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# On Full Orthogonal Designs in Order 56

S. Georgiou\*, C. Koukouvinos\*, and Jennifer Seberry†

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## Abstract

We find new full orthogonal designs in order 56 and show that of 1285 possible  $OD(56; s_1, s_2, s_3, 56 - s_1 - s_2 - s_3)$  163 are known, of 261 possible  $OD(56; s_1, s_2, 56 - s_1 - s_2)$  179 are known. All possible  $OD(56; s_1, 56 - s_1)$  are known.

*Key words and phrases:* Construction, sequences, circulant matrices, amicable sets, orthogonal designs.

*AMS Subject Classification:* Primary 05B15, 05B20, Secondary 62K05.

## 1 Introduction

An *orthogonal design* of order  $n$  and type  $(s_1, s_2, \dots, s_u)$  ( $s_i > 0$ ), denoted  $OD(n; s_1, s_2, \dots, s_u)$ , on the commuting variables  $x_1, x_2, \dots, x_u$  is an  $n \times n$  matrix  $A$  with entries from  $\{0, \pm x_1, \pm x_2, \dots, \pm x_u\}$  such that

$$AA^T = \left( \sum_{i=1}^u s_i x_i^2 \right) I_n$$

Alternatively, the rows of  $A$  are formally orthogonal and each row has precisely  $s_i$  entries of the type  $\pm x_i$ . In [2], where this was first defined, it was mentioned that

$$A^T A = \left( \sum_{i=1}^u s_i x_i^2 \right) I_n$$

and so our alternative description of  $A$  applies equally well to the columns of  $A$ . It was also shown in [2] that  $u \leq \rho(n)$ , where  $\rho(n)$  (Radon's function) is defined by  $\rho(n) = 8c + 2^d$ , when  $n = 2^a b$ ,  $b$  odd,  $a = 4c + d$ ,  $0 \leq d < 4$ .

A weighing matrix  $W = W(n, k)$  is a square matrix with entries  $0, \pm 1$  having  $k$  non-zero entries per row and column and inner product of distinct rows zero. Hence  $W$  satisfies  $WW^T = kI_n$ , and  $W$  is equivalent to an orthogonal design  $OD(n; k)$ . The number  $k$  is called the *weight* of  $W$ . If  $k = n$ , that is, all the entries of  $W$  are  $\pm 1$  and  $WW^T = nI_n$ , then  $W$  is called an Hadamard matrix of order  $n$ . In this case  $n = 1, 2$  or  $n \equiv 0 \pmod{4}$ .

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Given the sequence  $A = \{a_1, a_2, \dots, a_n\}$  of length  $n$  the *non-periodic autocorrelation function*  $N_A(s)$  is defined as

$$N_A(s) = \sum_{i=1}^{n-s} a_i a_{i+s}, \quad s = 0, 1, \dots, n-1, \quad (1)$$

If  $A(z) = a_1 + a_2 z + \dots + a_n z^{n-1}$  is the associated polynomial of the sequence  $A$ , then

$$A(z)A(z^{-1}) = \sum_{i=1}^n \sum_{j=1}^n a_i a_j z^{i-j} = N_A(0) + \sum_{s=1}^{n-1} N_A(s)(z^s + z^{-s}), \quad z \neq 0. \quad (2)$$

Given  $A$  as above of length  $n$  the *periodic autocorrelation function*  $P_A(s)$  is defined, reducing  $i + s$  modulo  $n$ , as

$$P_A(s) = \sum_{i=1}^n a_i a_{i+s}, \quad s = 0, 1, \dots, n-1. \quad (3)$$

The following theorem which uses four circulant matrices in the Goethals-Seidel array is very useful in our construction for orthogonal designs.

**Theorem 1** [3, Theorem 4.49] *Suppose there exist four circulant matrices  $A, B, C, D$  of order  $n$  satisfying*

$$AA^T + BB^T + CC^T + DD^T = fI_n$$

*Let  $R$  be the back diagonal matrix. Then*

$$GS = \begin{pmatrix} A & BR & CR & DR \\ -BR & A & D^T R & -C^T R \\ -CR & -D^T R & A & B^T R \\ -DR & C^T R & -B^T R & A \end{pmatrix}$$

*is a  $W(4n, f)$  when  $A, B, C, D$  are  $(0, 1, -1)$  matrices, and an orthogonal design  $OD(4n; s_1, s_2, \dots, s_u)$  on  $x_1, x_2, \dots, x_u$  when  $A, B, C, D$  have entries from  $\{0, \pm x_1, \dots, \pm x_u\}$  and  $f = \sum_{j=1}^u (s_j x_j^2)$ .  $\square$*

