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CONTINUATION AND LIQUIDATION TIMING: A PARETO OPTIMAL APPROACH

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CONTINUATION AND LIQUIDATION TIMING:
A PARETO OPTIMAL APPROACH

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ABSTRACT

This paper highlights the nexus of interactions among financial, industrial and macroeconomic factors determining the Pareto optimal date in which the firm's claimants stop collaborating and force the firm into liquidation. The condition for optimal liquidation time summarises the effects of volatility of the aggregate consumer income and the overall price level, demand elasticity, the industry's concentration level and depreciation on the firm's going-concern value. It also takes into account the effects of the firm's level of indebtedness, forgone interest on alternative usage of the financial resources extended to the firm and the costs of risk bearing perceived by the firm's claimants from continued collaboration.

1 INTRODUCTION

Though financial distress is a necessary condition for bankruptcy it is not a sufficient one. It has been argued by Bulow and Shoven (1978) that the criterion for bankruptcy is a positive gain to the coalition of the firm's claimants from immediate liquidation and hence a firm's operation might be liquidated (continued) even when its going-concern value exceeds (is below) its liquidation value. Using a similar analytical framework, White (1980) has further examined the social efficiency properties of alternative priority rules in liquidation, including the 'me-first' rule, and has concluded that those rules are not socially efficient, except under very strong assumptions about the payment to the bondholder and the interest rates on bonds. In contrast, Ang and Chua (1980) have stressed the important role of the 'me-first' rule in bankruptcy-liquidation decision. They have argued that as long as the firm's claimants are value maximisers and the 'me-first' rule is not violated the liquidation of a firm in a financial distress should take place when the firm's liquidation value exceeds its going-concern value, no matter what coalitions of the firm's claimants are formed.

In all of the aforementioned studies of the liquidation decision the outcomes have a zero-one characteristic. That is, either an immediate liquidation of the firm, or a continuation of its operation. The length of the continuation period has not been analysed. A possible justification for ignoring that aspect is that new information is accumulated as time progresses and, therefore, it might be sub-optimal to determine the duration of the insolvent firm's continued operation in advance. However, when alternatives are time-dependent and span more than a single period, high-priority claimants need to know in advance the length of the period in which their

collaboration with low-priority claimants in continuing the insolvent firm's operation is required. This information is essential for identifying the alternatives given up and assessing compensation for the forgone income from those alternatives, as well as for the excessive risk bearing. Hence, it is useful to study this neglected issue.

The conceptual analysis of the continuation period and liquidation timing developed in this paper refers to a financially distressed firm operating in a oligopolistic industry and facing a random demand for its product due to fluctuations of aggregate consumer income and overall price level. In order to simplify the mathematical analysis, the definition of the firm's claimants includes a stockholder and a bondholder (or any other lender) only. Both claimants have unbiased expectations about the firm's future operating profits; but may differ with regard to the degree of absolute risk aversion, assessment of the level of uncertainty involved in the firm's future operation and priority on the firm's liquidation proceeds. It is assumed that the financial crisis arises from the firm's current inability to pay back the bond value which matures in the present period. In liquidation, bond principal claim and interest payments would be paid first. It is also assumed that the firm's immediate liquidation value is at least as large as the firm's liabilities. Due to uncertainty about future returns from continuation and the existence of alternatives, the bondholder (or any other lender), unless sufficiently compensated, is not willing to extend the bond (or loan) maturity period and calls for an immediate liquidation; whereas the stockholder considers backing the firm as long as the expected returns from doing so sufficiently exceeds the costs of keeping the firm solvent. These costs consist of the costs of risk bearing and the compensation payments required for keeping the bondholder at least as well off as under

immediate liquidation while extending the bond's maturity period. Of course, when these costs exceed the stockholder's expected returns from continuation, the stockholder also prefers an immediate liquidation of the firm's assets. Thus, an interior solution to the problem of optimal liquidation time exists when the net returns to the stockholder from continuation are non negative.

Along these lines the paper continues as follows. The second section describes the claimants' returns from continuation. The third section presents the claimants' evaluations of those net returns and the collaboration decision problem. The fourth section derives the optimal collaboration period and the condition for immediate liquidation. The fifth section displays and interprets geometrically the optimal collaboration period under various assumptions about the claimants' rates of time preference. The sixth section discusses the comparative statics' properties of the collaboration period. Following Altman (1971) and Levy and Bar-Niv (1987), who have found that the corporate failure rate in the US is correlated with the fluctuations of the GNP and the overall price level, the seventh section incorporates the effects of macroeconomic conditions, as well as industrial conditions, on the firm's expected profits and going-concern value and consequently on the collaboration period.

2 CLAIMANTS' RETURNS FROM CONTINUATION

The conceptual framework uses the following notations:

T = the firm's liquidation date;
t = a continuous time index, $0 \leq t \leq T$;
S_0 = the liquidation value of the firm's productive assets at $t = 0$;

B_0 = the bond (or any other liabilities) claim against the firm at $t = 0$;
$\pi(t)$ = the firm's operating profit at t;
δ = a fixed rate of depreciation of the firm's productive assets;
i = the bond's contracted interest rate;
γ = the bondholder's subjective discounting rate;
ρ = the stockholder's subjective discounting rate;
C = the present value of the compensation payment to the bondholder for continuation to T;
$y_S(T)$ = the (random) present value of the net returns to the stockholder from continuation to T; and
$y_b(T)$ = the (random) present value of the net returns to the bondholder from continuation to T.

The collaboration period $(0,T)$ is determined by a consensus reached by the firm's claimants. It is assumed, for simplicity, that the sale price of the firm's productive assets remains the same over time and that there is no income tax. In this case, the present expected value of the net returns to the stockholder from keeping the firm solvent during the time interval $(0,T)$ is the sum of the discounted operating profits, minus the depreciation costs, plus the salvage value of the firm's assets, minus the liabilities at the end of the period and minus the compensation payment to the bondholder:

$$E[y_S(T)] = \int_0^T e^{-\rho t} \{E[\pi(t)] - \delta S_0\} dt + S_0 - B_0 e^{(i-\rho)T} - C. \qquad (1)$$

The first and the second terms on the right-hand side of equation 1 are constructed under the assumption that, due to its financial difficulties, the firm's net investment is zero. The

third term on the right-hand side of equation 1 reflects the assumption that in a case of continuation the firm receives a grace period (O,T). That is, the interest on bond is accumulated and paid at T. The firm's liabilities are discounted by the stockholder's subjective rate, ρ. Cash or other liquid assets, are not included in equation 1. Their inclusion will not change the analysis considerably. It is assumed in the following that the firm's instantaneous expected profit remains the same as time progresses. Thus, the discounted expected value of the net returns to the stockholder from keeping the firm solvent during the time interval $(0, T)$ can be equivalently rendered as

$$E[y_s(T)] = \frac{1}{\rho}[E(\pi) - \delta S_0](1 - e^{-\rho T}) + S_0 - B_0 e^{(i-\rho)T} - C. \qquad (2)$$

Correspondingly, the present value of the expected net returns to the bondholder from continuation are

$$E[y_b(T)] = B_0 e^{(i-\gamma)T} + C. \qquad (3)$$

Here, the first term on the right-hand side indicates the value of the principal and the accumulated interest discounted by the bondholder's rate of return on the alternative transaction; and the second term on the right-hand side represents the value of the compensation received by the bondholder for continuation to T.

