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# Two new complex orthogonal space time codes for 8 transmit antennas

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**Abstract:** Two new constructions of complex orthogonal space-time block codes of order 8 based on the theory of amicable orthogonal designs are presented and their performance compared with that of the standard code of order 8. These new codes are suitable for multi-modulation schemes where the performance can be sacrificed for a higher throughput.

**Category:** Information Theory, Communication

**Keywords:** Space-Time Codes, Orthogonal Designs, Radiocommunications, Block Codes

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## 1. Introduction

The transmit antenna diversity can be accomplished with the use of space-time codes (STCs) [1]. Out of the STCs, space-time block codes (STBCs) developed from the amicable orthogonal designs [2] lead to the simplest receiver structures and minimum processing delays if combined with modulation schemes having complex signal constellations, like in the case of quaternary phase shift keying (QPSK) or quadrature amplitude modulation (QAM). The simplest STBC based on amicable orthogonal designs is an Alamouti code [1] providing a transmission rate of 1 for the two transmit antenna system. STBCs based on amicable orthogonal designs, referred usually to as complex orthogonal STBCs, for a larger number of transmit antennas cannot provide the transmission rate of 1 but they are attractive anyway as they can provide a full diversity for the given number of transmit antennas and are usually simple to decode. There exist some complex orthogonal STBCs designs for 4 transmit antennas and 8 transmit antennas e.g. [3] providing the rates of  $\frac{3}{4}$  and  $\frac{1}{2}$ , respectively. They are usually based on those amicable orthogonal designs where each variable is represented just once in a design. Hence, the code matrices have many zeros (50% for 8 transmit antennas) resulting in many time slots when no useful information is being transmitted. In the letter, we introduce two new complex orthogonal STBCs for 8 transmit antennas having less

unused time slots. In the first of these new codes, one of the variables is repeated twice and in the second code, one variable is repeated 4 times per every transmit antenna. These properties can be further exploited to increase the code rate over  $\frac{1}{2}$  using more sophisticated modulation schemes, where the higher number of information bits are associated with those signals that appear in more than one time slot per each antenna.

## 2. New Codes and Their Performance

Orthogonal STBCs that can be used with complex signal constellations can be constructed using complex orthogonal designs (CODs) defined as follows.

**Definition 1:** A complex orthogonal design (COS)  $\mathbf{X}$  of order  $n$  is an  $n \times n$  matrix on the complex indeterminates  $s_1, \dots, s_t$ , with entries chosen from  $0, \pm s_1, \dots, \pm s_t$ , their complex conjugates  $\pm s_1^*, \dots, \pm s_t^*$ , or their product with  $i = \sqrt{-1}$ , such that

$$\mathbf{X}^H \mathbf{X} = \left( \sum_{k=1}^t |s_k|^2 \right) \mathbf{I}_n \quad (1)$$

where  $\mathbf{X}^H$  denotes the Hermitian transposition of  $\mathbf{X}$  and  $\mathbf{I}_n$  is the identity matrix of order  $n$ .

CODs are strongly connected to the amicable orthogonal designs (AODs), which has been explained in [1]. The detailed theory of amicable orthogonal designs (AODs), including limitations on the number of different variables for the given design order can be found in [4]. Drawing from the presented there theory of the existence of AODs, we found two new CODs of order 8. The first one, corresponding to the AOD(8; 1,1,2,2; 1,1,2,2) is of the form:

$$\begin{bmatrix}
s_1 & s_2 & \frac{s_3}{\sqrt{2}} & \frac{s_3}{\sqrt{2}} & 0 & 0 & \frac{s_4}{\sqrt{2}} & \frac{s_4}{\sqrt{2}} \\
-s_2^* & s_1^* & \frac{s_3}{\sqrt{2}} & -\frac{s_3}{\sqrt{2}} & 0 & 0 & \frac{s_4}{\sqrt{2}} & -\frac{s_4}{\sqrt{2}} \\
\frac{s_3}{\sqrt{2}} & \frac{s_3}{\sqrt{2}} & -s_1^R + is_2^I & -s_2^R + is_1^I & \frac{s_4}{\sqrt{2}} & \frac{s_4}{\sqrt{2}} & 0 & 0 \\
\frac{s_3}{\sqrt{2}} & -\frac{s_3}{\sqrt{2}} & s_2^R + is_1^I & -s_1^R - is_2^I & \frac{s_4}{\sqrt{2}} & -\frac{s_4}{\sqrt{2}} & 0 & 0 \\
0 & 0 & \frac{s_4}{\sqrt{2}} & \frac{s_4}{\sqrt{2}} & s_1 & s_2 & -\frac{s_3}{\sqrt{2}} & -\frac{s_3}{\sqrt{2}} \\
0 & 0 & \frac{s_4}{\sqrt{2}} & -\frac{s_4}{\sqrt{2}} & -s_2^* & s_1^* & -\frac{s_3}{\sqrt{2}} & \frac{s_3}{\sqrt{2}} \\
\frac{s_4}{\sqrt{2}} & \frac{s_4}{\sqrt{2}} & 0 & 0 & -\frac{s_3}{\sqrt{2}} & -\frac{s_3}{\sqrt{2}} & -s_1^R + is_2^I & -s_2^R + is_1^I \\
\frac{s_4}{\sqrt{2}} & -\frac{s_4}{\sqrt{2}} & 0 & 0 & -\frac{s_3}{\sqrt{2}} & \frac{s_3}{\sqrt{2}} & s_2^R + is_1^I & -s_1^R - is_2^I
\end{bmatrix}, \quad (2)$$

