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Elastic visco-plastic consolidation modelling of soft clays improved by geosynthetic vertical drains

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ABSTRACT: In this paper, an elastic viscoplastic (creep) model is incorporated into the consolidation equation to calculate settlements and excess pore pressures. A finite difference formulation for fully coupled axisymmetric consolidation is adopted to model both vertical and radial consolidation of a multi-layered soil. The effect of smear, non-linear stiffness and varying soil permeability with void ratio can be included in the analysis. The formulation is easily executed in an electronic spreadsheet such as Excel, and it is then validated by existing solutions. Finally, the analysis of a selected case history employing the writers' formulation is employed to analyse an embankment constructed on the Sunshine Motorway, Queensland, Australia stabilised with geosynthetics vertical drains. The embankment behaviour is analysed by the authors using an elastic viscoplastic model and compared with the field data. The analysis of this case history verifies the improved accuracy of the predictions due to creep effects, in contrast to the conventional predictions which disregarded creep.

1 INTRODUCTION

Geosynthetic prefabricated vertical drains (PVDs) and preloading have been employed widely to accelerate the consolidation process of soft clay. The main objective is to accelerate consolidation by shortening the drainage path. The behavior of soft clay foundations improved by vertical drains is usually predicted using closed-form solutions (Barron, 1948). The solutions were subsequently modified by Yoshikuni and Nakanodo (1974), Hansbo (1981) and Indraratna and Redana (2000). In these solutions, many simplifying assumptions including constant soil stiffness, permeability and time independent behaviour were made. Flexible numerical formulation can incorporate accurate soil behaviour for the consolidation problems, where the effects of non-linear stiffness, varying permeability and creep behaviour can be included explicitly.

In this paper, an elastic viscoplastic model is incorporated in the consolidation equation to calculate settlements and excess pore pressures. A finite difference formulation (FD) for fully coupled axisymmetric consolidation is adopted to model one-dimensional consolidation combining both vertical and radial drainage. It is then validated in contrast to existing solutions. Finally, the analysis is executed to analyse an embankment constructed on the Sunshine Coast, Queensland, Australia stabilised with PVDs.

2 ONE-DIMENSIONAL CONSOLIDATION EQUATION

The governing equation for the process of one-dimensional consolidation of a saturated soil considering both vertical and horizontal drainage can be expressed by (Barron, 1948):

$$-\frac{\partial \varepsilon_z}{\partial t} = \frac{k_v}{\gamma_w}\frac{\partial^2 u}{\partial z^2} + \frac{k_h}{\gamma_w}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) \quad (1)$$

where k_v and k_h are the coefficient of permeability for vertical and horizontal respectively, γ_w is the unit weight of water, ε_z is the vertical strain, u is the excess pore pressure, r and z are the radial coordinate axis and the vertical coordinate axis, respectively and t is time.

3 CREEP MODEL

Under constant effective stress, Bjerrum (1967) assumed that creep occurs both during and after primary consolidation. In this model, the settlement

can be divided into 2 parts: (a) an instant compression due to a reduction in void ratio, and (b) a delayed compression representing the volume reduction at unchanged effective stress (Fig. 1). Subsequently, Yin and Graham (1989) proposed the elastic visco-plastic model (EVP) based on the equivalent time line concept, which is able to capture soil behaviour observed in the field and laboratory. In this paper, the EVP model will be adopted (Yin and Graham, 1989). The strain at any given effective stress can be given by:

$$\varepsilon_z = \varepsilon_{z0}^{ep} + \frac{\lambda}{\nu}\ln\frac{\sigma_x'}{\sigma_{zo}'} + \frac{\psi}{\nu}\ln\frac{t_0 + t_e}{t_0} \tag{2}$$

where, ε_{z0}^{ep} is the strain at the reference point, ψ/ν is determined from the slope of creep strain plotted against ln(t_e), and t_e is equivalent time. This can be determined at the end of standard oedometer test. It is noted that the above parameters in Equation (2) can be determined from multi-stage loading oedometer test.