**Corollary 1** *If there are four sequences  $A, B, C, D$  of length  $n$  with entries from  $\{0, \pm x_1, \pm x_2, \pm x_3, \pm x_4\}$  with zero periodic or non-periodic autocorrelation function, then these sequences can be used as the first rows of circulant matrices which can be used in the Goethals-Seidel array to form an  $OD(4n; s_1, s_2, s_3, s_4)$ . We note that if the non-periodic autocorrelation function is zero, then there are sequences of length  $n + m$  for all  $m \geq 0$ .  $\square$*

This method for constructing orthogonal designs was used in [1, 5].

Throughout this paper we will use the definition and notation of Koukouvinos, Mitrouli, Seberry and Karabelas [5].

A pair of matrices  $A, B$  is said to be amicable (anti-amicable) if  $AB^T - BA^T = 0$  ( $AB^T + BA^T = 0$ ). Following [7] a set  $\{A_1, A_2, \dots, A_{2n}\}$  of square real matrices is said to be *amicable* if

$$\sum_{i=1}^n \left( A_{\sigma(2i-1)} A_{\sigma(2i)}^T - A_{\sigma(2i)} A_{\sigma(2i-1)}^T \right) = 0 \quad (4)$$

for some permutation  $\sigma$  of the set  $\{1, 2, \dots, 2n\}$ . For simplicity, we will always take  $\sigma(i) = i$  unless otherwise specified. So

$$\sum_{i=1}^n (A_{2i-1}A_{2i}^T - A_{2i}A_{2i-1}^T) = 0. \quad (5)$$

Clearly a set of mutually amicable matrices is amicable, but the converse is not true in general. Throughout this paper  $R_k$  denotes the back diagonal identity matrix of order  $k$ .

Let  $\{A_i\}_{i=1}^8$  be an amicable set of circulant matrices of order  $t$ , satisfying the additive property for  $(s_1, s_2, \dots, s_k)$ . Then the Kharaghani array

$$H = \begin{pmatrix} A_1 & A_2 & A_4 R_n & A_3 R_n & A_6 R_n & A_5 R_n & A_8 R_n & A_7 R_n \\ -A_2 & A_1 & A_3 R_n & -A_4 R_n & A_5 R_n & -A_6 R_n & A_7 R_n & -A_8 R_n \\ -A_4 R_n & -A_3 R_n & A_1 & A_2 & -A_8^T R_n & A_7^T R_n & A_6^T R_n & -A_5^T R_n \\ -A_3 R_n & A_4 R_n & -A_2 & A_1 & A_7^T R_n & A_8^T R_n & -A_5^T R_n & -A_6^T R_n \\ -A_6 R_n & -A_5 R_n & A_8^T R_n & -A_7^T R_n & A_1 & A_2 & -A_4^T R_n & A_3^T R_n \\ -A_5 R_n & A_6 R_n & -A_7^T R_n & -A_8^T R_n & -A_2 & A_1 & A_3^T R_n & A_4^T R_n \\ -A_8 R_n & -A_7 R_n & -A_6^T R_n & A_5^T R_n & A_4^T R_n & -A_3^T R_n & A_1 & A_2 \\ -A_7 R_n & A_8 R_n & A_5^T R_n & A_6^T R_n & -A_3^T R_n & -A_4^T R_n & -A_2 & A_1 \end{pmatrix}$$

is an  $OD(8t; s_1, s_2, \dots, s_k)$ .

The Kharaghani array which uses amicable sets of eight matrices is also very useful in our constructions for orthogonal designs.

The following lemma applies a lemma given in Georgiou, Koukouvinos, Mitrouli and Seberry [1] to determine the number of possible tuples to be searched determining the size of search space for orthogonal designs in order 56.

**Lemma 1** *Let  $n = 4m = 56$  be the order of an orthogonal design then the number of cases which must be studied to determine whether all orthogonal designs exist is*

- (i)  $\frac{1}{4}n^2 = 784$  when 2-tuples are considered;
- (ii)  $\frac{n-2}{72}(2n^2 + 7n + 6) = 5004$  when 3-tuples are considered;
- (iii)  $\frac{1}{576}(n^4 + 6n^3 - 2n^2 - 24n + 64) = 18890$  when 4-tuples are considered.

