Radial consolidation of clay using compressibility indices and varying horizontal permeability

B. Indraratna* C. Rujikiatkamjorn†
I. Sathananthan‡

*University of Wollongong, indra@uow.edu.au
†University of Wollongong, cholacha@uow.edu.au
‡University of Wollongong

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Buddhima Indraratna

PhD, DIC, MSc (Lond.), FIEAust., FASCE

Professor of Civil Engineering, Faculty of Engineering,
University of Wollongong, Wollongong City, NSW 2522, Australia
Tel: +61-2-4221-3046; Fax: +61-2-4221-3238
E-mail: indra@uow.edu.au

Cholachat Rujikiatkamjorn

BEng (Hons), MEng (AIT),

Doctoral Candidate, Civil Engineering, Faculty of Engineering,
University of Wollongong, Wollongong City, NSW 2522, Australia
Tel: +61-2-4221-5886; Fax: +61-2-4221-3238
E-mail: cr27@uow.edu.au

Iyathurai Sathananthan, BSc (Hons)
Doctoral Candidate, Civil Engineering Discipline, Faculty of Engineering,
University of Wollongong, Wollongong City, NSW 2522, Australia.
Tel: +61-2-4221-4588; Fax: +61-2-4221-3238
E-mail: is77@uow.edu.au

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Buddhima Indraratna, Cholachat Rujikiatkamjorn and Iyathurai Sathananthan

Abstract

A system of vertical drains with surcharge load to accelerate consolidation by shortening the drainage path is one of the most popular methods of soft ground improvement. The conventional radial consolidation theory (including smear and well resistance) have been commonly employed to predict the behaviour of vertical drains in soft clay. Its mathematical formulation is based on the small strain theory, and for a given stress range, a constant volume compressibility (m_v) and a constant coefficient of lateral permeability (k_h) are assumed. However, the value of m_v varies along the consolidation curve over a wide range of applied pressure (Δp). In the same manner, k_h also changes with the void ratio (e). In this paper, the writers have replaced m_v with the compressibility indices (C_c and C_r), which define the slopes of the e-logσ' relationship. Moreover, the variation of horizontal permeability coefficient (k_h) with void ratio (e) during consolidation is represented by the e-logk_h relationship that has a slope of C_k. In contrast to the conventional analysis , the current study highlights the influence of the C_c/C_k (or C_r/C_k) ratio and the preloading increment ratio ($\Delta p/\sigma'_i$) on the consolidation process. The analytical predictions are compared with the experimental results using a large scale consolidation chamber, and these predictions show good agreement with the measured data. Finally, an embankment case history taken from Muar Plains, Malaysia is analysed based on the current solution, and compared with field measurements.

Keywords: Compressibility; Embankments; Permeability; Soft soils; Soil consolidation; Vertical drains

Introduction

Preloading of soft clay over vertical drains is one of the most commonly employed methods to increase the shear strength of soil and to reduce its post-construction settlement. Since most compressible soils are characterized by very low permeability and considerable thickness, time required to achieve the desired settlement or shear strength can sometimes be too long to support the need for rapid construction (Johnson 1970). Using vertical drains, the drainage length is considerably shortened from the thickness of the soft soil layer (e.g. 10-30 m in the vertical direction, depending the geology and thickness of the compressible layer as well as the extent of loaded area) to half of the drain spacing in the horizontal direction (typical drain spacing is 1.0-2.0 m, (Indraratna et al. 1997). It is important to note that, for most soft clay deposits, the horizontal permeability is higher than the vertical permeability, hence, the rapid radial drainage accelerates the consolidation process (Jamiolkowski et al. 1983). This system has been used successfully to improve foundation soils for embankments, airports and highways (Indraratna and Redana 2000; Li and Rowe 2002).