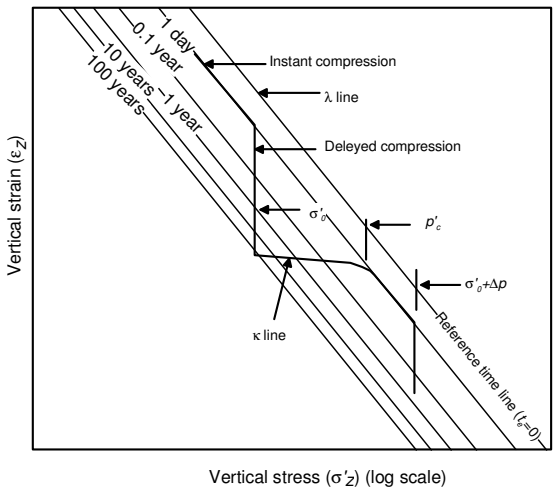

Figure 1. Principle for evaluating creep model (Bjerrum, 1967)

From Eq. (2), the equivalent time can be calculated by:

$$t_e = -t_0 + t_0 \exp\left[\left(\varepsilon_z - \varepsilon_{z0}^{ep}\right)\frac{\nu}{\psi}\right]\left(\frac{\sigma_x'}{\sigma_{zo}'}\right)^{-\lambda/\psi} \tag{3}$$

The incremental strain rate ($\partial\varepsilon_z/\partial t$) based on Fig. 1 can be written as:

$$\frac{\partial\varepsilon_z}{\partial t} = \frac{\kappa}{\nu\sigma'_z}\frac{\partial\sigma'_z}{\partial t} + \frac{\psi}{\nu}\frac{1}{t_0 + t_e} \tag{4}$$

Substituting Eq.(3) into Eq. (4), the elastic visco-plastic model can be obtained:

$$\frac{d\varepsilon_z}{dt} = \frac{\kappa}{\nu\sigma'_z}\frac{d\sigma'_z}{dt} + \frac{\psi}{t_0\nu}\exp\left[-\left(\varepsilon_z - \varepsilon_{z0}^{ep}\right)\frac{\nu}{\psi}\right]\left(\frac{\sigma_x'}{\sigma_{zo}'}\right)^{\lambda/\psi} \tag{5}$$

Equation (5) can be rewritten in terms of excess pore pressure by:

$$\frac{\partial\varepsilon_z}{\partial t} = \frac{\kappa}{\nu(\sigma - u)}\frac{\partial(\sigma - u)}{\partial t} + \frac{\psi}{t_0\nu}\exp\left[-\left(\varepsilon_z - \varepsilon_{z0}^{ep}\right)\frac{\nu}{\psi}\right]\left(\frac{\sigma - u}{\sigma_{zo}'}\right)^{\lambda/\psi} \tag{6}$$

where, σ is the total vertical stress.

If the total vertical stress is constant and ε_{z0}^{ep} is assumed to be zero, Equation (6) becomes:

$$\frac{\partial\varepsilon_z}{\partial t} = -m_v\frac{\partial u}{\partial t} + g(u, \varepsilon_z) \tag{7a}$$

In the above expression,

$$m_v = \frac{\kappa}{\nu(\sigma - u)} \tag{7b}$$

$$g(u, \varepsilon_z) = \frac{\psi}{t_0\nu}\exp\left[-\left(\varepsilon_z\right)\frac{\nu}{\psi}\right]\left(\frac{\sigma - u}{\sigma_{zo}'}\right)^{\lambda/\psi} \tag{7c}$$

4 FINITE DIFFERENCE FORMULATION OF 1-D CONSOLIDATION

Equation (7) can now be combined with Equation (1) to represent the elastic viscoplastic response as follows:

$$m_v\frac{\partial u}{\partial t} - g(u, \varepsilon_z) = \frac{k_v}{\gamma_w}\frac{\partial^2 u}{\partial z^2} + \frac{k_h}{\gamma_w}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) \tag{8}$$

It can be seen that Equation (8) is a nonlinear partial differential equation for 1-D consolidation considering both vertical and horizontal drainage under instantaneous loading. It does not have a general solution. Therefore, employing Crank-Nicolson implicit finite difference procedure, Equation (8) can be approximated by:

$$\begin{aligned} m_{v,i,j,t}\frac{u_{i,j,t+\Delta t} - u_{i,j,t}}{\Delta t} - g(u, \varepsilon_z)_{i,j} = \\ \frac{k_{v,i,j}}{\gamma_w}\frac{1}{2\Delta z^2}\begin{pmatrix}\left(u_{i+1,j,t+\Delta t} - 2u_{i,j,t+\Delta t} + u_{i-1,j,t+\Delta t}\right) \\ +\left(u_{i+1,j,t} - 2u_{i,j,t} + u_{i-1,j,t}\right)\end{pmatrix} \\ +\frac{k_{h,i,j}}{\gamma_w}\begin{pmatrix}\frac{1}{2\Delta r^2}\begin{pmatrix}\left(u_{i,j+1,t+\Delta t} - 2u_{i,j,t+\Delta t} + u_{i,j-1,t+\Delta t}\right) \\ +\left(u_{i,j+1,t} - 2u_{i,j,t} + u_{i,j-1,t}\right)\end{pmatrix} \\ +\frac{1}{4r\Delta r}\begin{pmatrix}\left(u_{i,j+1,t+\Delta t} - u_{i,j-1,t+\Delta t}\right) \\ +\left(u_{i,j+1,t} - u_{i,j-1,t}\right)\end{pmatrix}\end{pmatrix} \end{aligned} \tag{9}$$

