1 (Wolfe),

T2 denotes such an AODs can be constructed using Theorem 3.2 (Street),

T3 denotes such an AODs can be constructed using Theorem 3.3.

S denotes such an AODs is found by exhaustive search.

	1111	1114	1122	2222	1112	1124	1222	1113	1223	1115	1123	1133
1111	C	$\otimes$	$\otimes$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
1114		T2	U	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
1122			T1	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
2222				$\star$S	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
1112					T1	$\times$	$\star$S	$\times$	$\times$	$\times$	$\times$	$\times$
1124						$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
1222							U	$\times$	$\times$	$\times$	$\times$	$\times$
1113								U	$\times$	$\times$	$\times$	$\times$
1223									$\times$	$\times$	$\times$	$\times$
1115										$\times$	$\times$	$\times$
1123											$\otimes$	$\times$
1133												$\times$

Table 4.1 4-4-variables type of AODs

	1111	1114	1122	2222	1112	1124	1222	1113	1223	1123	1115	1133
111	$\diamond$	$\diamond$	U	$\times$	$\diamond$	$\times$	U	U	$\times$	$\times$	$\times$	$\times$
114	$\otimes$	$\diamond$	$\star\diamond$	$\times$	U	$\times$	U	U	$\times$	$\times$	$\times$	$\times$
122	$\otimes$	U	$\diamond$	$\times$	$\diamond$	$\times$	U	U	$\times$	$\times$	$\times$	$\times$
224	$\times$	$\times$	U	C	$\times$	C	U	$\times$	$\times$	$\times$	$\times$	$\times$
112	$\diamond$	U	$\diamond$	$\times$	$\diamond$	$\times$	$\star$S	$\times$	$\times$	$\otimes$	$\times$	$\times$
124	$\otimes$	$\diamond$	U	$\times$	U	$\times$	U	$\times$	$\times$	$\otimes$	$\times$	$\times$
222	$\otimes$	U	$\diamond$	$\star\diamond$	U	U	U	$\times$	$\times$	$\otimes$	$\times$	$\times$
113	$\times$	$\times$	$\times$	$\times$	$\diamond$	$\times$	U	U	$\times$	$\otimes$	$\times$	$\times$
134	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
223	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	U	$\times$	$\times$	$\otimes$	$\times$	$\times$
123	$\otimes$	U	$\diamond$	$\times$	$\times$	$\times$	$\times$	U	$\times$	$\otimes$	$\times$	$\times$
115	$\otimes$	$\diamond$	U	$\times$	$\times$	$\times$	$\times$	$\times$	U	$\otimes$	$\times$	$\times$
133	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	T1	$\times$	$\times$
116	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
125	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
233	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$

Table 4.2 3-4-variables type of AODs

	111	114	122	224	112	124	222	113	223	134	123	115	133	116	125	233
111	$\diamond$	$\diamond$	T3	$\times$	T3	$\diamond$	$\star$T3	$\diamond$	U	$\times$	U	$\diamond$	U	$\times$	$\times$	$\times$
114		T3	$\star$T3	T3	T3	T3	T3	U	U	$\times$	$\diamond$	$\diamond$	U	$\times$	$\times$	U
122			T3	$\star$T3	T3	$\star$T3	T3	$\diamond$	U	$\times$	$\diamond$	U	U	$\times$	$\times$	U
224				T3	T3	T3	T3	$\times$	U	$\diamond$	U	$\times$	U	$\diamond$	$\diamond$	C
112					T3	$\star$T3	T3	$\diamond$	$\star\diamond$	$\times$	$\diamond$	U	$\diamond$	$\times$	$\times$	U
124						T3	$\star$T3	U	U	$\times$	U	$\diamond$	$\diamond$	$\times$	$\times$	U
222							T3	U	U	U	$\diamond$	U	U	U	U	U
113								$\diamond$	U	$\times$	U	U	$\diamond$	$\times$	$\times$	$\times$
223									U	$\times$	U	U	$\diamond$	$\times$	$\times$	U
134										$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
123											$\diamond$	U	$\diamond$	$\times$	$\times$	U
115												$\diamond$	$\diamond$	$\times$	$\times$	$\times$
133													$\diamond$	$\times$	$\times$	$\times$
116														$\times$	$\times$	$\times$
125															$\times$	$\times$
233																U

Table 4.3 3-3-variables type of AODs

It is worth mentioning that amicable orthogonal designs are also important in the construction of orthogonal designs. Using our new designs $AODs(8;2,3;1,1,1,3)$ given in [11], $AODs(8;1,1,2;1,2,2,2)$ and Theorem 5.97 in [1] gives us the following new orthogonal designs in order 32: $OD(32;1,1,2,2,2,2,3,6,6)$, $OD(32;2,2,2,3,7,9)$, $OD(32;1,1,1,1,2,2,2,3,6)$ and $OD(32;1,1,1,2,2,2,3,9)$.

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