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# Measuring Multiple Parameters in a Self-mixing Optical Feedback System

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**Abstract**—We propose a new approach that yields the values of multiple parameters at the same time for self-mixing optical feedback interferometric systems. These parameters are the linewidth enhancement factor of Semiconductor Lasers, the optical feedback level factor as well as vibration information of a target including the frequency and the amplitude. The method is based on optical feedback interferometry. Its effectiveness has been confirmed by computer simulations and experiments.

**Keywords**—Linewidth enhancement factor; optical feedback; self-mixing interferometry; semiconductor laser; fitting algorithm; vibration measurement

## I. INTRODUCTION

The self-mixing optical feedback interferometric (SMOFI) effect occurs when a small fraction of the light emitted by a semiconductor laser (SL) is backscattered or reflected by an external target and re-enters the laser active cavity, resulting in the modulation of both the amplitude and the frequency of the lasing field. Initially, this back-reflection was seen as a major source of disturbance, strongly changing both frequency and amplitude of the lasing field. Presently, sensors based on this physical phenomenon using low-cost commercial laser diodes (LD) have been designed for a variety of applications such as dimensional control, vibration measurements, blood flow measurement, sound reproduction, angle measurements or even mass-market mobile telephones [1].

The existing vibration measurement methods based on SMOFI effect usually achieve a basic accuracy of a half-wavelength (typically 400nm). With suitable signal processing techniques, the accuracy could be improved up to 40 nm. However, to process the signal, the two parameters called linewidth enhancement factor (LEF)  $\alpha$  and optical feedback level factor  $C$  need to be known. Measuring  $\alpha$  has been an active research topic and extensive work has been conducted [2]. However, most existing approaches require sophisticated system setups or involve many complicated procedures, which thus make implementation difficult [3,4]. With regard to the measurement of  $C$ , there has been not much work reported yet. So when using the SMOFI based measurement, people usually assumed that LEF is known and choose it to be a value around

6, and that  $C$  is constant during measurement whose value is obtained by a rough evaluation before the measurement [5]. Obviously such assumptions bring out error to the results. In fact, it is difficult to keep a fixed  $C$  during test. For achieving a more accurate measurement, we should measure  $\alpha$  and  $C$  in real time so that to realize other parameters measurements. Hence it is significant to develop a technique that yields values of all the parameters related to the moving target as well as the SL's itself based on SMOFI effect.

Recently, two new approaches [6,7] have been proposed for the measuring  $\alpha$  based on SMOFI effect. One is in the cases of moderate feedback with  $1 < C < 3$ , one is in weak feedback regime with  $C < 1$ .

The paper produces an efficient algorithm for extracting values of  $\alpha$  and  $C$  as well as the vibration information, using the observed data from the SMOFI system. The proposed algorithm is characterized by robustness and immunization to noisy environment and high efficiency.

## II. BASIC THEORY

There are two alternative and equivalent methods for describing self-mixing optical feedback interferometric effects: the Long and Kobayashi equations based approach [8] and the three-mirror cavity based approach [9]. Both approaches yield the same description about the behavior of a single-mode SL with optical feedback, given by the following equations:

$$\phi_F(\tau) = \phi_0(\tau) - C \cdot \sin[\phi_F(\tau) + k] \quad (1)$$

$$P(\phi_F(\tau)) = P_0 [1 + mG(\phi_F(\tau))] \quad (2)$$

$$G(\phi_F(\tau)) = \cos(\phi_F(\tau)) \quad (3)$$

where  $k = \arctan(\alpha)$  and  $\alpha$  is linewidth enhancement factor;  $\phi_0(\tau) = \omega_0 \tau$  and  $\phi_F(\tau) = \omega_F(\tau) \tau$ , where  $\omega_0$  and  $\omega_F(\tau)$  are the angular frequencies of the SL without and with feedback respectively;  $\tau = 2L/c$ , where  $L$  is the length of the external cavity and  $c$  the speed of light;  $C$  is the feedback factor.