where, the subscripts i and j are the node coordinate ranging from i = 1,2,3,…m in vertical direction and from j = 1,2,3,…n in horizontal direction, respectively, Δz, Δr, and Δt are $z_{i+1} - z_i$, $z_{j+1} - z_j$ and $t_{i+\Delta t} - t_t$, respectively (Fig. 2).

$$\begin{aligned} u_{i,j,t+\Delta t} = \frac{\Delta T_{vi,j}}{2\left(1 + \Delta T_{vi,j} + \Delta T_{hi,j}\right)}\left(\begin{pmatrix}u_{i+1,j,t+\Delta t} + u_{i-1,j,t+\Delta t} \\ +u_{i+1,j,t} + u_{i-1,j,t}\end{pmatrix}\right) \\ +\frac{\left(1 - \Delta T_{vi,j} - \Delta T_{hi,j}\right)}{\left(1 + \Delta T_{vi,j} + \Delta T_{hi,j}\right)}u_{i,j,t} \\ +\frac{\Delta T_{hi,j}}{2\left(1 + \Delta T_{vi,j} + \Delta T_{hi,j}\right)}\begin{pmatrix}\left(1 + \frac{1}{2r/\Delta r}\right)\begin{pmatrix}u_{i,j+1,t+\Delta t} \\ +u_{i,j+1,t}\end{pmatrix} \\ +\left(1 - \frac{1}{2r/\Delta r}\right)\begin{pmatrix}u_{i,j-1,t+\Delta t} \\ +u_{i,j-1,t}\end{pmatrix}\end{pmatrix} \\ +\frac{\Delta t}{\left(1 + \Delta T_{vi,j} + \Delta T_{hi,j}\right)m_{v,i,j,t}}g(u, \varepsilon_z)_{i,j} \end{aligned} \tag{10a}$$

where, $\Delta T_{hi,j} = \dfrac{\Delta t}{m_{v,i,j,t}}\dfrac{k_{h,i,j}}{\gamma_w\Delta r^2}$ (10b)

$$\Delta T_{vi,j} = \frac{\Delta t}{m_{v,i,j,t}} \frac{k_{v,i,j}}{\gamma_w \Delta z^2} \tag{10c}$$

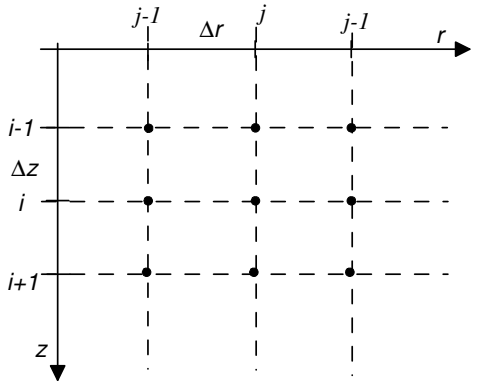

Figure 2. Location of finite difference node

At the permeability interface (i.e. smear zone and undisturbed zone), the excess pore pressure at the boundary (u^*) has to satisfy the continuity of flow to prevent sudden drop of excess pore pressure, hence:

$$u^* = \frac{\frac{k_h}{k_s}\frac{r_{i,j+1}}{r_{i,j-1}} u_{i,j+1,t} + u_{i,j-1+t}}{1 + \frac{r_{i,j+1}}{r_{i,j-1}}} \tag{11}$$

Vertical strain at each node can be calculated based on Equation (7a) as follows:

$$(\varepsilon_z)_{i,j,t+\Delta t} = (\varepsilon_z)_{i,j,t} - m_{vi,j,t}(u_{i,j,t+\Delta t} - u_{i,j,t}) + \Delta t(g(u,\varepsilon_z))_{i,j} \tag{12}$$

The settlement along the radial distance at a given time (ρ_t) can be calculated by:

$$\rho_t = \left(0.5(\varepsilon_z)_{0,j,t} + \sum_{i=1}^{m-1}(\varepsilon_z)_{i,j,t} + 0.5(\varepsilon_z)_{n,j,t}\right)\Delta z \tag{13}$$