## 2 New full orthogonal designs from smaller orders

**Theorem 2** *There are  $OD(56; s_1, s_1, 65-s_1, 56-s_1)$  constructed using the full  $OD(28; s_1, 28-s_1)$  given in [2, 5, 6] for:*

$$\begin{array}{cccc} (1, 1, 27, 27) & (5, 5, 23, 23) & (9, 9, 19, 19) & (13, 13, 15, 15) \\ (2, 2, 26, 26) & (6, 6, 22, 22) & (10, 10, 18, 18) & (14, 14, 14, 14) \\ (3, 3, 25, 25) & (7, 7, 21, 21) & (11, 11, 17, 17) & \\ (4, 4, 24, 24) & (8, 8, 20, 20) & (12, 12, 16, 16) & \end{array}$$

**Proof.** We use the amicable orthogonal designs of type  $AOD(2; (1, 1), (1, 1))$  in order two with the two variable designs in order 28 to obtain the desired designs in order 56.  $\square$

**Theorem 3** *There are full  $OD(56; s_1, s_2, s_3, 56 - s_1 - s_2 - s_3)$  constructed using the full  $OD(28; s_1, s_2, 28 - s_1 - s_2)$  and  $OD(28; s_1, s_2, s_3, 28 - s_1 - s_2 - s_3)$  designs in order 28 for the 4-tuples given in Table 2*

(1, 1, 2, 52)	(2, 2, 13, 39)	(2, 13, 13, 28)	(4, 8, 22, 22)	(7, 14, 14, 21)
(1, 1, 4, 50)	(2, 2, 14, 38)	(2, 13, 15, 26)	(4, 9, 9, 34)	(8, 8, 8, 32)
(1, 1, 6, 48)	(2, 2, 16, 36)	(2, 14, 14, 26)	(4, 12, 20, 20)	(8, 8, 10, 30)
(1, 1, 12, 42)	(2, 2, 18, 34)	(2, 16, 18, 20)	(4, 13, 13, 26)	(8, 8, 16, 24)
(1, 1, 16, 38)	(2, 2, 25, 27)	(2, 16, 19, 19)	(4, 14, 19, 19)	(8, 8, 18, 22)
(1, 1, 18, 36)	(2, 2, 26, 26)	(2, 18, 18, 18)	(4, 16, 18, 18)	(8, 8, 20, 20)
(1, 1, 26, 28)	(2, 3, 3, 48)	(3, 3, 12, 38)	(4, 17, 17, 18)	(8, 10, 10, 28)
(1, 2, 2, 51)	(2, 3, 12, 39)	(3, 3, 14, 36)	(5, 5, 10, 36)	(8, 10, 18, 20)
(1, 2, 3, 50)	(2, 3, 15, 36)	(3, 3, 20, 30)	(5, 5, 18, 28)	(8, 12, 18, 18)
(1, 2, 16, 37)	(2, 4, 25, 25)	(3, 5, 12, 36)	(5, 10, 18, 23)	(8, 14, 14, 20)
(1, 2, 17, 36)	(2, 6, 6, 42)	(4, 4, 4, 44)	(5, 15, 18, 18)	(8, 16, 16, 16)
(1, 2, 26, 27)	(2, 6, 12, 36)	(4, 4, 8, 40)	(6, 6, 6, 38)	(9, 9, 10, 28)
(1, 3, 16, 36)	(2, 6, 18, 30)	(4, 4, 12, 36)	(6, 6, 8, 36)	(9, 9, 18, 20)
(1, 3, 26, 26)	(2, 6, 24, 24)	(4, 4, 16, 32)	(6, 7, 7, 36)	(9, 10, 10, 27)
(1, 6, 12, 37)	(2, 8, 8, 38)	(4, 4, 20, 28)	(6, 10, 10, 30)	(9, 10, 18, 19)
(1, 6, 13, 36)	(2, 8, 10, 36)	(4, 7, 7, 38)	(6, 12, 18, 20)	(9, 11, 18, 18)
(1, 7, 12, 36)	(2, 9, 9, 36)	(4, 8, 8, 36)	(6, 12, 19, 19)	(10, 10, 16, 20)
(1, 18, 18, 19)	(2, 9, 18, 27)	(4, 8, 12, 32)	(6, 14, 18, 18)	(10, 10, 18, 18)
(2, 2, 2, 50)	(2, 12, 18, 24)	(4, 8, 18, 26)	(6, 15, 15, 20)	(10, 14, 14, 18)
(2, 2, 8, 44)	(2, 12, 21, 21)	(4, 8, 20, 24)	(7, 7, 14, 28)	(14, 14, 14, 14)

Table 1: Full 4-variable  $OD(56; s_1, s_2, s_3, 56 - s_1 - s_2 - s_3)$  constructed from full three and four variable designs in order 28.