The power emitted by the SL is given by Equation (2) where  $P(\phi_F(\tau))$  and  $P_0$  are the power emitted by the SL with and without the external cavity respectively.  $P(\phi_F(\tau))$  is also called self-mixing interference (SMI) signal. It is seen that with the external cavity, the emitted power deviated from  $P_0$  by a factor of  $mG(\phi_F(\tau))$  where  $m$  is called modulation index (typical  $m \approx 10^{-3}$ ), and  $G(\phi_F(\tau))$  is called the interferometric function which gives the effect of the external cavity length to the emitted power.

Consider the case where the external target is subjected a simple harmonic vibration that is,

$$L(t) = L_0 + \Delta L_0 \sin(2\pi ft) \quad (4)$$

where  $L_0$  is the initial length of the external cavity,  $f$  is the vibration frequency,  $\Delta L_0$  is vibration amplitude, and  $t$  is time. From equations (1), (3) and (4) and using the relationship  $\tau(t) = \frac{2L(t)}{c}$ , we have

$$\begin{aligned} \phi_F(\tau_0) + \phi_F[\Delta\tau(t)] &= \phi_0(\tau_0) + \phi_0[\Delta\tau(t)] \\ &\quad - C \sin\{\phi_F(\tau_0) + \phi_F[\Delta\tau(t)] + k\} \end{aligned} \quad (5)$$

$$G(\phi_F(\tau)) = \cos(\phi_F(\tau_0) + \phi_F[\Delta\tau(t)]) \quad (6)$$

where  $\tau_0 = \frac{2L_0}{c}$ ,  $\Delta\tau(t) = \frac{2\Delta L_0 \sin(2\pi ft)}{c}$ .

By adjusting the external cavity slightly, an initial  $L_0$  can be found which is according to the maxim of the emitted power  $P(\phi_F(\tau))$ . In this case  $\phi_F(\tau_0) = 2p\pi$ , where  $p$  is positive integer. From equation (5),  $\phi_0(\tau_0) = C \cdot \sin(k)$ . Hence Equations (5) and (6) are simplified as

$$\begin{aligned} \phi_F[\Delta\tau(t)] &= C \cdot \sin(k) + \phi_0[\Delta\tau(t)] \\ &\quad - C \cdot \sin\{\phi_F[\Delta\tau(t)] + k\} \end{aligned} \quad (7)$$

$$G(\phi_F(\tau)) = \cos(\phi_F[\Delta\tau(t)]) \quad (8)$$

In order to describe the measurement algorithm in section III, by sampling the above equations, we have the discrete-time equations as follows:

$$\phi_0(n) = \phi_{00} \sin\left(\frac{2\pi n}{N_0}\right) \quad (9)$$

$$\phi_F(n) = C \cdot \sin(k) + \phi_0(n) - C \cdot \sin[\phi_F(n) + k] \quad (10)$$

$$G(n) = \cos(\phi_F(n)) \quad (11)$$

where  $n$  is the discrete time index,  $\phi_{00} = \frac{4\pi\Delta L_0}{\lambda_0}$ ,  $N_0 = \frac{f_s}{f}$  and  $f_s$  is the sampling frequency.

Equations (9)-(11) are the model describing the behavior of an SMOFI system with an external target subject to simple harmonic vibration. Clearly from the model, we should be able to determine the parameters  $N_0$ ,  $\phi_{00}$ ,  $C$  and  $k$  based on the signal waveform  $G(n)$ . Then we can work out the vibration frequency  $f$  and amplitude  $\Delta L_0$  from the target as well as the parameter  $\lambda_0$  and  $C$  related to SL.

### III. THE MEASURING ALGORITHMS OF THE PARAMETERS

The proposed approach consists of three steps. The first step is to obtain  $G(n)$  from the observed SMI signal  $P(n)$  using Equation (2). Then the second step is to get  $f$  and a rough  $\phi_{00}$ , by using auto-correlation and phase unwrapping methods. Finally, data fitting techniques are used to yield the values of  $C$ ,  $k$  and  $\phi_{00}$  respectively. Some technical details associated with the above steps are described as follows.