The average excess pore pressure at a given depth can be determined by:

$$\bar{u}_i = \sum_{j=0}^{n-1} \frac{2u_{i,j}\Delta r(r_{i,j+1} + r_{i,j})/2}{r_e^2 - r_w^2} \tag{14}$$

The average excess pore pressure at a given radius is given by:

$$\overline{u_j} = \sum_{i=0}^{m-1} \frac{2u_{i,j}\Delta r(r_{i+1,j} + r_{i,j})/2}{r_e^2 - r_w^2} \tag{15}$$

To simulate the permeable boundary (i.e., at the top surface or at the drain interface), the pore pressure can be set to zero or equivalent to drain discharge capacity. The initial value of $(\varepsilon_z)_{i,j}$, $u_{i,j}$ and $\sigma_{i,j}$ at time $t = 0$ are required prior to iteration.

5 MODEL VALIDATION

The above FD scheme can be readily executed in an electronic spreadsheet such as Excel. The program was employed to analyze and compare with the available solutions including 1D Terzaghi vertical consolidation and Barron's radial consolidation both with and without smear effect (Fig. 3). It is noted that, in the following analysis, the discharge capacity (q_w) of the drain is assumed to be high enough for well resistance to be neglected. Vertical and horizontal permeability was determined from 1-D consolidation tests to be appoximately 1×10^{-10} m/s, respectively. For the radial consolidation, the equivalent drain diameter (d_w) was taken to be 60 mm. The smear diameter (d_s) or smear zone width ($2b_s$) was 208mm, based on laboratory testing by Indraratna and Redana (1997). The ratio of the undisturbed permeability to the smear zone permeability was assumed to be 3.0. A good agreement between the analytical and numerical could be found and confirmed the reliability of the FD formulation.

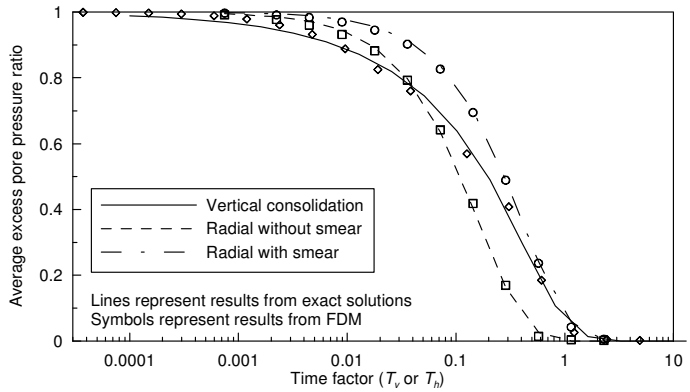

Figure 3. Finite difference and analytical model result.

6 APPLICATION TO CASE HISTORY

The Subshine Motorway is located in Maroochy Shire, Queensland, Australia. Subsoil layer at this site is composed of very soft, highly compressible, saturated organic marine clays of high sensitivity, which present difficulties in developing the new alignment. A trial embankment with PVDs (25 × 50 m^2)to reduce consolidation time was contructed to assess the feasibility of this method.

In this study, the soil profile at the site has been divided into 3 sublayers. The subsoil is relatively uniformed, consisting of a silty clay layer (2.5 m depth) overlying very soft to soft silty clay extending from 2.5 m to 5.0 m depth. A 6 m thick, medium silty clay layer underlies the soft silty clay layer. The groundwater level is at the ground surface. A trial embankment was constructed on the soft marine clay with 11m long PVD at a spacing of 2m. The adopted parameters of 3 subsoil layers obtained from the laboratory tests are listed in Table 1. The embankment load was applied in 4 stages up to a maximum height of 2.85 m (the unit weight of surcharge fill equals to 20kN/m^3). The stages of loading for both embankments are illustrated in Fig. 4. According to the laboratory tests conducted by Indraratna and Redana (1998), the ratio between horizontal and vertical permeability within the smear zone was set to 1. Outside the smear zone, the horizontal permeability was taken to be twice that of

the vertical permeability. The diameter of the smear zone was about 5 times the vertical drain diameter.

Based on the analysis, Figure 5 illustrates the comparison between the predicted surface settlement at the centreline and the measured data. The predicted result incorporating creep agrees well with the measured results. In contrast, the conventional analysis underestimates the measured result after 30 days when the working effective stress has passed the preconsolidation pressure .

Table 1. Elasto-viscoplatic parameters used in the analysis.