**Theorem 4** *There are  $OD(56; s_1, s_1, 2s_2, 2s_3, 56 - 2s_1 - 2s_2 - 2s_3)$  constructed using the Multiplication Theorem [3, Lemma 4.11] with the full  $OD(28; s_1, s_2, s_3, 28 - s_1 - s_2 - s_3)$  given in [2, 5, 6] for the values given in Table 2.*

(1, 1, 2, 2, 50)	(2, 2, 8, 8, 36)	(2, 9, 9, 18, 18)	(7, 7, 14, 14, 14)
(1, 1, 2, 16, 36)	(2, 2, 13, 13, 26)	(4, 4, 4, 8, 36)	(8, 8, 8, 16, 16)
(1, 1, 2, 26, 26)	(2, 2, 16, 18, 18)	(4, 4, 8, 8, 32)	(8, 8, 10, 10, 20)
(1, 1, 6, 12, 36)	(2, 3, 3, 12, 36)	(4, 4, 8, 20, 20)	(9, 9, 10, 10, 18)
(1, 1, 18, 18, 18)	(2, 6, 6, 6, 36)	(4, 8, 8, 18, 18)	
(2, 2, 2, 25, 25)	(2, 6, 12, 18, 18)	(5, 5, 10, 18, 18)	

Table 2: Full 5-variable designs in order 56 from full 4-variable designs in order 28.

In table 3 we present the new amicable sets of eight matrices which can be used in the Kharaghani array to construct some new full orthogonal designs in order 56.

Type	$A_1$ $A_3$ $A_5$ $A_7$	$A_2$ $A_4$ $A_6$ $A_8$	ZERO
(1,1,25,29)	$(a, d, d, \bar{d}, d, \bar{d}, \bar{d})$ $(b, d, d, \bar{d}, d, \bar{d}, \bar{d})$ $(\bar{d}, d, d, d, d, d, d)$ $(b, b, b, \bar{b}, b, \bar{b}, \bar{b})$	$(\bar{b}, b, b, b, b, b, b)$ $(b, d, d, \bar{d}, d, \bar{d}, \bar{d})$ $(c, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(b, b, b, \bar{b}, b, \bar{b}, \bar{b})$	PAF n=7
(1,2,3,25,25)	$(a, d, d, \bar{d}, d, \bar{d}, \bar{d})$ $(a, d, d, \bar{d}, d, \bar{d}, \bar{d})$ $(\bar{d}, d, d, d, d, d, d)$ $(e, h, h, \bar{h}, h, \bar{h}, \bar{h})$	$(a, d, d, \bar{d}, d, \bar{d}, \bar{d})$ $(\bar{h}, h, h, h, h, h, h)$ $(g, h, h, \bar{h}, h, \bar{h}, \bar{h})$ $(e, h, h, \bar{h}, h, \bar{h}, \bar{h})$	PAF n=7
(1,2,8,45)	$(\bar{a}, b, b, a, b, a, a)$ $(a, b, b, \bar{a}, b, \bar{a}, \bar{a})$ $(d, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(c, b, b, \bar{b}, b, \bar{b}, \bar{b})$	$(b, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(b, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(d, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(\bar{b}, b, b, b, b, b, b)$	PAF n=7
(1,2,13,40)	$(\bar{a}, b, b, a, b, a, a)$ $(\bar{b}, \bar{a}, \bar{a}, b, \bar{a}, b, b)$ $(c, a, a, \bar{a}, a, \bar{a}, \bar{a})$ $(d, b, b, \bar{b}, b, \bar{b}, \bar{b})$	$(a, a, a, \bar{a}, a, \bar{a}, \bar{a})$ $(a, a, a, \bar{a}, a, \bar{a}, \bar{a})$ $(c, a, a, \bar{a}, a, \bar{a}, \bar{a})$ $(\bar{a}, a, a, a, a, a, a)$	PAF n=7
(1,2,14,39)	$(a, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(d, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(c, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(\bar{b}, \bar{a}, \bar{a}, b, \bar{a}, b, b)$	$(d, a, a, \bar{a}, a, \bar{a}, \bar{a})$ $(b, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(\bar{b}, b, b, b, b, b, b)$ $(\bar{a}, b, b, a, b, a, a)$	PAF n=7
(1,2,19,34)	$(\bar{a}, b, b, a, b, a, a)$ $(c, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(d, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(b, a, a, \bar{a}, a, \bar{a}, \bar{a})$	$(a, b, b, \bar{a}, b, \bar{a}, \bar{a})$ $(a, a, a, \bar{a}, a, \bar{a}, \bar{a})$ $(\bar{a}, a, a, a, a, a, a)$ $(c, a, a, \bar{a}, a, \bar{a}, \bar{a})$	PAF n=7
(1,3,8,19,25)	$(\bar{a}, b, b, a, b, a, a)$ $(a, b, b, \bar{a}, b, \bar{a}, \bar{a})$ $(e, h, h, \bar{h}, h, \bar{h}, \bar{h})$ $(d, b, b, \bar{b}, b, \bar{b}, \bar{b})$	$(e, h, h, \bar{h}, h, \bar{h}, \bar{h})$ $(e, h, h, \bar{h}, h, \bar{h}, \bar{h})$ $(b, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(\bar{h}, h, h, h, h, h, h)$	PAF n=7
(1,3,13,14,25)	$(a, d, d, \bar{d}, d, \bar{d}, \bar{d})$ $(\bar{f}, \bar{e}, \bar{e}, f, \bar{e}, f, f)$ $(f, f, f, \bar{f}, f, \bar{f}, \bar{f})$ $(\bar{d}, d, d, d, d, d, d)$	$(\bar{e}, f, f, e, f, e, e)$ $(a, d, d, \bar{d}, d, \bar{d}, \bar{d})$ $(a, d, d, \bar{d}, d, \bar{d}, \bar{d})$ $(g, e, e, \bar{e}, e, \bar{e}, \bar{e})$	PAF n=7
(1,10,18,27)	$(a, d, d, \bar{d}, d, \bar{d}, \bar{d})$ $(a, d, d, \bar{d}, d, \bar{d}, \bar{d})$ $(c, d, d, \bar{d}, d, \bar{d}, \bar{d})$ $(\bar{a}, b, b, a, b, a, a)$	$(d, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(d, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(\bar{d}, d, d, d, d, d, d)$ $(a, b, b, \bar{a}, b, \bar{a}, \bar{a})$	PAF n=7