It is assumed that the claimants' expectations about their net returns from continuation are unbiased, but, naturally, reflect increased uncertainty as the collaboration period expands. More specifically, the claimants' expected net returns from continuation to T, $y_s^e(T)$ and $y_b^e(T)$, are assumed to be normally distributed with means which are equal to the

theoretical values presented by equation 2 and equation 3, and with variances which are proportional to the collaboration's period, $\sigma_s^2 T$ and $\sigma_b^2 T$, respectively. This is equivalent to arguing that the expected net returns to the stockholder and bondholder from continuation to T can be approximated by Wiener processes (Brownian motion).

3 CLAIMANTS' PREFERENCES AND THE COLLABORATION DECISION PROBLEM

It is postulated that both claimants maximise expected utility and that their preferences on the expected net returns can be represented by utility functions which, in order to simplify the mathematical analysis, reflect constant degrees of absolute risk aversion R_s and R_b, respectively:

$$U_j(y_j^e(T)) = 1 - exp\{- R_j y_j^e(T)\} \text{ for } j = s, b. \qquad (4)$$

With this specification of U, the expected utility functions can be displayed as

$$E(U_j) = 1 - \int_{-\infty}^{\infty} exp\{-R_j y_j^e(T)\}\ \phi_j(y_j^e(T))\ dy_j^e(T) = 1 - m(-R_j)$$

$$\text{for } j = s, b \qquad (5)$$

where m is the *moment-generating function* associated with the distribution of $y_j^e(T)$.

Given that $y_j^e(T)$ is normally distributed we obtain

$$E(U_j) = 1 - exp\{-R_j E[y_j^e(T)] + 0.5\, R_j^2 \sigma_j^2 T\} \text{ for } j = s, b. \qquad (6)$$

Since the bondholder prefers an immediate liquidation (that is, $T = 0$) upon continuation of the firm's operation, unless sufficiently compensated, the Pareto optimal collaboration period can be found by specifying the stockholder's decision problem as follows

$$\max_{T} E[U_s(y_s^e(T))]$$

subject to

$$E[U_b(y_b^e(T))] = E[U_b(y_b^e(0))]. \qquad (7)$$

The constraint reflects the compensation payment, C, required to keep the bondholder at least as well off as under immediate liquidation. In view of equations 6 and 3, this constraint can be rendered as

$$R_b[B_0 e^{(i-\gamma)T} + C] - 0.5R_b^2\sigma_b^2 T = R_b B_0. \qquad (8)$$

Hence,

$$C = B_0[1 - e^{(i-\gamma)T}] + 0.5\, R_b\, \sigma_b^2 T. \qquad (9)$$

That is, in order to postpone the liquidation of the firm to T, the bondholder should be paid the forgone interest differential between his or her alternative financial investment and the firm's bond plus the costs of risk-bearings which are equal to the product of the perceived level of uncertainty associated with his or her net returns from continuation to T and his or her degree of absolute risk aversion.

Summing up, the Pareto optimal collaboration period can be found by solving the stockholder decision problem

$$\max_{T} \{1 - exp\{-R_s E[y_s(T)] + 0.5R_s^2\sigma_s^2 T\}\}$$

where $E[y_S(T)]$ is given by equation 2 and C by equation 9.

4 OPTIMAL COLLABORATION PERIOD AND THE CONDITION FOR IMMEDIATE LIQUIDATION

Since maximising $\{1-exp\{-R_S E[y_S(T)]+0.5R_S{}^2\sigma_S{}^2T\}\}$ is equivalent to maximising the risk deducted expected return from continuation to the stockholder $\{E[y_S(T)]-0.5R_S\sigma_S{}^2T\}$, the Pareto optimal collaboration period (T^o) should satisfy the first-order condition

$$[E(\pi)-\delta S_0]e^{-\rho T^o}=[(i-\rho)e^{(i-\rho)T^o}+(\gamma-i)e^{(i-\gamma)T^o}]B_0+0.5(R_b\sigma_b^2+R_s\sigma_s^2) \qquad (10)$$

and the second-order condition

$$H=-\rho\,[E(\pi)-\delta S_0]\,e^{-\rho T^o}-(i-\rho)^2 B_0 e^{(i-\rho)T^o}+(i-\gamma)^2 B_0 e^{(i-\rho)T^o}<0. \qquad (11)$$

Furthermore, by setting T^o to be equal to zero in equation 10 we obtain that the condition for immediate liquidation is:

$$E(\pi)-\delta S_0=(\gamma-\rho)B_0+0.5(R_b\sigma_b^2+R_s\sigma_s^2) \qquad (12)$$

That is, the financially distressed firm should be immediately forced into liquidation if the expected profit after depreciation obtained from an infinitesimal continuation of the firm's operation (the term on the left-hand side) is just equal to the cost of doing so in terms of the forgone interest on the alternative financial investment for the bondholder, discounted by the stockholder rate of time preference, and the

costs of risk-bearing for both claimants. Of course, immediate liquidation is also optimal when the expected profit gained from infinitesimal continuation is overweighed by the costs.

5 GEOMETRICAL INTERPRETATION OF THE SOLUTION UNDER VARIOUS ASSUMPTIONS ABOUT THE CLAIMANTS' RATES OF TIME PREFERENCE

The left-hand side of equation 10 indicates the marginal benefit from continuation (*MBFC*) to the stockholder, whereas the right-hand side is the marginal cost of continuation (*MCOC*). The slope of the *MBFC* curve in the *T*-$ plane is given by the first term on the left-hand side of inequality 11:

$$\frac{dMBFC}{dT} = -\rho[E(\pi) - \delta S_0]e^{-\rho T} \tag{13}$$

In the following we consider the nontrivial case where the expected instantaneous profit after depreciation, $E(\pi) - \delta S_0$, is positive and hence the *MBFC* curve is downward sloping.

