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EFFICIENCY AND EQUITY CONSIDERATIONS IN ALLOCATING REMUNERATIONS UNDER UNCERTAINTY: THE CASE OF AUSTRALIAN UNIVERSITIES

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ABSTRACT

By considering the heterogeneity of the labour force, uncertainty about performances, and risk and inequality aversion, and by applying a skill-wise optimal-control method, an efficient rule for allocating salary loadings in a regulated place of employment is derived. This rule indicates that salary loadings may vary monotonically, but not necessarily increase, with skill. Only when the marginal costs of risk-bearing and inequality are smaller than the expected marginal performance, loadings should monotonically increase with skill.

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1. INTRODUCTION

Following the decision of the Australian Industrial Relations Commission to change the Academic Salaries Award from paid-rates to a minimum-rates awards, many universities in Australia have offered a range of salary loadings in order to attract and retain academic staff members. Under the prevailing practice, salary loadings have not been available to all, and in many cases the criterion for their receipt has been neither clear nor academically sound. While some academic staff have become better off, the distribution of salary loadings has not been a zero-sum game due to the existence of negative externalities and deadweight loss stemming from inequality and from sub-efficient allocation of limited financial resources. Consequently, the policy of offering salary loadings has been opposed by many union and non-union academic members. However, the rigidities inherent in the academic-salary system and the substantial 'brain-drain' process generated by the relative decline in the level of academic salaries during the last two decades (Marginson, 1991) imply that a remuneration scheme might increase the degree of flexibility in rewarding academic staff and, if appropriately designed and implemented, such a scheme might also improve the composition of the academic staff.

The purpose of this paper is to construct conceptually a salary-loading rule which does not entail the aforementioned deficiencies and which enables a university to attract and retain a better pool of academic staff. The design of a rule for allocating salary loadings is a classical economic problem. It has been shown by Arrow (1963) that any group-ordering with the desirable properties of being complete, reflexive, transitive, coinciding with unanimous preferences, and responsive to people's ranking of any two possible allocations irrespective of their ranking of alternative allocations, is inconsistent with democracy. The impossibility of aggregating individual preferences in a perfect way implies that there is no *ideal rule* for distributing remuneration. In view of this impossibility, the present paper develops an *efficient rule* for allocating remuneration within a group of workers in which skill is unequally and continuously distributed.

The loading-allocation rule developed in this paper is efficient in the sense that it is derived within a framework of a skill-wise optimal-control problem in which eligibility is not limited and remuneration serves as an instrument for attracting and retaining a pool of academic staff whose overall academic performance is the highest for a given degree of egalitarianism. The conceptual framework incorporates the problem of uncertainty about future performance of academic staff members and the associated costs of risk-bearing for the institution. It also takes into account the effect of income differentials between academics and alternative places of employment on a university's ability to attract and retain academic staff.

2. CONCEPTUAL FRAMEWORK AND THE OPTIMAL-CONTROL PROBLEM

The conceptual framework described below is a generic one and can be applied to other enterprises with few modifications. It regards academic performance as the quality and quantity of academic product supplied within a given period, and defines academic skill as the ability to fulfil academic tasks. Since academic product is a composit and abstract good, which includes the yields on teaching, research and administrative activities, there are considerable problems in assessing and measuring academic skill and academic performance. Consequently, the application of the derived rule for efficient allocation of remuneration requires that universities ought to assign weights to the various components of the academic product and ought to define the measuring methods of those components. Moreover, since the distribution of academic skill changes with the recruitment of new staff, learning by doing and on the job training, the assessment of the distribution of skill within the academic staff should be revised periodically.

Let h denote a continous index of academic skill lying within the unit interval (0, 1), where 0 and 1 indicate, in relative terms, the lowest and highest levels of academic skill, respectively. It is assumed that the academic skill level of each member of the academic staff is measured at the beginning of the planning period and that the academic performance of each member during that period is a function of his/her academic skill level as described by the following stochastic equation:

$$v(h) = \mu h - \sigma z(h). \qquad (1)$$

Here, μ is a positive parameter denoting the expected academic performance of a member endowed with the highest level of skill; μh is the expected academic performance of a member endowed with a fraction h of the highest skill level; σ is the standard-deviation coefficient associated with a Brownian motion (Wiener) process within the skill interval, z(h). This process combines the effects of excluded factors such as changes in skill, effort and workplace's environment during the period, as well as the effects of the state of the world and luck, on a member's performance. Moreover, this Wiener process indicates that the level of uncertainty about a member's performance incerases with the member's skill level. The underlying rationale is that a more skillful member is likely to be either assigned to, or voluntarily engaged in, more difficult and challenging academic activities and tasks.