Table 3: New full orthogonal designs in order 56 constructed from new amicable sets of eight matrices.

Type	$A_1$ $A_3$ $A_5$ $A_7$	$A_2$ $A_4$ $A_6$ $A_8$	ZERO
(1,14,14,27)	$(a, d, d, \bar{d}, d, \bar{d}, \bar{d})$ $(b, d, d, \bar{d}, d, \bar{d}, \bar{d})$ $(c, d, d, \bar{d}, d, \bar{d}, \bar{d})$ $(\bar{b}, \bar{a}, \bar{a}, b, \bar{a}, b, b)$	$(d, a, a, \bar{a}, a, \bar{a}, \bar{a})$ $(d, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(\bar{d}, d, d, d, d, d, d)$ $(\bar{a}, b, b, a, b, a, a)$	PAF n=7
(1,20,35)	$(\bar{a}, a, a, a, a, a, a)$ $(a, a, a, \bar{a}, a, \bar{a}, \bar{a})$ $(a, b, b, \bar{a}, b, \bar{a}, \bar{a})$ $(b, b, b, \bar{b}, b, \bar{b}, \bar{b})$	$(d, a, a, \bar{a}, a, \bar{a}, \bar{a})$ $(\bar{a}, b, b, a, b, a, a)$ $(a, a, a, \bar{a}, a, \bar{a}, \bar{a})$ $(b, b, b, \bar{b}, b, \bar{b}, \bar{b})$	PAF n=7
(2,2,8,8,18,18)	$(\bar{a}, b, b, a, b, a, a)$ $(a, b, b, \bar{a}, b, \bar{a}, \bar{a})$ $(d, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(h, f, f, \bar{f}, f, \bar{f}, \bar{f})$	$(\bar{e}, f, f, e, f, e, e)$ $(e, f, f, \bar{e}, f, \bar{e}, \bar{e})$ $(d, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(h, f, f, \bar{f}, f, \bar{f}, \bar{f})$	PAF n=7
(2,4,22,28)	$(\bar{a}, a, a, a, \bar{a}, a, a)$ $(f, e, h, \bar{h}, \bar{h}, h, h)$ $(a, \bar{a}, a, a, a, \bar{a}, \bar{a})$ $(a, a, \bar{a}, a, \bar{a}, a, a)$	$(\bar{f}, h, h, h, \bar{h}, h, h)$ $(a, a, a, \bar{a}, \bar{a}, a, a)$ $(f, \bar{e}, h, h, h, \bar{h}, \bar{h})$ $(f, h, \bar{h}, h, \bar{h}, h, h)$	PAF n=7
(3,22,31)	$(\bar{a}, b, b, a, b, a, a)$ $(a, b, b, \bar{a}, b, \bar{a}, \bar{a})$ $(d, a, a, \bar{a}, a, \bar{a}, \bar{a})$ $(\bar{a}, b, b, a, b, a, a)$	$(d, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(d, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(a, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(\bar{b}, \bar{a}, \bar{a}, b, \bar{a}, b, b)$	PAF n=7
(4,4,4,4,10,10,10,10)	$(b, c, a, c, d, d, \bar{d})$ $(b, \bar{c}, a, \bar{c}, \bar{d}, \bar{d}, d)$ $(b, d, \bar{a}, d, \bar{c}, \bar{c}, c)$ $(b, \bar{d}, \bar{a}, \bar{d}, c, c, \bar{c})$	$(f, g, e, g, h, h, \bar{h})$ $(f, \bar{g}, e, \bar{g}, \bar{h}, \bar{h}, h)$ $(f, \bar{h}, \bar{e}, \bar{h}, g, g, \bar{g})$ $(f, h, \bar{e}, h, \bar{g}, \bar{g}, g)$	NPAF n=7