Barron (1948) introduced an analytical solution for radial consolidation of soil without the smear effect. Subsequently, Hansbo (1981) incorporated smear effect and well resistance into Barron's formulation. Since a small strain theory is employed in Hansbo's theory, a constant coefficient of volume compressibility (m_v) and a constant coefficient of horizontal permeability (k_h) were assumed for a given stress range. In contrast, for a relatively large applied stress range, it is known that both soil permeability and soil volume compressibility coefficients decrease as a result of physical reduction in void ratio during the consolidation process (Tavenas et al. 1983; Seah et al. 2004). The stress state in relation to the preloading (surcharge) and preconsolidation pressure is essential to predict the actual settlement (Casagrande1932; Holtz and Kovacs 1981; Burland 1990; Indraratna and Balasubramaniam 1993). In this paper, the e-logσ' relationship is used to determine the compressibility indices (C_c and C_r), and the e-logk_h relationship is used to represent permeability variation. In contrast, in the conventional radial consolidation (Barron 1948), the parameters m_v and k_h were not changed as a function of the void ratio for a given stress range. The smear effect is considered, but the well resistance is neglected. It can be noted that, for most common length of drains (less than 20 m long), the well resistance is not significant (Holtz et al. 1991; Indraratna et al. 1994; Hansbo 1997). The effects of the compressibility indices, the variation of soil permeability and the magnitude of preloading are examined through the consolidation process. Subsequently, the writers' model is verified using laboratory testing. Finally, the predictions of settlements and excess pore pressures are compared with field measurements at the centerline of embankments obtained readily from settlement plates and piezometers.

Vertical Drain Theory

Barron (1948) presented a comprehensive solution to the problem of radial consolidation by drain wells. He studied two extreme cases, namely (a) free strain and (b) equal strain. The 'free strain hypothesis' assumes that the load is uniform over a circular zone of influence for each vertical drain, and that the differential settlements occurring over this zone have no effect on the redistribution of stresses by arching of the fill load. The 'equal vertical strain hypothesis' on the other hand, assumes that arching occurs in the upper layer during the consolidation process without any differential settlement in the clay layer. The arching effect implies a more or less rigid boundary at the surface of the soil layer being consolidated with vertical drains, assuming that the vertical strain is uniform in the horizontal plane of the soil. Nevertheless for all practical purposes, the average consolidation obtained in both these cases is nearly the same, and the solution obtained from the second assumption is simpler than the first case (Barron 1948). Therefore, it has been common to use the equal vertical strain in most radial drainage-consolidation analyses.

The key assumptions for the conventional equal vertical strain solution (Hansbo 1981) are also applicable to a unit cell defined by any two adjacent centerlines between drains, and they are:

(1) Soil is fully saturated and homogeneous, and laminar flow thorough the soil (Darcy's law) is adopted. At the outer boundary of the unit cell, flow is not allowed to occur (Fig. 1), and for relatively long vertical drains, only the radial (horizontal) flow is permitted to occur (i.e. no vertical flow).

(2) Soil strain is uniform at the upper boundary of the unit cell the small strain theory is valid.

In this paper, the discharge capacity (q_w) of the drain is assumed to be high enough for well resistance to be neglected. Holtz et al. (1989) suggested that as long as the working discharge capacity of PVD exceeds say 150 m^3/year after installation, the effect on consolidation due to well resistance (e.g. folding, increased lateral pressure, siltation, etc.) may not be significant. Indraratna and Redana (2000) described that well resistance in long term becomes significant for PVD with q_w less than 40-60 m^3/year. Hansbo (1981) considered the smear effect which occurs during the vertical drain installation. Average degree of consolidation for vertical drains can be given by:

[1a] $$U_r = 1 - exp\left(\frac{-8T_h}{\mu}\right)$$

[1b] in which, $$\mu = ln\frac{n}{s} + \frac{k_h}{k_s} ln\, s - 0.75$$

where, U_r = average degree of consolidation due to radial drainage, μ = a group of parameters representing the geometry of the vertical drain system and smear effect, $n = d_e/d_w$, s = d_s/d_w, d_e = equivalent diameter of cylinder of soil around drain, d_s = diameter of smear zone and d_w = diameter of drain well. In Eq. (1b), k_h = average horizontal permeability in the undistrubed zone (m/s), and k_s = average horizontal permeability in the smear zone (m/s). T_h is the dimensionless time factor for consolidation due to radial drainage.