#### A. Measuring $N_0$

From Equations (9)-(11) it is seen that  $N_0$  is the fundamental period of  $G(n)$ . Hence  $N_0$  can be estimated by considering the the auto-correlation function of  $G(n)$ , given as follows

$$r_G(m) = \frac{1}{N} \sum_{n=0}^{N-1-m} G_N(n) \cdot G_N(n+m) \quad (12)$$

where  $N$  is the data length of  $G(n)$  being used.  $m$  is time delay which varies from  $-(N-1)/2$  to  $(N-1)/2$ . An example of an SMI signal and the autocorrelation defined by Equation (12) is given in Figure 1. Obviously,  $N_0$  can be obtained by detecting the positions of the peaks in  $r_G(m)$ .

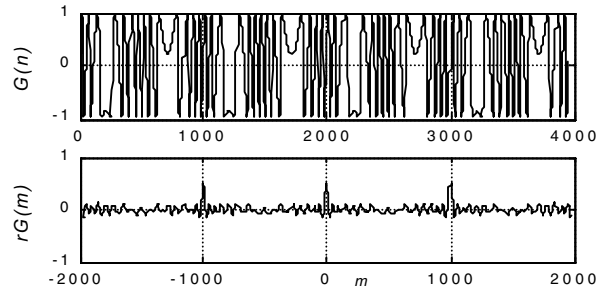


Figure 1. The autocorrelation result of  $G(n)$

#### B. Measuring vibrating amplitude

In order to get the vibration amplitude  $\Delta L$ , or  $\phi_{00}$ , we should get the phase  $\phi_F(n)$ . Considering that Hilbert transform

of  $G(n)$  is that  $H[G(n)] = -\sin(\phi_F(n))$ ,  $\phi_F(n)$  can be calculated by:

$$\phi_F(n) = \text{tg}^{-1}\left(-\frac{H[G(n)]}{G(n)}\right) \quad (13)$$

However Equation (13) will yield phases that are wrapped into  $[-\pi, \pi]$ , and phase unwrapping must be done in order to retrieve the true phase. The unwrapping is performed by changing absolute jumps greater than  $\pi$  to their  $2\pi$  complement.

From equation (10), we may see, if removing the constant component, the difference between  $\phi_0(n)$  and  $\phi_F(n)$  is  $C \cdot \sin[\phi_F(n) + k]$ . By using  $\tilde{\phi}_{00}$  to denote the amplitude of fluctuation of  $\phi_F(n)$ , the maximal error is  $2C$  if we use  $\tilde{\phi}_{00}$  to estimate  $\phi_{00}$ . This is a rough measurement for  $\phi_{00}$  prepared for the next section. Considering the phase unwrapping operation  $\tilde{\phi}_{00}$  can be estimated by:

$$\tilde{\phi}_{00} = \frac{1}{4(N-N_0)} \sum_{n=1}^{N-N_0} [\text{Unwra}(\phi_F(n-N_0)) - \text{Unwra}(\phi_F(n))] \quad (14)$$

where  $\text{Unwra}[\phi_F(n)]$  denotes the unwrapping operation of  $\phi_F(n)$ .

#### C. Measuring $C$ , $k$ and

Under the case of knowing  $N_0$ , and a rough  $\tilde{\phi}_{00}$ ,  $C$  and  $k$  as well as an accurate  $\phi_{00}$  can be estimated by a similar data fitting method proposed in reference [10], in which a gradient-based optimization algorithm achieves the best data-to-theoretical model fitting. We define an objective function as:

$$F(\hat{\phi}_{00}, \hat{k}, \hat{C}) = \sum_{n=1}^N \left\{ G(n) - \hat{G}(n, \hat{\phi}_{00}, \hat{k}, \hat{C}) \right\}^2 \quad (15)$$

where  $\hat{G}(n, \hat{\phi}_{00}, \hat{k}, \hat{C})$  are the values based on computation using models (9)-(11) incorporating the estimated values of  $\hat{\phi}_{00}$  and  $\hat{C}$  and  $\hat{k}$ .  $\hat{\phi}_{00}$  and  $\hat{C}$  and  $\hat{k}$  are considered as optimal if the above objective function is minimized.