(4,6,46)	$(c, \bar{c}, \bar{c}, c, c, b, \bar{a})$ $(c, \bar{c}, c, c, c, \bar{c}, c)$ $(\bar{c}, c, \bar{c}, c, a, b, c)$ $(\bar{c}, c, c, c, c, b, \bar{c})$	$(\bar{c}, c, \bar{c}, c, a, b, c)$ $(\bar{c}, c, c, c, c, b, \bar{c})$ $(c, \bar{c}, \bar{c}, c, c, b, \bar{a})$ $(c, \bar{c}, c, c, c, \bar{c}, c)$	PAF n=7
(4,7,21,24)	$(a, a, \bar{a}, a, a, a, d)$ $(f, f, \bar{f}, f, \bar{e}, f, \bar{f})$ $(f, f, \bar{f}, f, \bar{e}, \bar{f}, f)$ $(\bar{d}, \bar{d}, d, a, \bar{a}, a, a)$	$(f, f, \bar{f}, f, e, f, f)$ $(\bar{a}, \bar{a}, a, a, d, a, \bar{a})$ $(\bar{a}, \bar{a}, a, \bar{d}, a, \bar{d}, a)$ $(f, f, \bar{f}, \bar{f}, e, \bar{f}, \bar{f})$	NPAF n=7
(7,7,7,7,7,7,7,7)	$(\bar{a}, a, a, g, a, e, c)$ $(\bar{g}, g, g, \bar{a}, g, c, \bar{e})$ $(\bar{e}, e, e, \bar{c}, e, \bar{a}, g)$ $(\bar{b}, b, b, d, b, f, \bar{h})$	$(\bar{f}, f, f, \bar{h}, f, b, \bar{d})$ $(\bar{h}, h, h, f, h, d, b)$ $(\bar{d}, d, d, \bar{b}, d, \bar{h}, f)$ $(\bar{c}, c, c, e, c, \bar{g}, \bar{a})$	NPAF n=7

Table 3 (cont.)

Type	$A_1$	$A_2$	ZERO
	$A_3$	$A_4$	
	$A_5$	$A_6$	
	$A_7$	$A_8$	
(7,7,18,24)	$(a, a, \bar{a}, a, c, a, d)$ $(b, b, \bar{b}, \bar{b}, a, \bar{b}, \bar{b})$ $(b, b, \bar{b}, \bar{b}, \bar{a}, \bar{b}, b)$ $(b, b, \bar{b}, b, a, b, b)$	$(b, b, \bar{b}, b, \bar{a}, b, \bar{b})$ $(\bar{a}, \bar{a}, a, a, d, a, \bar{c})$ $(\bar{c}, \bar{c}, c, \bar{d}, a, \bar{d}, a)$ $(\bar{d}, \bar{d}, d, c, \bar{a}, c, a)$	NPAF n=7
(8,11,37)	$(\bar{a}, b, b, a, b, a, a)$ $(a, b, b, \bar{a}, b, \bar{a}, \bar{a})$ $(\bar{b}, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(c, b, b, \bar{c}, b, \bar{c}, \bar{c})$	$(c, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(c, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(c, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(\bar{c}, b, b, c, b, c, c)$	PAF n=7
(11,14,31)	$(\bar{a}, b, b, a, b, a, a)$ $(c, a, a, \bar{a}, a, \bar{a}, \bar{a})$ $(\bar{c}, b, b, c, b, c, c)$ $(c, b, b, \bar{c}, b, \bar{c}, \bar{c})$	$(\bar{b}, \bar{a}, \bar{a}, b, \bar{a}b, b)$ $(a, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(c, b, b, \bar{b}, b, \bar{b}, \bar{b})$ $(c, b, b, \bar{b}, b, \bar{b}, \bar{b})$	PAF n=7