Figure 1 shows the unit cell adopted for the analysis and the patterns of the vertical drain. The equivalent drain diameter (d_e) is a function of drain spacing (d) and its configuration. It is equal to 1.05d and 1.128d for triangular and square patterns, respectively (Fig. 1). Rixner et al. (1986) indicated that the triangular spacing gives more uniform settlement than the square pattern, while the square pattern is more convienient to control in the field. The installed drain pattern also depends on how they are installed in the field, equipment used, etc. The symbol d_w is the equivalent band drain diameter or the actual diameter of sand drain (m). In the case of prefabricated vertical drain (PVD), d_w can be determined by $2(a+b)/\pi$, where a and b are the width and thickness of PVD, respectively (Hansbo 1979).

Other assumptions for the proposed analytical solution

Apart from the two assumptions stated earlier for Hansbo (1981) theory, the additional assumptions made in the writers' analysis are summarised below:

- During the consolidation process, at a given depth, the relationship between the average void ratio and the logarithm of average effective stress in the normally consolidated range (Fig. 2a) can be expressed by: $\bar{e} = e_0 - C_c \log(\sigma'/\sigma'_i)$. If the current vertical effective stress (σ') is smaller than p'_c, the recompression index (C_r) is used instead of C_c for the overconsolidated range.

- In radial drainage, the horizontal permeability of soil decreases with the average void ratio (Fig. 2b). The relationship between these two parameters can be commonly found by (Tavenas et al. 1983): $\bar{e} = e_0 + C_k \log(k_h / k_{hi})$ The permeability index (C_k) is generally considered to be independent of stress history (p'_c) (e.g. Nagaraj et al. 1994).

Proposed Analytical Solution

Laboratory testing has shown that during the consolidation process, a variation of soil volume compressibility and soil permeability can be found, which imparts a direct influence on the shape of e-logσ' and e-logk_h (Lekha et al. 2003). In addition, in the field, the nature of subsoil stress history (normally consolidated or lightly overconsolidated soil) gives different consolidation responses (Seah and Juirnarongrit 2003). Therefore, in order to predict the behaviour of a vertical drain system more accurately, it is necessary to incorporate the relationships of e-logσ' and e-logk_h with radial consolidation, and then find a new solution for the radial consolidation.

The flow rate in the unit cell can be expressed by Darcy's law as:

[2] $$\frac{\partial Q}{\partial t} = \frac{k_h}{\gamma_w}\frac{\partial u}{\partial r}A_{cs}$$

where, Q is the flow in soil mass, A_{cs} is the cross sectional area of the flow at distance r which is equal to $2\pi r(dz)$ for an element thickness of dz.

The rate of changing volume of soil mass is given by:

[3] $$\frac{\partial V}{\partial t} = \frac{\partial \varepsilon}{\partial t}\pi\left(r_e^2 - r^2\right)dz$$

where, V is the volume of the soil mass, and ε is the volumetric strain.

The flow rate in the unit cell is equal to the rate of volume change of soil mass, therefore,

[4] $$\frac{k_h}{\gamma_w}\left\{\frac{\partial u}{\partial r}\right\}2\pi r dz = \frac{\partial \varepsilon}{\partial t}\pi\left(r_e^2 - r^2\right)dz$$

where, k_h = average coefficient of permeability in undisturbed zone. Rearranging Eq. (4) gives the following equation for the pore pressure gradient in the undisturbed soil domain outside the smear zone.

[5] $$\frac{\partial u}{\partial r} = \frac{\gamma_w}{2k_h}\frac{\partial \varepsilon}{\partial t}\left(\frac{r_e^2 - r^2}{r}\right) \qquad r_e \geq r \geq r_s$$

In the smear (disturbed) zone, the corresponding pore pressure gradient is then given by:

[6] $$\frac{\partial u_s}{\partial r} = \frac{\gamma_w}{2k_s}\frac{\partial \varepsilon}{\partial t}\left(\frac{r_e^2 - r^2}{r}\right); \qquad r_s \geq r \geq r_w$$

where, u_s = excess pore pressure in smear zone, k_s = average permeability in smear zone and γ_w = unit weight of water.