Similarly, the slope of the *MCOC* curve in the *T*-$ plane is equal to the sum of the second and third terms on the left-hand side of the second-order condition 11, and hence

$$\frac{dMCOC}{dT} \gtreqless 0 \ \text{as} \ \frac{i-\rho}{i-\gamma} \gtreqless e^{(\rho-\gamma)T} \tag{14}$$

or, equivalently,

$$\frac{dMCOC}{dT} \gtreqless 0 \ \text{as} \ \gamma \gtreqless \rho. \tag{15}$$

That is, if the bondholder' rate of time preference is greater, equal to, or smaller than the stockholder's rate of time preference, the *MCOC* curve is upward sloping, flat, or downward sloping, respectively, as displayed by Figures 1a to

1d. These figures has been drawn under the assumption that the expected instantaneous profit after depreciation (that is, the left hand side of equation 12 and the intercept of the *MBFC* curve) exceeds the instantaneous cost of continuation (that is, the right-hand side of equation 12 and the intercept of the *MCOC* curve). As argued earlier, under the alternative assumption that the expected instantaneous profit net of depreciation is equal to, or smaller than, the instantaneous costs of continuation, the stockholder prefers an immediate liquidation of the firm. Figures 1a to 1d indicate that the period of collaboration is longer the smaller the discrepancy between the bondholder's and the stockholder's rates of time preference, and can even be infinite when ρ is sufficiently larger than γ so that the *MCOC* curve is downward sloping and lying entirely below the *MBFC* curve. This conclusion is valid as long as ρ is held constant while γ is lowered and moderates the slope of the *MCOC* curve as we move from Figure 1a to Figure 1d. The underlying rationale is that γ represents the bondholder's opportunity costs, and, hence, the lower γ the smaller the compensation payment required for keeping the bondholder as well of as under immediate liquidation, and, subsequently, the longer the period of continuation desired by the stockholder.