Table 3 (cont.)

**Remark 1** We note that amicable sets of eight matrices of type  $(4, 4, 4, 4, 10, 10, 10, 10)$  and  $(7, 7, 7, 7, 7, 7, 7, 7)$  which are used for constructing OD's in order 56 are also found in [4].

(1, 2, 3, 50)	(1, 14, 16, 25)	(2, 18, 18, 18)	(4, 4, 20, 28)	(4, 12, 20, 20)	(8, 10, 14, 24)
(1, 2, 25, 28)	(2, 2, 8, 44)	(3, 3, 25, 25)	(4, 4, 24, 24)	(4, 13, 14, 25)	(8, 10, 18, 20)
(1, 3, 8, 44)	(2, 2, 16, 36)	(3, 8, 19, 26)	(4, 8, 8, 36)	(4, 14, 14, 24)	(8, 12, 18, 18)
(1, 3, 13, 39)	(2, 2, 18, 34)	(3, 8, 20, 25)	(4, 8, 10, 34)	(4, 14, 18, 20)	(8, 14, 14, 20)
(1, 3, 14, 38)	(2, 2, 26, 26)	(3, 9, 19, 25)	(4, 8, 14, 30)	(4, 16, 18, 18)	(10, 10, 10, 26)
(1, 3, 19, 33)	(2, 3, 25, 26)	(3, 13, 14, 26)	(4, 8, 18, 26)	(7, 7, 7, 35)	(10, 10, 12, 24)
(1, 3, 25, 27)	(2, 4, 25, 25)	(3, 13, 15, 25)	(4, 8, 19, 25)	(7, 7, 14, 28)	(10, 10, 14, 22)
(1, 5, 25, 25)	(2, 8, 8, 38)	(3, 14, 14, 25)	(4, 8, 20, 24)	(7, 7, 21, 21)	(10, 10, 16, 20)
(1, 8, 19, 28)	(2, 8, 10, 36)	(4, 4, 4, 44)	(4, 10, 10, 32)	(7, 14, 14, 21)	(10, 10, 18, 18)
(1, 8, 22, 25)	(2, 8, 18, 28)	(4, 4, 8, 40)	(4, 10, 12, 30)	(8, 8, 10, 30)	(10, 12, 14, 20)
(1, 11, 19, 25)	(2, 8, 20, 26)	(4, 4, 10, 38)	(4, 10, 14, 28)	(8, 8, 18, 22)	(10, 14, 14, 18)
(1, 13, 14, 28)	(2, 10, 18, 26)	(4, 4, 14, 34)	(4, 10, 18, 24)	(8, 8, 20, 20)	(14, 14, 14, 14)
(1, 13, 17, 25)	(2, 16, 18, 20)	(4, 4, 18, 30)	(4, 10, 20, 22)	(8, 10, 10, 28)	

Table 4: Full 4-variable  $OD(56; s_1, s_2, s_3, 56 - s_1 - s_2 - s_3)$  constructed from full designs presented in table 2.

### 3 Full designs with even parameters

We note that Seberry [8] showed that if all  $OD(n; x, y, n-x-y)$  exist then all  $OD(2n; z, w, 2n-z-w)$  exist for  $s \geq 0$  an integer. In particular if all  $OD(2^t p; x, y, 2^t p - x - y)$  exist, for some odd integer  $p$ , then all  $OD(2^{t+s} p; z, w, 2^{t+s} p - z - w)$  exist for  $s \geq 0$  an integer we observe

**Lemma 2** *If all  $OD(2^t p; 2x, 2y, 2^t p - 2x - 2y)$  exist, for some odd integer  $p$ , then all  $OD(2^{t+s} p; 2z, 2w, 2^{t+s} p - 2z - 2w)$  exist for  $s \geq 0$  an integer.*