The excess pore pressure in the smear zone can now be obtained by integration of Eq. (6) in the r direction and using the boundary condition, $u = 0$ at $r = r_w$:

[7] $$u_s = \frac{\gamma_w}{2k_s}\frac{\partial \varepsilon}{\partial t}\left(r_e^2\, ln\frac{r}{r_w} - \frac{r^2 - r^2{}_w}{2}\right); \qquad r_s \geq r \geq r_w$$

Meanwhile, u_s at $r = r_s$ can be determined from:

[8] $$u_{s,r=r_s} = \frac{\gamma_w}{2k_s}\frac{\partial \varepsilon}{\partial t}\left(r_e^2\, ln\frac{r_s}{r_w} - \frac{r_s^2 - r^2{}_w}{2}\right)$$

Integrating Eq. (5) in the r direction with the boundary condition $u = u_{r=r_s}$ at $r = r_s$ yields:

[9] $$u - u_{r=r_s} = \frac{\gamma_w}{2k_h}\frac{\partial \varepsilon}{\partial t}\left(r_e^2\, ln\frac{r}{r_s} - \frac{r^2 - r^2{}_s}{2}\right); \qquad r_e \geq r \geq r_s$$

Assuming that the excess pore pressure at the outer boundary of the smear zone ($u_{r=r_s}$) is equal to the excess pore pressure at the inner boundary of the undisturbed zone ($u_{s,r=r_s}$), as shown in Fig. 1, then:

[10] $$u = \left\{\frac{\gamma_w}{2k_h}\frac{\partial \varepsilon}{\partial t}\left(r_e^2\, ln\frac{r}{r_s} - \frac{r^2 - r^2{}_s}{2}\right) + \frac{\gamma_w}{2k_{h,s}}\frac{\partial \varepsilon}{\partial t}\left(r_e^2\, ln\frac{r_s}{r_w} - \frac{r_s^2 - r^2{}_w}{2}\right)\right\}; r_e \geq r \geq r_s$$

Let $\overline{u_t}$ be the average excess pore pressure of the smear and intact zones, at depth z and for a given time, t, hence:

[11] $$\overline{u_t} = \frac{\int_0^l \int_{r_s}^{r_e} u 2\pi r dr dz + \int_0^l \int_{r_w}^{r_s} u_s 2\pi r dr dz}{\pi\left(r_e^2 - r_w^2\right) l}$$

Substituting Eqs. (7) and (10) into Eq. (11) gives the following expression for the average excess pore pressure $\overline{u_t}$ at any time, knowing that $d_e = 2r_e$ and $\partial\varepsilon = \partial\overline{e}/1 + e_0$. Therefore,

[12] $$\bar{u}_t = \frac{\partial \bar{e}}{\partial t}\frac{d_e^2 \mu}{8k_h(1+e_0)}\gamma_w$$

Substituting $R_u = \bar{u}_t / \Delta p$ in Eq. (12) and further modifying

[13] $$R_u = \frac{\partial \bar{e}}{\partial \sigma'}\frac{\partial(\sigma - \bar{u}_t)}{\partial t}\frac{k_{hi}}{k_h}\frac{d_e^2 \mu}{8k_{hi}(1+e_0)\Delta p}\gamma_w$$

The dimensionless parameter R_u is useful when the role of applied surcharge on pore water pressure is considered, and also when comparisons are made between two or more embankments during construction.

Since preloading pressure (Δp) is assumed to be an instantaneous loading on the top of unit cell and total stress (σ) is constant throughout consolidation process $\partial\sigma / \partial t = 0$. Simplifying Eq. (13):

[14] $$\frac{\partial R_u}{\partial T_h} = -\frac{8}{\mu}\frac{m_{vi}}{m_v}\frac{k_h}{k_{hi}}R_u,$$ and

[15] $$T_h = \frac{c_{hi}t}{d_e^2}$$

where, $m_v = (\partial\bar{e}/\partial\sigma')/(1+e_0)$, $m_{vi} = (\partial\bar{e}/\partial\sigma'_{t=0})/(1+e_0)$ and $c_{hi} = \dfrac{k_{hi}(1+e_0)}{\gamma_w \partial\bar{e}/\partial\sigma'_{t=0}}$

The void ratio-effective stress and the void ratio-permeability relations for normally consolidated clays can be express as (Tavenas et al. 1983):