Equation 1 implies that the academic performance of a member of an academic staff endowed with academic skill level h is normally distributed with mean

$$E[v(h)] = \mu h \tag{2}$$

and variance

$$Var[v(h)] = \sigma^2 h. \tag{3}$$

Let N denote the size of the academic labour force and p(h) the density function of skill within the entire academic labour force. Then the number of people endowed with academic skill h is given by:

$$n(h) = p(h)N. \tag{4}$$

In accordance with Todaro's (1969) hypothesis it is assumed that the share of the labour force with skill level h attracted to universities, s, increases with the expected income differential (after loadings) between the academics and alternative places of employment. Therefore, the number of people endowed with skill level h attracted to academics, m(h), is given by the supply function:

$$m(h) = s(wh + \ell(h))n(h) \tag{5}$$

where s is a function of the aforementioned expected income differential with $s'>0$ indicating the mobility of labour, w is the expected salary (before loadings) differential between the academics and the alternative places of employment for the most skillful people, wh is the expected salary differential for a person endowed with a fraction h of the highest skill level, and $\ell(h)$ is the salary loadings offered by universities to individuals endowed with skill level h. When w is negative and substantially large in absolute terms and exceeds the nonpecuniary benefits associated with academic employment, loadings might serve as an efficient instrument to alleviate the 'brain-drain' problem. Since it is in the interest of universities to set $\ell(h)$ as low as possible, there is no excess supply of labour at any level of h and hence m(h) is equal to the number of people with skill level h employed by the university. In order to obtain a solution to the following optimal-control problem it is assumed that s, and subsequently m, is a concave function of $\ell(h)$.

Let the university's benefits from the academic performance of an individual endowed with skill level h be given by 1-*exp*{-2Rv(h)} and recall that v(h) is normally distributed. Then, the expected benefits from v(h) can be rendered as:

$$u[v(h)] = exp\,\{E[v(h)] - RVar[v(h)]\}. \tag{6}$$

(See Freund, 1956, for details, and Hammond, 1974, for general properties.)

In this context, R denotes the institution's degree of risk aversion and RVar [v(h)] indicates the costs of risk bearing associated with the uncertain future performance of individuals endowed with a skill level h. It is postulated that universities distribute a predetermined amount of money as salary loadings for attracting and retaining a pool of academic staff whose aggregate risk-adjusted academic performance is the highest for a given rate of equality preference ϕ. A university's degree of egalitarianism may reflect its managers' rate of substitution between efficiency of performance and equality as well as the membership structure and bargaining power of the academic staff's trade union. It is likely that ϕ receives a relatively large value when the academic staff union has a substantial bargaining power and is dominated by low-rank members and there is a positive correlation between rank and skill.

Consequently, the efficient distribution of salary loadings can be found by solving the following optimal-control problem defined over the finite skill interval (0,1):

$$\underset{\{\ell(h)\}}{\text{maximising}} \int_0^1 e^{-\phi h}\, u[v(h)]m(h)dh \tag{7}$$

subject to the loading-budget constraint

$$\int_0^1 \ell(h)m(h)dh = B. \tag{8}$$

3. SOLUTION AND THE EFFICIENT LOADING RULE

The solution to the above skill-wise optimal-control problem can be considerably simplified by substituting the loading-budget constraint into the objective function in the following way. Equation 8 indicates that the loading budget B can be interpreted as a depleting financial resource over the skill interval. That is,

$$\frac{dB}{dh} = -\ell(h)m(h) \tag{9}$$

and hence

$$m(h) = -\frac{dB/dh}{\ell(h)}. \tag{10}$$

By substituting equation 10 for m(h) in the objective function 7, a university's problem of allocating salary loadings can be rendered as a standard problem of calculus of variations:

$$\underset{\{\ell(h)\}}{\text{maximising}} \int_0^1 e^{-\phi h}u[v(h)]\left[\frac{-dB/dh}{\ell(h)}\right] dh. \tag{11}$$

By virtue of Euler equation, the necessary condition for maximum sum of equality-wise discounted and risk-adjusted academic performance is

$$\frac{d}{dh}[e^{-\phi h}u(h)/\ell(h)] = 0 \tag{12}$$

and by taking the above differential and then multiplying both sides of the resultant equation by $[\ell(h)/u(h)]e^{\phi h}$, the following 'no-arbitrage' condition is obtained:

$$\frac{d\ell(h)/dh}{\ell(h)} = \frac{du/dh}{u(h)} - \phi. \tag{13}$$

In recalling equations 6, 2 and 3, the 'no-arbitrage' condition can be expressed further as

$$\frac{d\ell(h)/dh}{\ell(h)} = \mu - (R\sigma^2 + \phi). \tag{14}$$

This 'no-arbitrage' condition for allocating remuneration indicates that the rate of change in the salary loadings within a heterogeneous group of workers should be equal to the difference between the expected rate of return on academic skill and the marginal costs accruing to the institution in terms of risk-bearing and foregone equality.