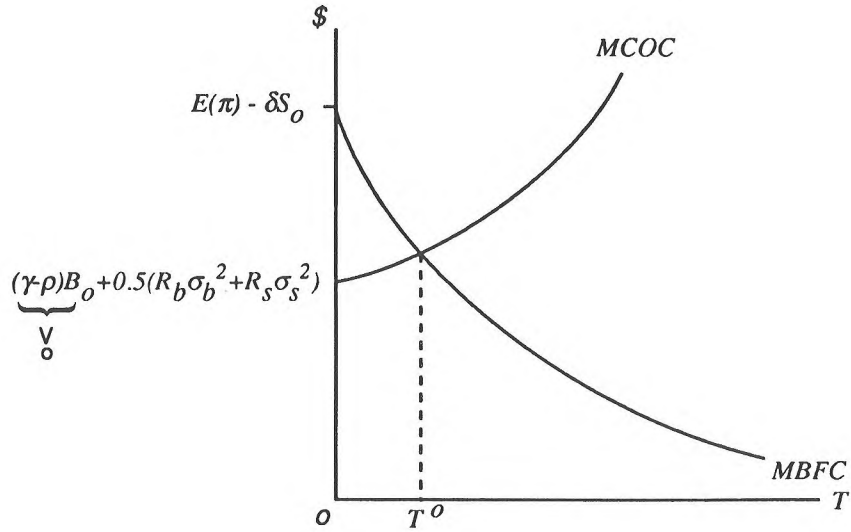

Figure 1a: $\gamma > \rho$

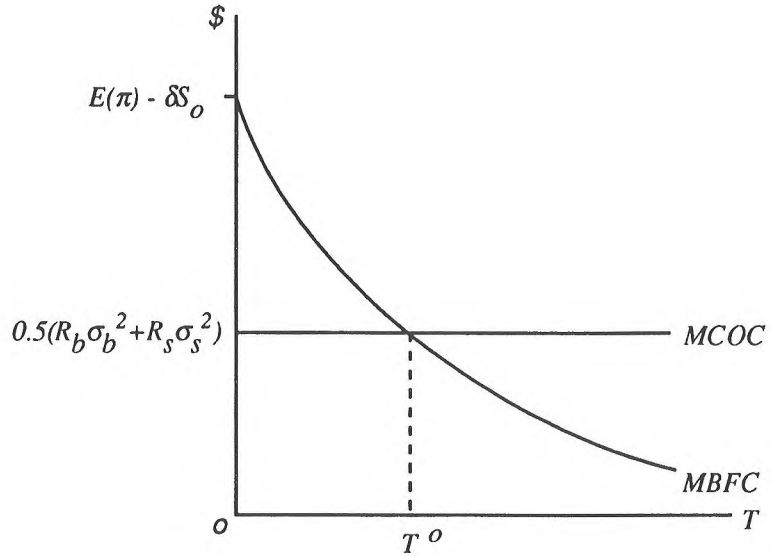

Figure 1b: $\gamma = \rho$

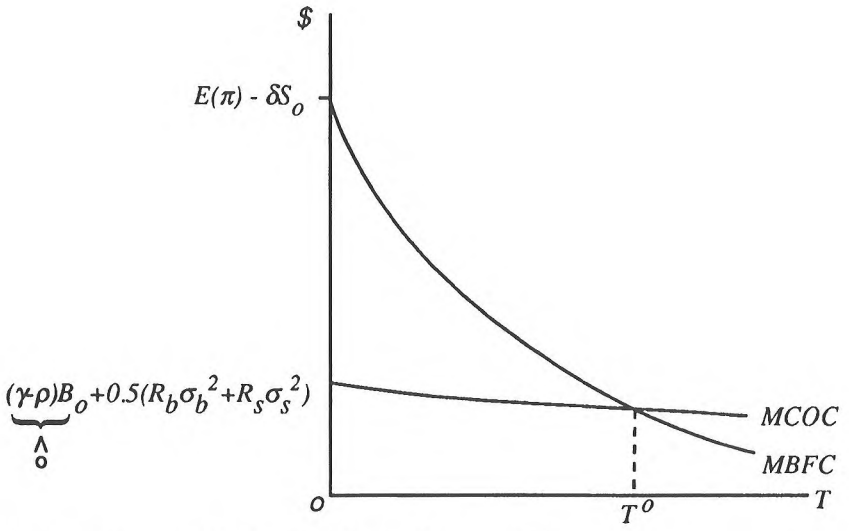

Figure 1c: $\gamma < \rho$, finite collaboration period

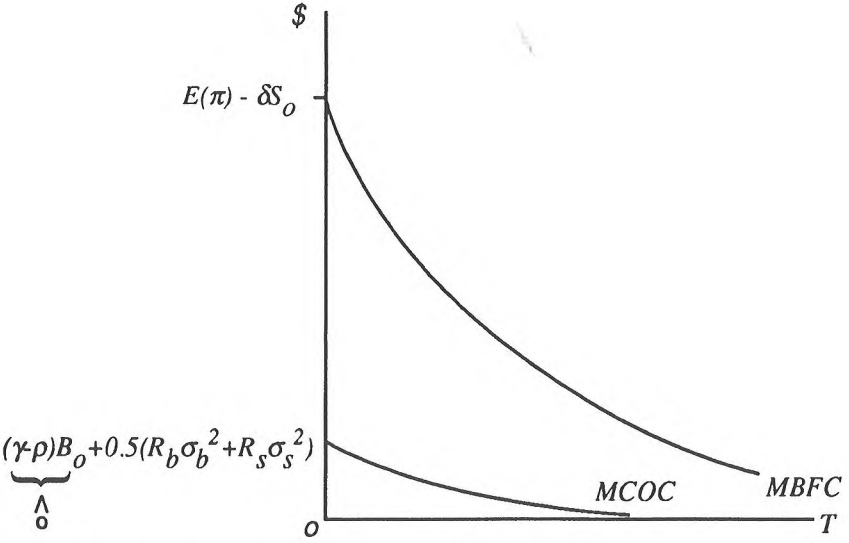

Figure 1d: $\gamma << \rho$, infinite collaboration period

It is important to note, however, that if we also allow ρ to rise within the open unit interval *(0,1)* the *MCOC* curve is flattened and shifted downward as the discounted compensation costs diminish, whereas the *MBFC* curve is steepened and tilted toward the origin as the discounted expected future profit diminishes. Thus, the net effect of an hypothetical rise in the stockholder's rate of time preference on the collaboration period is not clear, *a-priori*, as displayed by Figure 2a and Figure 2b.

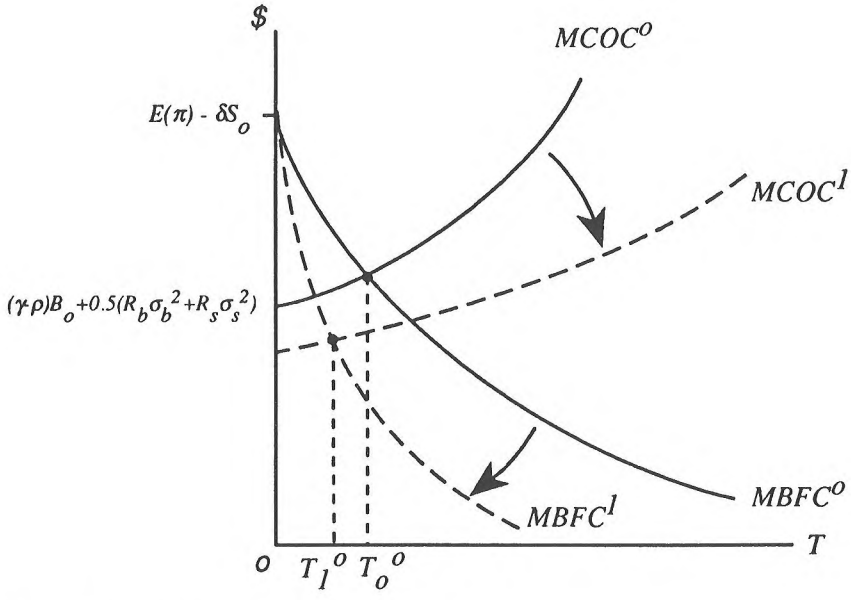

Figure 2a: A rise in ρ shortens T^0

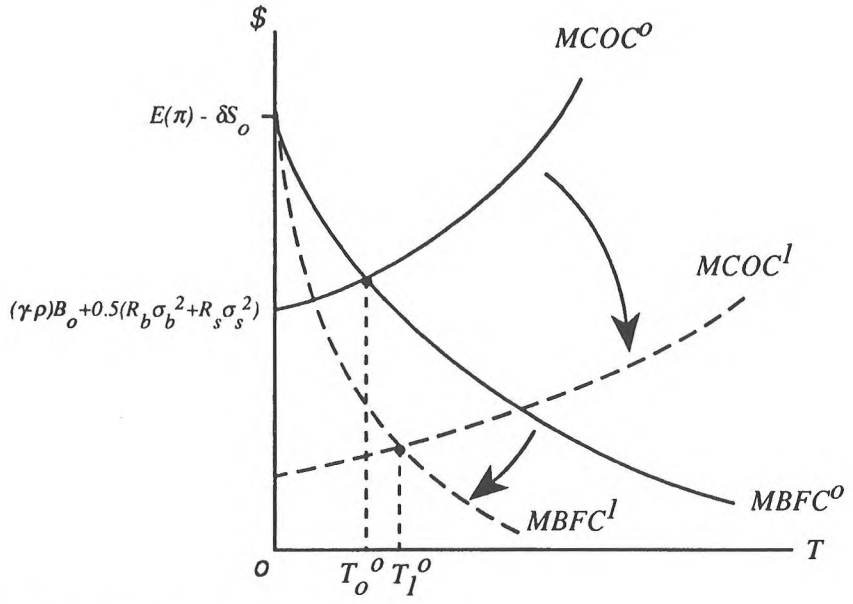

Figure 2b: A rise in ρ lengthens T^0

6 COMPARATIVE STATICS

Due to the complexity of equation 10, a closed-form solution to the Pareto optimal collaboration period cannot be obtained. Nevertheless, the effects of the firm's initial level of indebtedness and capital stock, depreciation rate, expected profit and uncertainty on the continuation period can be obtained by taking the total differential equation 10 and considering the second-order condition 11 as summarised in propositions 1 to 5 and displayed diagramatically by shifting the *MBFC* and *MCOC* curves. The proofs to these propositions are provided by the Appendix.

Proposition 1 (*the effect of the firm's initial liabilities* B_0): If the bondholder's time preference rate exceeds the stockholder's

time preference rate (that is, $\gamma > \rho$), the greater the firm's initial liabilities the shorter the collaboration period. In contrast, if the stockholder's rate of time preference exceeds the bondholder's rate of time preference, the greater the firm's initial liabilities the longer the collaboration period. If, however, both claimants have the same rate of time preference, the collaboration period is not affected by the firm's initial liabilities.