[16] $$\bar{e} = e_0 - C_c \log\left(\frac{\sigma'}{\sigma'_i}\right)$$

[17] $$\bar{e} = e_0 + C_k \log\left(\frac{k_h}{k_{hi}}\right)$$

Differentiating Eq. (16) with respect to the effective stress (σ') gives:

[18] $$\frac{m_{vi}}{m_v} = 1 + \Delta p / \sigma'_i - R_u \Delta p / \sigma'_i$$

Combining Eqs. (16) and (17) gives:

[19] $$\frac{k_h}{k_{hi}} = \left(\frac{\sigma'}{\sigma'_i}\right)^{-C_c / C_k} = \left(\frac{m_{vi}}{m_v}\right)^{-C_c / C_k}$$

Therefore:

[20] $$\frac{k_h}{k_{hi}} = \left(1 + \Delta p / \sigma'_i - R_u \Delta p / \sigma'_i\right)^{-C_c / C_k}$$

Substituting Eqs. (18) and (20) into (14) yields:

[21] $$\frac{\partial R_u}{\partial T_h} = -\frac{8}{\mu} P R_u$$

where, $P = \left(1 + \Delta p / \sigma'_i - R_u \Delta p / \sigma'_i\right)^{1 - C_c / C_k}$

It can be seen that Eq. (21) is a nonlinear partial differential equation for radial consolidation under instantaneous loading, incorporating the e-logσ' and e-logk_h relations. The nonlinear differential Eq. (21) with variable R_u does not have a general solution and P varies from $\left(1 + \frac{\Delta p}{\sigma'_i}\right)^{1 - C_c / C_k}$ to 1. Hence, it can be assumed to have an average value given by:

[22] $$P = P_{av} = 0.5\left(1 + \left\{1 + \frac{\Delta p}{\sigma'_i}\right\}^{1 - C_c / C_k}\right)$$

Incorporating the above assumption for P_{av}, Eq. (21) can be written as:

[23] $$\frac{\partial R_u}{\partial T_h *} = -\frac{8}{\mu} R_u$$

where, $T_h *$ is the modified time factor, defined by:

$$[24] \quad T_h{}^* = P_{av}T_h = 0.5\left(1+\left\{1+\frac{\Delta p}{\sigma'_i}\right\}^{1-C_c/C_k}\right)T_h$$

Integrating the above equation (23) subject to the boundary condition that $\overline{u_t} = \Delta p$ at $T_h{}^* = 0$, gives the following expression similar to the original equation (Eq. 1):

$$[25] \quad R_u = \exp\left(\frac{-8T_h{}^*}{\mu}\right)$$

Substituting $T_h{}^*$ in Eq. (25), the expression for excess pore pressure ratio for normally consolidated clay becomes:

$$[26a] \quad R_u = \exp\left(-4\left[1+\left\{1+\frac{\Delta p}{\sigma'_i}\right\}^{1-C_c/C_k}\right]\frac{T_h}{\mu}\right)$$

For overconsolidated soil, $\sigma' < p'_c$ (e.g. topmost layer close to surface)

$$[26b] \quad R_u = \exp\left(-4\left[1+\left\{1+\frac{\Delta p}{\sigma'_i}\right\}^{1-C_r/C_k}\right]\frac{T_h}{\mu}\right)$$

When the effective stress equals the preconsolidation pressure ($\sigma' = p'_c$), the corresponding time factor $T_{h,pc}$ can then be determined by:

$$[26c] \quad T_{h,pc} = \frac{\mu}{4\left[1+\left\{1+\frac{\Delta p}{\sigma'_i}\right\}^{1-C_r/C_k}\right]}\ln\left(\frac{\Delta p}{\sigma'_i + \Delta p - p'_c}\right)$$

When $\sigma' > p'_c$ along the slope of the normally consolidation curve (C_c), the expression for excess pore pressure ratio when $T_h > T_{h,pc}$ is given by:

$$[26d] \quad R_u = \left(\frac{\sigma'_i + \Delta p - p'_c}{\Delta p}\right)\exp\left(-4\left[1+\left\{1+\frac{\sigma'_i + \Delta p - p'_c}{p'_c}\right\}^{1-C_c/C_k}\right]\frac{(T_h - T_{h,pc})}{\mu}\right)$$

where, $T_h = \dfrac{c_{h,pc} t}{d_e^2}$, $c_{h,pc} = c_{hi} \left(\dfrac{p'_c}{\sigma_i'} \right)^{1-C_r/C_k}$