Iterative equations for optimal estimating the above parameters within equation (15) are:

$$\hat{\phi}_{00j} = \hat{\phi}_{00j-1} - \mu \frac{\partial F}{\partial \hat{\phi}_{00}} \Big|_{\hat{\phi}_{00}=\hat{\phi}_{00j-1}} \quad (16)$$

$$\hat{C}_j = \hat{C}_{j-1} - \mu \frac{\partial F}{\partial \hat{C}} \Big|_{\hat{C}=\hat{C}_{j-1}} \quad (17)$$

$$\hat{k}_j = \hat{k}_{j-1} - \mu \frac{\partial F}{\partial \hat{k}} \Big|_{\hat{k}=\hat{k}_{j-1}} \quad (18)$$

where  $\mu > 0$  is the step size and the subscript  $j$  refers to the iteration index for updating the parameters.

The gradients of  $F(\hat{\phi}_{00}, \hat{k}, \hat{C})$  with respect to parameters  $\hat{\phi}_{00}$  and  $\hat{C}$  and  $\hat{k}$  can be derived as follows:

$$\frac{\partial F}{\partial \hat{\phi}_{00}} = 2\hat{\phi}_{00} \sum_{n=1}^N \left\{ G(n) - \hat{G}(n, \hat{\phi}_{00}, \hat{k}, \hat{C}) \right\} \times \sin(\hat{\phi}_F(n)) \cdot \sin\left(\frac{2\pi n}{N_0}\right) \quad (19)$$

$$\frac{\partial F}{\partial \hat{C}} = 2 \sum_{n=1}^N \left\{ G(n) - \hat{G}(n, \hat{\phi}_{00}, \hat{k}, \hat{C}) \right\} \times \sin(\hat{\phi}_F(n)) \cdot \{ \sin(\hat{k}) - \sin[\hat{\phi}_F(n) + \hat{k}] \} \quad (20)$$

$$\frac{\partial F}{\partial \hat{k}} = 2\hat{C} \sum_{n=1}^N \left\{ G(n) - \hat{G}(n, \hat{\phi}_{00}, \hat{k}, \hat{C}) \right\} \times \sin(\hat{\phi}_F(n)) \cdot \{ \cos(\hat{k}) - \cos[\hat{\phi}_F(n) + \hat{k}] \} \quad (21)$$

where  $\hat{\phi}_F(n)$  are the values obtained by computing equations (9) and (10) incorporating the relevant iterative values of  $\hat{\phi}_{00}$ ,  $\hat{C}$  and  $\hat{k}$ .

#### IV. PERFORMANCE SIMULATION

Computer simulations are performed to test the effectiveness of the proposed algorithm. The proposed algorithm is summarized as:

- Step 1, create samples  $G(n)$  (for  $n=1, 2, \dots, N$ ) using equations (9)-(11).
- Step 2, obtain  $N_0$  by the auto-correlation function of  $G(n)$  using equation (12).
- Step 3, use equation (13) to obtain the phase  $\phi_F(n)$  from  $G(n)$  and unwrapped the result.
- Step 4, measure  $\phi_{00}$  roughly using equations (14), which is denoted as  $\tilde{\phi}_{00}$ .
- Step 5, set arbitrary initial values for  $C$  and  $k$ , and use  $\tilde{\phi}_{00}$  as the initial value of  $\phi_{00}$ , the evaluated parameters are marked as  $\hat{C}$  and  $\hat{k}$  and  $\hat{\phi}_{00}$ .
- Step 6, use Equations (9) and (10) to yield the evaluated  $\phi_F(n)$  incorporating the estimated values of  $\hat{\phi}_{00}$  and  $\hat{C}$  and  $\hat{k}$  as well as  $N_0$  gotten in step 2.
- Step 7, calculate the gradients using Equations (19)-(21).
- Step 8: Update  $\hat{C}$  and  $\hat{k}$  and  $\hat{\phi}_{00}$  using Equations (16)-(18). And then Go to Step 6 or stop.

Table I shows the simulation results for with the true parameters values given. In order to emulate the practical situation, a small white noise is also added with a signal-to-noise ratio of 20dB.