The geometrical interpretation of this proposition is as follows. When $\gamma > \rho$ the $MCOC$ is upward sloping. In which case, a rise in B_0 steepens the $MCOC$ curve and shifts it upward and therefore reduces T^o as displayed in Figure 3a. When $\gamma < \rho$ the MCOC is downward sloping. In which case, a rise in B_0 steepens the $MCOC$ curve and shifts it downward and therefore increases T^o as displayed in Figure 3b. However, when $\gamma = \rho$ the $MCOC$ curve is flat and its location is not affected by the size of B_0.

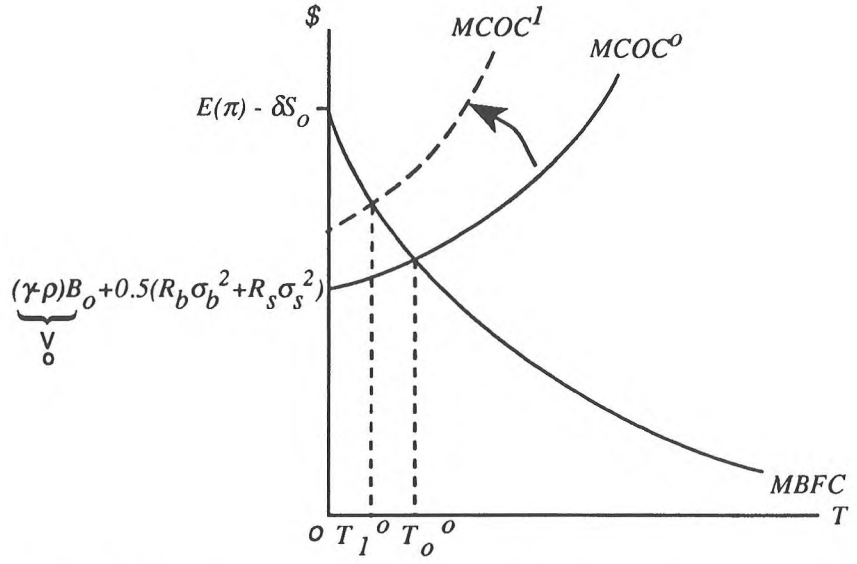

Figure 3a: $\gamma > \rho$, a rise in B_0 shortens T^0

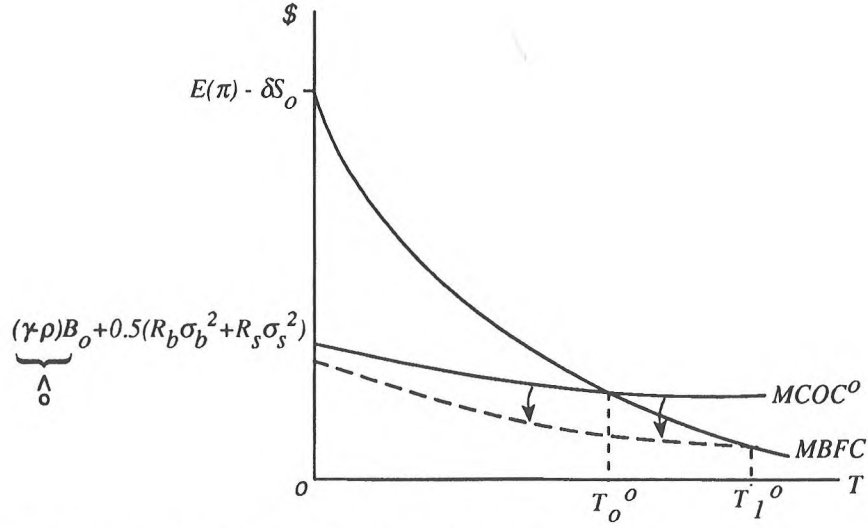

Figure 3b: $\gamma < \rho$, a rise in B_0 lengthens T^0

Proposition 2 (*The effect of the firm's liquidation value and depreciation rate*): For any combination of claimants' rate of time preference, the greater the firm's liquidation value, the shorter the collaboration period. This effect is fortified by the depreciation rate of the firm's productive capital.

Geometrically, this proposition can be explained as follows. An increase in either the firm's liquidation value or depreciation rate shifts the *MBFC* curve downward and consequently leads to a smaller T^o for all possible combinations of claimants' rates of time preference as displayed for instance by Figure 4.

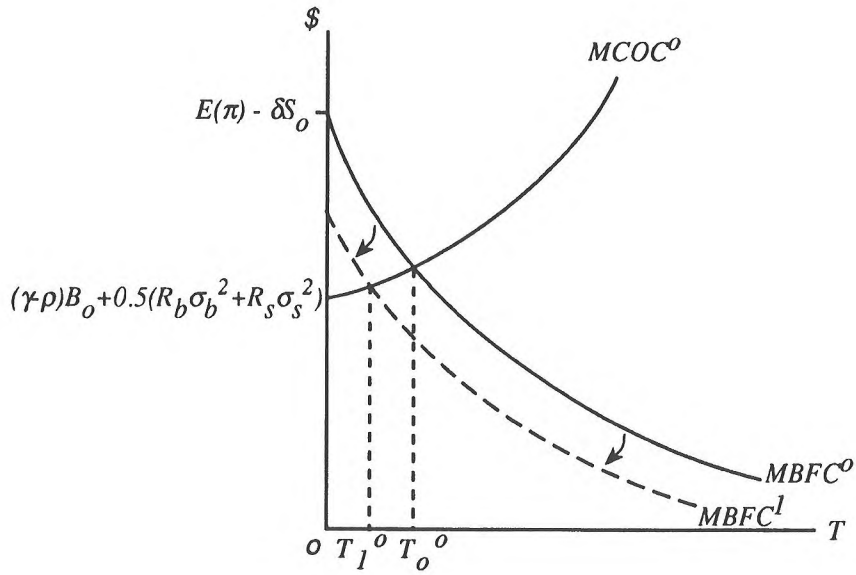

Figure 4: $\gamma < \rho$, a rise in either S_0 or δ shortens the collaboration period

Proposition 3 (The effect of the bond's contracted interest rate):

$$\frac{dT^o}{di} \gtreqless 0 \text{ as } [1 - (i-\rho)T^o]\, e^{(i-\rho)T^o} \lesseqgtr [1 + (i-\gamma)T^o]\, e^{(i-\gamma)T^o}$$

That is, the effect of the interest rate, i, on the collaboration period is not clear *a priori*. On the one hand, a high interest rate on the firm's bonds implies a high rate of debt accumulation for the firm (that is, a steeper *MCOC* curve) and hence discourages the stockholder from continuation. On the other hand, it enhances the attraction of the firm's bond to the bondholder relatively to alternative financial transactions and hence reduces the compensation payment required for the bondholder's collaboration (that is, a flatter *MCOC* curve).