where  $s_k^R$ , and  $s_k^I$  denote real and imaginary parts of  $s_k$ , respectively. The second design

is based on AOD(8; 1,1,1,4; 1,1,1,4) and is of the form:

$$\begin{bmatrix}
s_1 & 0 & s_3^R + is_2^I & s_2^R + is_3^I & \frac{s_4}{2} & \frac{s_4}{2} & \frac{s_4}{2} & \frac{s_4}{2} \\
0 & s_1 & -s_2^R + is_3^I & s_3^R - is_2^I & \frac{s_4}{2} & -\frac{s_4}{2} & \frac{s_4}{2} & -\frac{s_4}{2} \\
-s_3^R + is_2^I & s_2^R + is_3^I & s_1^* & 0 & \frac{s_4}{2} & \frac{s_4}{2} & -\frac{s_4}{2} & -\frac{s_4}{2} \\
-s_2^R + is_3^I & -s_3^R - is_2^I & 0 & s_1^* & \frac{s_4}{2} & -\frac{s_4}{2} & -\frac{s_4}{2} & \frac{s_4}{2} \\
-\frac{s_4}{2} & -\frac{s_4}{2} & -\frac{s_4}{2} & -\frac{s_4}{2} & s_1^R - is_3^I & s_2^* & s_3^R - is_1^I & 0 \\
-\frac{s_4}{2} & \frac{s_4}{2} & -\frac{s_4}{2} & \frac{s_4}{2} & -s_2 & s_1^R + is_3^I & 0 & s_3^R - is_1^I \\
-\frac{s_4}{2} & -\frac{s_4}{2} & \frac{s_4}{2} & \frac{s_4}{2} & -s_3^R - is_1^I & 0 & s_1^R + is_3^I & -s_2^* \\
-\frac{s_4}{2} & \frac{s_4}{2} & \frac{s_4}{2} & -\frac{s_4}{2} & 0 & -s_3^R - is_1^I & s_2 & s_1^R - is_3^I
\end{bmatrix} \quad (3)$$

If all the symbols of both new STBCs,  $S_1$  and  $S_2$  defined by the CODs given by (2) and (3), respectively, are associated with QPSK complex symbols, then the bit error rate (BER) performance in a Gaussian channel of both  $S_1$  and  $S_2$  is exactly the same as