TABLE I. EVALUATION RESULTS FOR THE PARAMETERS

| True values                 | $N_0$             | 500    | 1500   | 2000   | 1500   |
|-----------------------------|-------------------|--------|--------|--------|--------|
|                             | $\phi_{00}$       | 12     | 30     | 37     | 41     |
| C                           | 0.2               | 0.4    | 0.7    | 0.9    |        |
| $\alpha$                    | 1                 | 4      | 5      | 6      |        |
| Results and their Max Dev.% | $N_0$             | 500    | 1499   | 2000   | 1499   |
|                             | Dev.%             | 0      | 0      | 0      | 0      |
|                             | $\hat{\phi}_{00}$ | 12.001 | 30.001 | 36.100 | 41.000 |
|                             | Dev.%             | 0.012  | 0.002  | 0.001  | 0.001  |
|                             | $\hat{C}$         | 0.201  | 0.403  | 0.701  | 0.900  |
|                             | Dev.%             | 0.231  | 0.767  | 0.085  | 0.038  |
| $\hat{\alpha}$              | 0.998             | 3.963  | 4.983  | 6.001  |        |
| Dev.%                       | 0.165             | 0.934  | 0.334  | 0.011  |        |

### V. EXPERIMENT

The OFSMI experimental setup used for generating SMI signal is shown in Figure 2. The LD is biased with a dc current, a microscope objective focuses light on the metal plate. The metal plate vibrates in a simple harmonic way induced by a loudspeaker driven by a sine signal. The SMI signal is obtained from the monitor photodiode (PD) connected to a trans-impedance amplifier and then acquired by personal computer. The used A/D acquisition frequency is 20KHz. An experimental SMI signal is shown in Figure 3. The measured results are shown in table II.

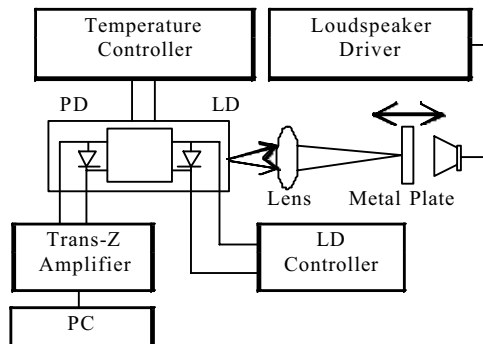


Figure 2. SMOFI experimental setup used for generating SMI signal

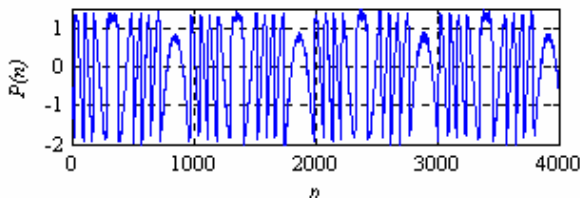


Figure 3. A SMI signal obtained from the above set-up

TABLE II. THE TEST RESULTS

| SMI Signal p-p (V) | 1.7       | 1.8        | 2.2       |
|--------------------|-----------|------------|-----------|
| $\alpha$           | 2.99±3.9% | 2.94±1.7%  | 3.21±1.8% |
| C                  | 0.47±1.8% | 0.65±2.8%  | 0.70±3.1% |
| f (Hz)             | 195±0.1%  | 195±0.1%   | 195±0.1%  |
| $\Delta L$ (nm)    | 1995±2.8% | 1989±0.15% | 1974±2.0% |

The tests are taken under the same loudspeaker driver signal (f=195Hz, Vp-p=10V) but with different feedback level C, that is different SMI signal amplitude(peak-peak value); Temperature controller is set at 25°C; the used LD operates at single mode with the injection current of 80mA. The tested objects are HL7851 LD and a vibration metal plate. At least 10 experiment waveforms are processed for each group of the measured parameters; Accuracy is calculated as the maximum deviation of measured results.

### VI. CONCLUSION

We have presented a new approach to measure all the values of the parameters related to the moving target as well as the SL's itself based on SMOFI effect. The effectiveness for the proposed algorithm has been confirmed from computer simulations and experiments.

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