Proposition 4 (The effect of uncertainty): If both claimants are risk averse, then for any combination of claimants' rate of time preference the greater the level of uncertainty about the claimants' net returns from continuation the shorter the collaboration period.

The underlying rationale of this proposition is that an increase in the level of uncertainty about the net returns raises the costs of risk bearing to the stockholder and hence moderates his/her inclination for continuation. Moreover, an increase in the costs of risk bearing to the bondholder raises the compensation payment in a case of continuation for keeping the bondholder as well of as under immediate liquidation and hence shortening the collaboration period. Geometrically, an increase in the costs of risk bearing shifts the *MCOC* curve upward and hence shorten the period of collaboration for any combination of claimants' rate of time preference as displayed for instance by Figure 5.

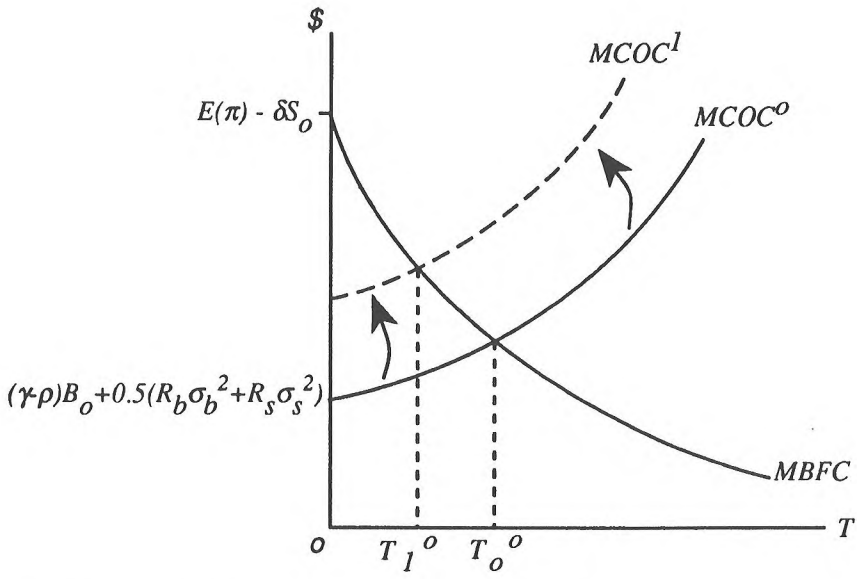

Figure 5: $\gamma > \rho$, a rise in the uncertainty level shortens the collaboration period

Proposition 5 (The effect of the firm's expected profit): For any combination of claimants' rates of time preference, the larger the firm's expected instantaneous profit the longer the collaboration period.

The underlying rationale of this proposition is straightforward. A more optimistic outlook about the firm's instantaneous profit leads to a larger marginal benefit from continuation of the firm operation and hence lengthens the collaboration period for any combination of the claimants' rates of time preference. Geometrically, despite steepening the *MBFC* curve, an increase in $E(\pi)$ shifts that curve upward as displayed for instance by Figure 6.

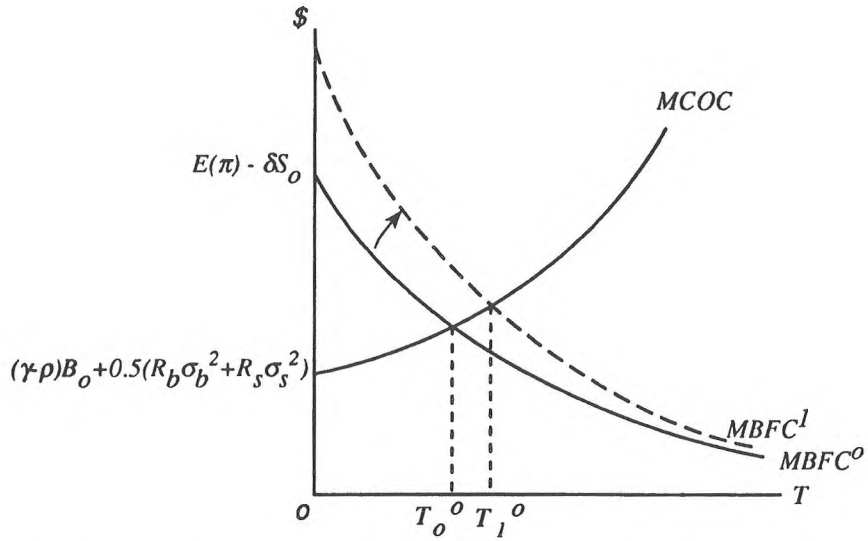

Figure 6: $\gamma > \rho$, a rise in the expected instantaneous profit lengthens the collaboration period

7 EFFECTS OF INDUSTRIAL AND MACROECONOMIC CONDITIONS ON THE COLLABORATION PERIOD

As indicated earlier, the firm's expected profits are a key factor in the determination of the net returns to the stockholders from continuation, and hence, in the determination of the length of the collaboration period. These expected profits might be affected by the industrial and global economic environment in which the firm operates. This section explores the possible effects of industrial and macroeconomic conditions on the liquidation decision and timing. It is assumed that the firm operates in a Cournot-Nash equilibrium where all firms are expected profit maximisers, take their competitors' supply as given (that is, zero conjectural variation), and have an identical and constant marginal cost of

production (that is, $mc_n(q_n) = mc$ for all q_n and for every $n = 1,...,N$). These firms are, however, different with regard to their financial (debt-equity) structure.

It is assumed further that the inverse demand function for the industry's product consists of an isoelastic deterministic part and a stochastic part due to uncertainty about the consumer aggregate income and the prices of all other goods:

$$p(Q, Y, P) = Q^{-1/\xi} + \tilde{p}\ (P, Y). \qquad (16)$$

Here, Q denotes the quantity demanded; ξ is the constant price elasticity of the deterministic part; Y and P are random variables with means $\bar{Y}$ and $\bar{P}$ and finite variances and covariance, denoting the aggregate consumer income and the overall price level of all the other goods, respectively. The stochastic part, $\tilde{p}$, is twice differentiable; $\tilde{p}_Y$ is positive (negative) in the case of a normal (inferior) good and equal to zero otherwise; and $\tilde{p}_p$ is positive (negative) when the industry's product is essentially a substitute (complementary) to the rest of the goods and zero otherwise. In order to introduce the effects of variations in the macroeconomic factors Y and P on the bankruptcy decision, a second-order Taylor approximation of the inverse demand function at the means' point $(\bar{Y}, \bar{P})$ is considered:

$$p(Q, Y, P) \cong Q^{-1/\xi} + \tilde{p}_Y(Y - \bar{Y}) + \tilde{p}_P(P - \bar{P}) + 0.5\tilde{p}_{YY}(Y - \bar{Y})^2$$
$$+ 0.5\tilde{p}_{PP}(P - \bar{P})^2 + 0.5\tilde{p}_{YP}(Y - \bar{Y})(P - \bar{P}). \qquad (17)$$