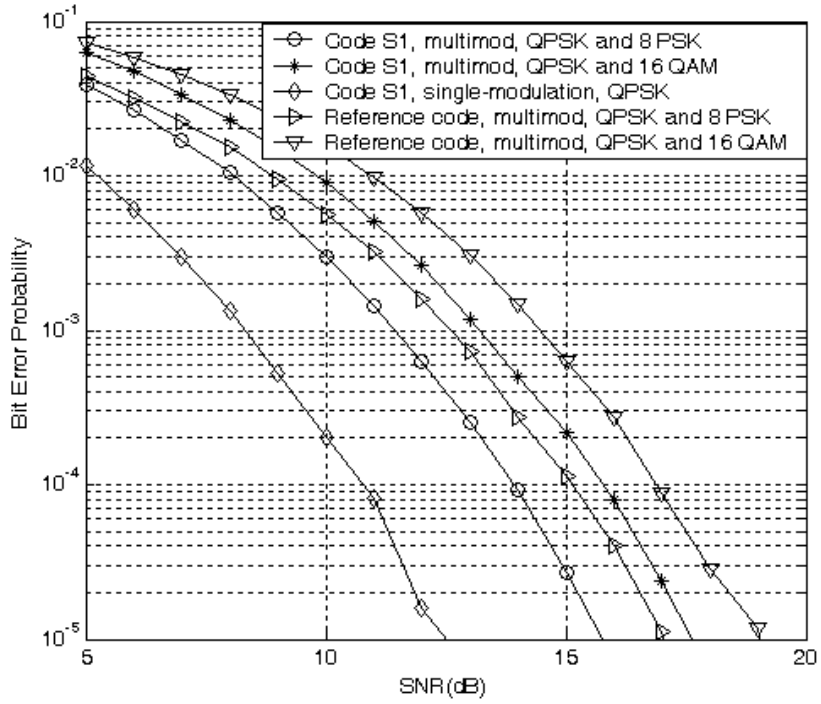
performance of the complex orthogonal STBC of order 8 described in [3]. The achieved code rate is also the same and equal to  $\frac{1}{2}$ . From (2) and (3) it is visible that in  $S_1$  and  $S_2$  some of the symbols are transmitted in more than a single time slot per given antenna. In fact, in  $S_1$ , symbols  $s_3$  and  $s_4$  are transmitted twice as often as  $s_1$  or  $s_2$ . In  $S_2$ , the symbol  $s_4$  is transmitted four times as often as  $s_1$ ,  $s_2$  or  $s_3$ . Thus, by associating  $s_3$  and  $s_4$  in  $S_1$  and  $s_4$  in  $S_2$  with symbols from multilevel complex modulation schemes and the remaining symbols in each of  $S_1$  and  $S_2$  with QPSK symbols, the overall code rates can be increased. Of course, there is a tradeoff between the rate increase and the BER performance. This is illustrated in Figure 1 and Figure 2 for  $S_1$  and  $S_2$ , respectively. In both figures, SNR (signal-to-noise ratio) is defined by the ratio between the total power received in each symbol time slot and the noise power at the receive antenna. Multi-modulation, using QPSK, 8PSK and 16 QAM constellations, is utilized and the bit error performance of the STBC given in [3] is also simulated to compare with the proposed codes. In simulation, the signal power per transmission in each symbol time slot is normalized to 1. Figures 1 and 2 show that the proposed codes associated with QPSK single-modulation provide a good bit error performance. Additionally, by sacrificing 3 dB and 5 dB, with respect to SNR at  $\text{BER}=10^{-5}$ , in case of  $S_1$ , and by 2.5 dB and 3 dB, in case of  $S_2$ , one can increase the code rate from  $\frac{1}{2}$  to higher code rates of  $\frac{5}{8}$  and  $\frac{3}{4}$ , in case of  $S_1$ , and of  $\frac{9}{16}$  and  $\frac{5}{8}$ , in case of  $S_2$ , respectively. Furthermore, at the same code rate, both proposed codes provide better bit error performance than the code in [3], by around 1 dB (QPSK+8 PSK) and 1 dB (QPSK+16 QAM), in case of  $S_1$ , and around 1.5 dB (QPSK+8 PSK) and 2.5 dB (QPSK+16 QAM), in case of  $S_2$ , respectively, at  $\text{BER}=10^{-5}$ .

### 3. Conclusions

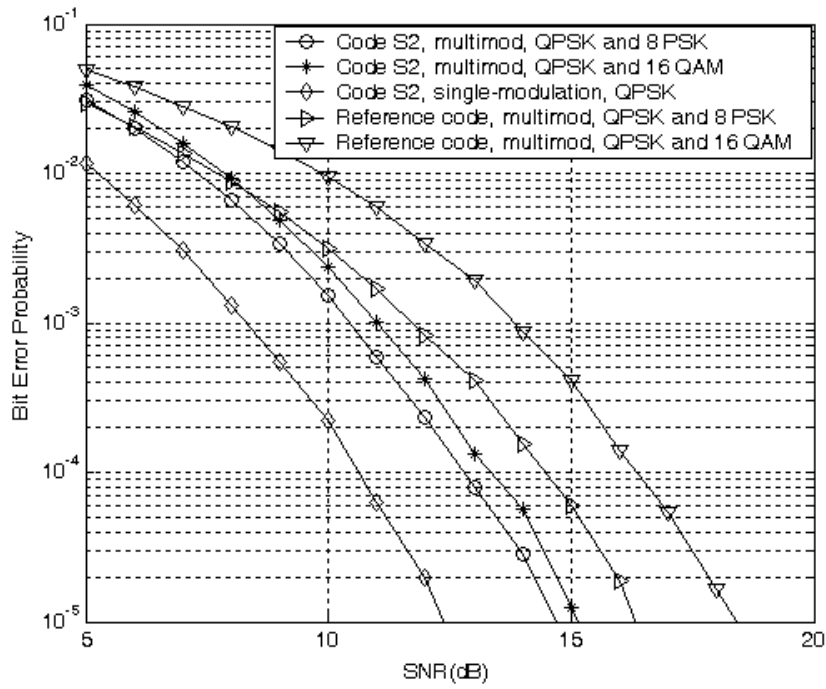
In the letter, we presented two new complex orthogonal STBCs for 8 transmit antennas that can provide higher data rates (up to  $\frac{3}{4}$ ) than other complex STBCs of the same order. This is achieved by employing multilevel modulation schemes for those code symbols that are transmitted more often than other symbols for which QPSK is used. This feature can be utilized in design of adaptive STBC schemes, where the code rate can be traded for BER and vice-versa.

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**Figure 1:** Bit error performance of  $S_1$  in single and multi-modulation schemes



**Figure 2:** Bit error performance of  $S_2$  in single and multi-modulation schemes

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**Figure 1:** Bit error performance of  $S_1$  in single and multi-modulation schemes

**Figure 2:** Bit error performance of  $S_2$  in single and multi-modulation schemes