$s_1, s_2, s_3$	$s_1, s_2, s_3$	$s_1, s_2, s_3$	$s_1, s_2, s_3$	$s_1, s_2, s_3$	$s_1, s_2, s_3$
(1, 9, 46)	(3, 24, 29)	(5, 19, 32)	(7, 15, 34)	(9, 15, 32)	(11, 22, 23)
(1, 23, 32)	(4, 5, 47)	(5, 20, 31)	(7, 16, 33)	(9, 16, 31)	(12, 13, 31)
(1, 24, 31)	(4, 11, 41)	(5, 21, 30)	(7, 17, 32)	(9, 17, 30)	(12, 15, 29)
(2, 5, 49)	(4, 15, 37)	(5, 22, 29)	(7, 19, 30)	(9, 21, 26)	(12, 17, 27)
(2, 7, 47)	(4, 23, 29)	(5, 24, 27)	(7, 20, 29)	(9, 23, 24)	(13, 16, 27)
(2, 11, 43)	(5, 6, 45)	(6, 9, 41)	(7, 22, 27)	(10, 11, 35)	(13, 19, 24)
(2, 23, 31)	(5, 7, 44)	(6, 11, 39)	(7, 23, 26)	(10, 13, 33)	(13, 20, 23)
(3, 4, 49)	(5, 8, 43)	(6, 16, 34)	(8, 9, 39)	(10, 15, 31)	(13, 21, 22)
(3, 6, 47)	(5, 9, 42)	(6, 17, 33)	(8, 13, 35)	(10, 17, 29)	(15, 17, 24)
(3, 7, 46)	(5, 11, 40)	(6, 21, 29)	(8, 15, 33)	(10, 21, 25)	(15, 19, 22)
(3, 10, 43)	(5, 13, 38)	(6, 23, 27)	(8, 17, 31)	(11, 12, 33)	(16, 17, 23)
(3, 11, 42)	(5, 14, 37)	(7, 8, 41)	(8, 21, 27)	(11, 13, 32)	(17, 19, 20)
(3, 21, 32)	(5, 16, 35)	(7, 9, 40)	(9, 12, 35)	(11, 15, 30)	
(3, 22, 31)	(5, 17, 34)	(7, 10, 39)	(9, 14, 33)	(11, 16, 29)	

Table 5: The existence of these 82 full  $OD(56; s_1, s_2, 72 - s_1 - s_2)$  is not yet established.

**Corollary 2** *If  $OD(56; 6, 16, 34)$  exist then all  $OD(2^{s+3}7; 2z, 2w, 2^{s+3}7 - 2z - 2w)$  exist for  $s \geq 0$  an integer.*

**Proof.** A search of full  $OD(56; x, y, 56 - x - y)$  show only the parameters indicated are as yet unsolved.  $\square$

## 4 Summary

We have found new designs in order 56 and shown that of 1285 possible  $OD(56; s_1, s_2, s_3, 56 - s_1 - s_2 - s_3)$  163 are known: of 261 possible  $OD(56; s_1, s_2, 56 - s_1 - s_2)$  179 are known; and all possible  $OD(56; s_1, 56 - s_1)$  are known.

## References

- [1] S. Georgiou, C. Koukouvinos, M. Mitrouli and J. Seberry, Necessary and sufficient conditions for three and four variable orthogonal designs in order 36, *J. Statist. Plann. Inference*, (to appear).
- [2] A.V.Geramita, J.M.Geramita, and J.Seberry Wallis, Orthogonal designs, *Linear and Multilinear Algebra*, 3 (1976), 281-306.
- [3] A.V.Geramita, and J.Seberry, *Orthogonal Designs: Quadratic Forms and Hadamard Matrices*, Marcel Dekker, New York-Basel, 1979.
- [4] W.H. Holzmann, and H. Kharaghani, On the Plotkin arrays, *Australas. J. Combin.*, to appear.
- [5] C.Koukouvinos, M.Mitrouli, J.Seberry, and P.Karabelas, On sufficient conditions for some orthogonal designs and sequences with zero autocorrelation function, *Australas. J. Combin.*, 13 (1996), 197-216.
- [6] C. Koukouvinos and J. Seberry, New orthogonal designs and sequences with two and three variables in order 28, *Ars Combinatoria*, 54 (2000), 97-108.

- [7] H. Kharaghani, Arrays for orthogonal designs, *J. Combin. Designs*, to appear.
- [8] J. Seberry Wallis, On the existence of Hadamard matrices, *J. Combin. Theory Ser. A*, 21 (1976), 188-195.