It is postulated that the firms' expected profit-maximisation problem is

$$\max_{q_n} E\{[p(Q, Y, P) - mc]q_n\}$$

subject to

$$Q = \sum_{n'=1}^{N} q_{n'} \quad . \tag{18}$$

The solution of this problem yields the Cournot-Nash expected product price

$$E(p^*) = \frac{1}{1 - 1/\xi N}\{mc - 0.5[\tilde{p}_{YY}Var(Y) + \tilde{p}_{PP}Var(P) + \tilde{p}_{YP}Cov(Y, P)]\} \tag{19}$$

as is shown in the Appendix.

The firm's expected instantaneous profit is given by

$$E[\pi(t)]_n = E[(p^* - mc)\, q_n^*] \tag{20}$$

and by recalling equation 22 and that q_n^* is a chosen quantity and all of the firms have an identical production operation, the individual firm's expected instantaneous profit can be expressed further as:

$$E[\pi(t)]_n = \frac{Q^*}{N}[E(p^*)-mc] = \frac{mcQ^*}{N(\xi N-1)} - \frac{0.5\xi Q^*}{\xi N-1}[\tilde{p}_{YY}Var(Y) + \tilde{p}_{PP}Var(P) + \tilde{p}_{YP}Cov(Y, P)] \tag{21}$$

where Q^* is the industry's volume of sales. *A priori*, the signs of the second derivatives of $\tilde{p}$ and $Cov(Y, P)$ are unknown and hence the effects of uncertainty about the macroeconomic

conditions, as well as the effects of the degree of industry concentration and price elasticity, on the firm's expected operating profit are not clear. When the overall effect of aggregate income and consumer price volatility on the industry product price (that is, $\tilde{p}_{YY}Var(Y) + \tilde{p}_{PP}Var(P) + \tilde{p}_{YP}Cov(Y, P)$) is negative (positive) the expected profit is greater (smaller) than that under certainty ($mcQ^*/N(\xi N-1)$). The discrepancy between the two decreases with the level of the price elasticity and the number of producers. Note further that the assumption of time-invariant expected profit underlying the transition from equation 1 to equation 2 requires that the industry's structure (for example, number of rival firms, marginal cost of production and demand elasticity), as well as the functional form of $\tilde{p}$ and the variances and covariance of Y and P, do not change as time passes.

The effects of the industrial and macroeconomic conditions on the collaboration period can now be obtained by virtue of equations 10 and 21. They are summarised by the following three propositions whose proofs are given in the Appendix.

Proposition 6 (The effects of aggregate price and income volatility):

$$\frac{dT^o}{dVar(Y)} \gtreqless 0 \text{ as } \tilde{p}_{YY} \lesseqgtr 0, \quad \frac{dT^o}{dVar(P)} \gtreqless 0 \text{ as } \tilde{p}_{PP} \lesseqgtr 0, \text{ and}$$

$$\frac{dT^o}{dCov(Y,P)} \gtreqless 0 \text{ as } \tilde{p}_{YP} \lesseqgtr 0.$$

This proposition indicates that the effects of the aggregate consumer price and income volatility crucially depend upon the signs of the second derivatives of the inverse demand function' stochastic part.. If the marginal effect of income on

the firm's product price (that is, the extent of being a normal good) rises (diminishes) with Y, the higher the volatility of aggregate income the shorter (longer) the optimal continuation period of the firm's operation. If the marginal effect of the aggregate consumer price on the firm's product price (that is, the substitution effect) increases (diminishes) with P, the higher the volatility of the consumer price level the shorter (longer) the optimal collaboration period. Finally, if the extent of being a normal good is positively (adversely) affected by a rise in the aggregate consumer price level, the larger the correlation between the aggregate income level and the consumer price level the shorter (longer) the optimal collaboration period.

Proposition 7 (The effect of the price elasticity of the demand for the firm's product):

$$\frac{dT^o}{d\xi} \gtreqless 0 \text{ as } 0.5[\,\tilde{p}_{YY}Var(Y) + \tilde{p}_{PP}Var(P) + \tilde{p}_{YP}Cov(Y,P)] \gtreqless mc.$$

This proposition indicates that if the effect of the aggregate consumer income and price volatility on the expected price of the insolvent firm's product is grater (smaller) than the firm's marginal cost of production, the more elastic the demand for the firm's product, the longer (shorter) the optimal continuation period of the firm's operation.

Proposition 8 (The effect of the industry's level of concentration):

$$\frac{dT^o}{d\left(\frac{1}{N}\right)} \gtreqless 0 \text{ as } mc \gtreqless 0.5[\,\tilde{p}_{YY}Var(Y) + \tilde{p}_{PP}Var(P) + \tilde{p}_{YP}Cov(Y,P)][\xi^2 N/(2\xi N-1)].$$

This proposition indicates that if the firm's marginal costs

of production are larger (smaller) than the effect of the aggregate consumer income and price volatility on the expected price of the insolvent firm's product adjusted to the industry size and demand elasticity, the more concentrated the industry the longer (shorter) the optimal continuation period of the firm's operation.

8 SUMMARY

The analysis of the optimal continuation of an insolvent firm's operation was conducted within a framework in which the firm's managers are sheer profit maximisers; the firm's claimants have different priority in liquidation and unbiased expectations about their net returns from continuation, but variances that are proportionate to the length of the continuation period, and maximise their expected utility from those net returns. The financial crisis arises from the firm's current inability to pay back the bond value (or loan) that has matured. The Pareto optimal period of continuation of the firm's operation was found by maximising the stockholder's expected utility from the net returns from continuation while maintaining the bondholder's utility level equal to that under immediate liquidation with an adequate compensation payment. The solution to this problem displayed the effects of the firm's initial assets and liabilities and their rates of depreciation and accumulation, the claimants' rates of time preference and attitudes toward risk, the firm's expected profits and macroeconomic and industrial conditions on the optimal continuation period of the firm's operation and liquidation timing.

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APPENDIX

Proof of Proposition 1: Assuming that all other things remain the same, the total differential of the necessary condition 10 with respect to B_0 and T^o implies

$$\frac{dT^o}{dB_0} = \frac{(1-\rho)\, e^{(i-\rho)T^o} - (i-\gamma)\, e^{(i-\gamma)T^o}}{H}$$

and since $H < 0$, $\frac{dT^o}{dB_0} \gtreqless 0$ *as* $i \lesseqgtr \rho + (i-\gamma)\, e^{(\rho-\gamma)T^o}$.