Depth (m)	λ $\times10^{-2}$	κ $\times10^{-3}$	k_h $\times10^{-4}$ m/day	ψ $\times10^{-3}$	e_0	p'_c (kPa)
0-2.5	27	27	2.8	10	1.9	20
2.5-5	48	48	3.3	15	3.1	31
5-11	26	26	1.2	8	1.8	66

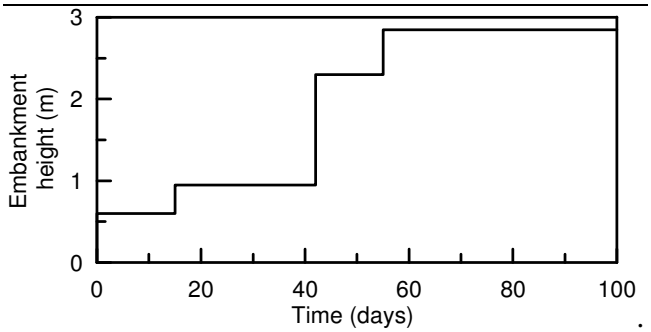

Figure 4. Multistage loading for embankment.

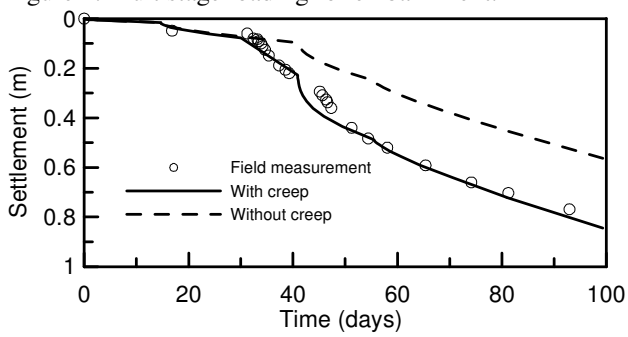

Figure 5. Surface settlements

The comparison in Fig. 6 shows that the elasto viscoplastic soil behaviour contributes to retarding the dissipation of the excess pore pressure. This is because, at a given mean effective stress, the viscous nature of clay causes additional soil compression for a certain period of time without further excess pore water dissipation (Fig. 7). However, the still high excess pore pressure (field data) suggests that there are other mechanisms that may be responsible for the retarded excess pore pressure dissipation.

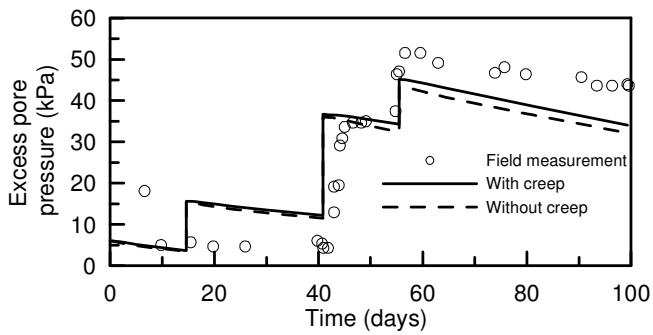

Figure 6. Excess pore pressure

7 CONCLUSIONS

In this paper, the settlements and excess pore pressures of the test embankment stabilised by geosynthetic vertical drains at the Sunshine Motorway, Australia are simulated using a finite difference scheme with an elasto viscoplastic (creep) model. The predicted settlements using creep model agrees well with the measured data. The discrepancy of the excess pore pressure between the prediction and the measurement is reduced by the application of the creep model. The retarded excess pore pressure may be attributed to other factors such as drain folding and high lateral earth pressure.

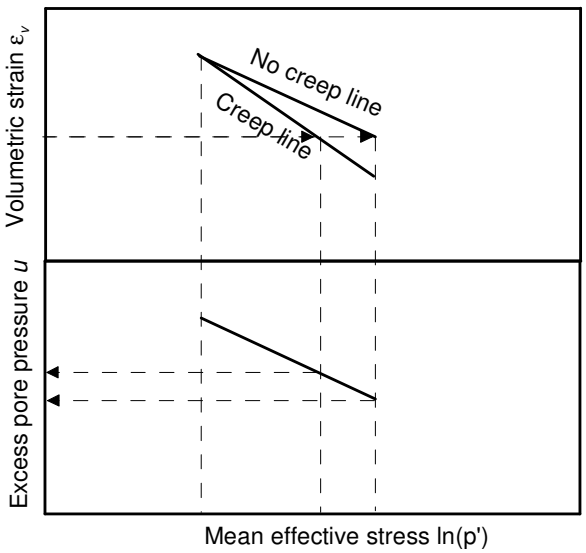

Figure 7. Excess pore pressure retardation due to viscous nature of clay.

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