The 'no-arbitrage' condition constitutes a first-order linear differential equation whose solution gives the efficient salary-loading rule:

$$\ell(h) = \ell_0 \, exp\{(\mu\text{-}R\sigma^2 - \phi)h\}. \qquad (15)$$

Here, ℓ_0 denotes the loadings' base level, i.e., the amount of salary loadings for the least skilled worker. By substituting the right-hand sides of equations 4, 5 and 15 for m(h) and ℓ(h) into equation 8, ℓ_0 should satisfy the loading-budget constraint:

$$\int_0^1 \ell_0 \, exp\{(\mu\text{-}R\sigma^2\text{-}\phi)h\}s(wh + \ell_0 \, exp\,\{(\mu\text{-}R\sigma^2\text{-}\phi)h\})p(h)Ndh = B. \qquad (16)$$

Equation 16 indicates that ℓ_0 increases with B but decreases with (μ-Rσ_2-ϕ), w and N.

The solution to the optimal-control problem indicates that if the discounted expected performance (i.e., the expected performance adjusted for risk and inequality) of the most skillful member were equal to zero, salary loadings should be equally distributed. In any other case, they may vary monotonically, buy may not necessarily increase with academic skill. Only when the discounted expected performance is positive should salary loadings increase monotonically with skill. The possible patterns of efficient distribution of loadings are depicted in Figure 1 below.

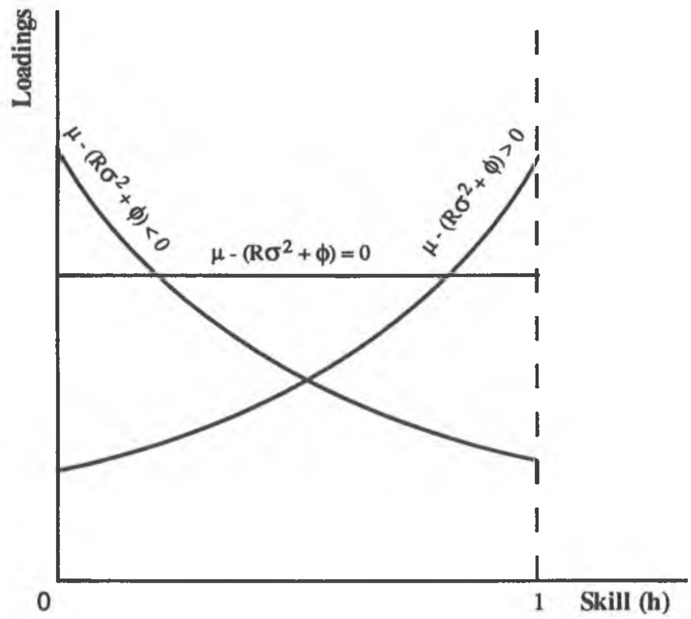

Figure 1: Possible patterns of efficient allocations of salary-loadings

4. IMPLICATIONS FOR THE DISTRIBUTION OF LOADINGS AND THE SKILL-STRUCTURE

It has been suggested by Sen (1973), Pyatt (1976) and Yitzhaki (1979) that the costs of diverging from equal distribution can be measured in terms of depression and relative deprivation. Highly intensified feelings of relative deprivation might lead to a deterioration of the working environment and subsequently to lower overall performances. Equation 15 indicates that an equal distribution of salary loadings coincides with the efficient allocation if $\mu - R\sigma^2 - \phi = 0$ and/or the labour force is homogeneous. In any other case, the efficient distribution of salary loadings corresponds to the distribution of skill within the labour force. The intensity of the resultant relative deprivation feelings can be controlled by the choice of ϕ.

By taking the expectation of both sides of equation 15 we obtain the mean salary-loadings as

$$E(\ell) = \ell_0\, E[e^{\beta h}] = \ell_0\, \psi_h(\beta) \tag{17}$$

where

$$\beta \equiv \mu - R\sigma^2 - \phi \tag{18}$$

and $\psi_h(\beta)$ is the moment-generating function of h evaluated at β.