Proof of Proposition 2: Assuming that all other things remain the same, the total differential of the necessary condition 10 with respect to S_0 and T^o implies

$$\frac{dT^o}{dS_0} = \frac{\delta e^{-\rho T^o}}{H} < 0$$

since $H<0$.

Proof of Proposition 3: Assuming that all other things remain the same, the total differential of the necessary condition 10 with respect to i and T^o implies:

$$\frac{dT^o}{di} = \frac{\left\{[1-(i-\rho)T^o]e^{(i-\rho)T^o} - [1+(i-\gamma)T^o]e^{(i-\gamma)T^o}\right\}B_0}{H}$$

and since $H < 0$,

$$\frac{dT^o}{di} \gtreqless 0 \text{ as } [1 - (i-\rho)T^o]\, e^{(i-\rho)T^o} \gtreqless [1 + (i-\gamma)T^o]\, e^{(i-\gamma)T^o}$$

Proof of Proposition 4: Assuming that all other things remain the same, the total differential of the necessary condition 10 with respect to $0.5R_s\sigma_s^2$ and T^o implies

$$\frac{dT^o}{d(0.5R_s\sigma_s^2)} = \frac{1}{H} < 0$$

and similarly,

$$\frac{dT^o}{d(0.5R_b\sigma_b^2)} = \frac{1}{H} < 0.$$

Proof of Proposition 5: Assuming that all other things remain the same, the total differential of the necessary condition 10 with respect to $E(\pi)$ and T^o implies

$$\frac{dT^o}{dE(\pi)} = \frac{-e^{-\rho T^o}}{H} > 0$$

since $H<0$.

The Solution to the Firm's Expected Profit-Maximisation Problem

Given the second-order Taylor approximation of the inverse demand function, the firm's objective function can be displayed as

$$\max_{q_n} E\{[Q^{-1/\xi} + \tilde{p}_Y(Y - \bar{Y}) + \tilde{p}_P(P - \bar{P}) + 0.5\,\tilde{p}_{YY}(Y - \bar{Y})^2$$

$$+ 0.5\,\tilde{p}_{PP}(P - \bar{P})^2 + 0.5\,\tilde{p}_{YP}(Y - \bar{Y})(P - \bar{P}) - mc]\,q_n\}.$$

Taking the expectation of the objective function and recalling that the choice variable q_n is deterministic and that $E(Y - \bar{Y}) = 0 = E(P - \bar{P})$, the objective function can be expressed further as

$$\max_{q_n} \{Q^{-1/\xi} q_n + 0.5[\tilde{p}_{YY}Var(Y) + \tilde{p}_{PP}Var(P) + \tilde{p}_{YP}Cov(Y,P)]q_n - mc\,q_n\}.$$

The first-order condition for maximum is

$$-(1/\xi)Q^{*-1/\xi}\,q_n^*/Q^* + Q^{*-1/\xi} - mc + 0.5\,[\tilde{p}_{YY}Var(Y) + \tilde{p}_{YY}Var(P) + \tilde{p}_{YP}Cov(Y,P)] = 0$$

and since $Q^{*-1/\xi} = E(p^*)$ and $q_n^*/Q^* = 1/N$ (all firms have identical production operation)

$$E(p^*) = \frac{1}{1 - 1/\xi N}\{mc - 0.5[\tilde{p}_{YY}Var(Y) + \tilde{p}_{PP}Var(P) + \tilde{p}_{YP}Cov(Y,P)]\}.$$

Proof of Proposition 6: By virtue of proposition 5 and equation 21

$$\frac{dT^0}{dVar(Y)} = \frac{dT^O}{dE(\pi)}\frac{dE(\pi)}{dVar(Y)} = -\frac{0.5\xi Q^*}{\xi N - 1}\frac{dT^o}{dE(\pi)}\tilde{p}_{YY} \underset{<}{\overset{>}{=}} 0 \text{ as } \tilde{p}_{YY} \underset{>}{\overset{<}{=}} 0,$$

$$\frac{dT^0}{dVar(P)} = \frac{dT^O}{dE(\pi)}\frac{dE(\pi)}{dVar(P)} = -\frac{0.5\xi Q^*}{\xi N - 1}\frac{dT^o}{dE(\pi)}\tilde{p}_{PP} \underset{<}{\overset{>}{=}} 0 \text{ as } \tilde{p}_{PP} \underset{>}{\overset{<}{=}} 0,$$

and

$$\frac{dT^0}{dCov(Y,P)} = \frac{dT^O}{dE(\pi)} \frac{dE(\pi)}{dCov(Y,P)} = -\frac{0.5\xi Q^*}{\xi N - 1} \frac{dT^o}{dE(\pi)} \tilde{p}_{YP} \gtrless 0$$

as $\tilde{p}_{YP} \lessgtr 0$.

Proof of Proposition 7: By virtue of proposition 5 and equation 21

$$\frac{dT^o}{d\xi} = \frac{Q^*\{mc - 0.5\,[\,\tilde{p}_{YY}\,Var(Y) + \tilde{p}_{PP}\,Var(P) + \tilde{p}_{YP}\,cov(Y,P)]\}\; e^{-\rho T^o}}{(\xi N-1)^2\; H}$$

and since $H<0$,

$$\frac{dT^o}{d\xi} \gtreqless 0 \text{ as } 0.5\,[\,\tilde{p}_{YY}Var(Y) + \tilde{p}_{PP}Var\,(P) + \tilde{p}_{YP}Cov(Y,P)] \gtreqless mc.$$

Proof of Proposition 8: By virtue of proposition 5 and equation 21

$$\frac{dT^o}{dN} = \frac{Q^*\{(2\xi N-1)\,mc - 0.5\,\xi^2\,N^2\,[\tilde{p}_{YY}\,Var(Y) + \tilde{p}_{PP}\,Var(P) + \tilde{p}_{YP}\,cov(Y,P)]\}\, e^{-\rho T^o}}{N^2(\xi N-1)^2\; H}$$

and recalling that $H<0$,

$$\frac{dT^o}{d\left(\frac{1}{N}\right)} \gtreqless 0 \text{ as } mc \gtreqless 0.5[\tilde{p}_{YY}Var(Y) + \tilde{p}_{PP}Var(P) + \tilde{p}_{YP}Cov(Y,P)][\xi^2 N/(2\xi N-1)].$$

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