Similarly, and in recalling that

$$\mathrm{Var}\,(\mathcal{L}) \;=\; E(\mathcal{L}^2) \;-\; [E(\mathcal{L})]^2 \tag{19}$$

we obtain that the variance of salary-loadings is given by

$$\mathrm{Var}\,(\mathcal{L}) = \mathcal{L}_0^2[\psi_h(2\beta) \;-\; \psi_h^2(\beta)]. \tag{20}$$

For example, if academic skill were normally distributed within the labour pool attracted and retained with mean α and variance θ, then

$$\psi(\beta) \;=\; exp\;\{\alpha\beta \;+\; 0.5\theta^2\beta^2\} \tag{21}$$

and the efficient salary-loadings are lognormally distributed with mean

$$E\,(\mathcal{L}) \;=\; \mathcal{L}_0\, e^{\alpha\beta + 0.5\theta^2\beta^2} \tag{22}$$

and variance

$$\mathrm{Var}\,(\mathcal{L}) \;=\; \mathcal{L}_0^2\, e^{2\alpha\beta}\,(e^{2\theta^2\beta^2} \;-\; e^{\theta^2\beta^2}). \tag{23}$$

In this example, if β is positive (negative), the greater the skill mean the greater (smaller) the salary-loadings' mean and variance. Furthermore, regardless of the sign of β, the higher the variance of skill the greater the mean and variance of salary-loadings.

The substitution of equation 15 into equation 5 implies that the supply of workers with a skill level h under the efficient allocation of salary loadings is given by:

$$m(h) = s(wh + \mathcal{L}_0\; exp\;\{(\mu - R\sigma^2 - \phi)h\})\; p(h)N \tag{24}$$

for every $h \in (0,1)$, where $\mathcal{L}_0$ is found from equation 16. In recalling that $\mathcal{L}_0$ declines with $\mu - R\sigma^2 - \phi$, the greater $\mu - R\sigma^2 - \phi$ the larger the number of high-skilled workers attracted and retained by the efficient loadings' scheme. Furthermore, since $s'>0$ and $\mathcal{L}_0$ increases with B, the number of people, at any skill level, attracted and employed by the institution increases with the loadings' budget. Thus, an increase in the budget for loadings enables an institution to practise a higher degree of scrutiny in selecting staff members. However, the selection process favours workers with a higher skill only if $\mu - R\sigma^2 - \phi$ is positive.

5. CONCLUDING REMARKS

This paper considers the issue of allocating remuneration in a regulated workplace as a skill-wise optimal-control problem. The solution to this problem gives the efficient rule for allocating salary loadings. This efficient rule indicates that salary loadings may vary monotonically, but may not necessarily increase, with academic skill. Only in the case where the marginal costs associated with a divergence from equal allocation of loadings, in terms of risk-bearing and foregone equality, are smaller than the expected marginal return, in terms of academic performance, the efficient loadings monotonically increase with the individual academic skill. In this case, the salary-loadings' mean and variance are positively related to the skill's mean and variance in the labour pool. The base level of loadings increases with the loading budget and decreases with the wage differential between the academics and alternative places of employment, the rate of return on skill, adjusted for risk and inequality, and the size of the potential groups of candidates.

As is the case with any analytically derived policy scheme, there are considerable problems in implementing the above-mentioned loading rule. The first problem is associated with the social admissibility of the underlying criterion of maximum sum of risk-adjusted performances discounted by the institution's rate of equality preference. The second problem is associated with the fact that the model's key variables (i.e., academic skill and academic performance), parameters (i.e., degree of risk aversion and rate of equality preference) and functions (i.e., valuation of performance and labour-mobility) are either compounds of several components or abstract, and their measurement and assessment involves *ad-hoc* definitions and strong assumptions.

Extensions of the analysis may consider: (i) the possibility that salary loadings affect effort and subsequently the mean and variance of academic performance; (ii) the opportunity cost of the budget for loadings; (iii) the oligopsonistic structure of the academic-labour market; and (iv) the costs stemming from the uncoordinated behaviour of universities in the academic-labour market..

REFERENCES

Arrow, K. (1963), *Social Choice and Individual Values*, New York: Wiley.

Marginson, S. (1991), 'Australian Academic Salaries—Trends and Relativities', *Australian Bulletin of Labour* 17 (1): 49-72.

Freund, R. J. (1956), 'An Introduction of Risk into a Risk-Programming Model', *Econometrica* 24: 253-263.

Hammond, S. (1974), 'Simplification of Choice under Uncertainty', *Management Science* 20: 1047-1072.

Pyatt, G. (1976), 'On the Interpretation and Disaggregation of Gini Coefficients', *Economic Journal* 86: 243-255.

Sen, A. K. (1973), *On Economic Inequality*, Oxford: Clarendon Press.

Todaro, M.P. (1969), 'A model of Labor Migration and Urban Unemployment in Less Developed Countries', *The American Economic Review* 59: 138-148.

Yitzhaki, S. (1979), 'Relative Deprivation and the Gini Coefficient', *Quarterly Journal of Economics* 93: 321-324.

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