By combining Eqns (9) and (12) the normalized excess pore pressure at any point within the smear zone can be found as:

$$[27a]\ \frac{u'}{\Delta p} = \frac{4k_h}{k_s} \frac{R_u}{\mu d_e^2} \left[r_e^2 \ln\left(\frac{r}{r_w} \right) - \frac{\left(r^2 - r_w^2\right)}{2} \right]$$

By combining Eqns (10) and (12) the normalized excess pore pressure at any point outside the smear zone is given by:

$$[27b]\ \frac{u}{\Delta p} = \frac{4R_u}{\mu d_e^2} \left\{ r_e^2 \ln\left(\frac{r}{r_s} \right) - \frac{\left(r^2 - r_s^2\right)}{2} + \frac{k_h}{k_s} \left[r_e^2 \ln\left(\frac{r_s}{r_w} \right) - \frac{\left(r_s^2 - r_w^2\right)}{2} \right] \right\}$$

When the value of C_c/C_k approaches unity, the writers' solution converges to that of Hansbo (1981) described earlier (see Eq. (1)), hence:

$$[28]\quad R_u = \exp\left(\frac{-8T_h}{\mu} \right)$$

Since the relationship between effective stress and strain is not linear, the average degree of consolidation can be defined either based on excess pore pressure (stress) (U_p) or based on strain (settlement at the top surface) (U_s). U_p indicates the rate of dissipation of excess pore pressure whereas U_p shows the rate of development of the surface settlement. Normally, $U_p \neq U_s$ except when the relationship between effective stress and strain is linear, which is Terzaghi's one-dimensional theory. Therefore, the average degree of consolidation based on excess pore pressure can be obtained as follows:

$$[29]\quad U_p = 1 - R_u$$

The average degree of consolidation based on settlement (strain) can be given by:

[30] $$U_s = \frac{\rho}{\rho_\infty}$$

The associated settlements (ρ) are evaluated by the following equations:

[30a] $$\rho = \frac{HC_r}{1+e_0}\log\left(\frac{\sigma'}{\sigma'_i}\right), \qquad \sigma'_i \leq \sigma' \leq p'_c$$

[30b] $$\rho = \frac{H}{1+e_0}\left[C_r \log\left(\frac{p'_c}{\sigma'_i}\right) + C_c \log\left(\frac{\sigma'}{p'_c}\right)\right], \qquad p'_c \leq \sigma' \leq \sigma'_i + \Delta p$$

[30c] $$\rho = \frac{HC_c}{1+e_0}\log\left(\frac{\sigma'}{\sigma'_i}\right) \text{ for normally consolidated clay}$$

It is noted that ρ_∞ can be obtained by substituting $\sigma' = \sigma'_i + \Delta p$ into the above equations.

where, ρ = settlement at a given time, ρ_c = total primary consolidation settlement, C_c = compression index, C_r = recompression index and H = compressible soil thiskness.

Depending on the location of the initial and final effective stresses with respect to the normally consolidated and overconsolidated domains,the following is a summary of the relavant computational steps.

(i) If both the initial and final effective stresses are in the normally consolidated range, Equations (26a) and (29) are employed to calculate U_p, whereas Equations (30) and (30c) are used to compute U_s.

(ii) If both the initial and final effective stresses are in the overconsolidated range, Equations (26b) and (29) are employed to calculate U_p, and Equations (30) and (30a) are used to determine U_s.

(iii) If the initial effective stress falls on the overconsolidated domain and the final effective stress is on the normally consolidated domain; Equations (26b)-(26d) and (29) are employed to calculate U_p, whereas Equations (30) (30a) and (30b) are employed to calculate U_s.

Comparisons of Hansbo (1981) with proposed solutions

In this section, the effects of the values of C_c/C_k and load increment ratio ($\Delta p/\sigma'_i$) are examined in the post p'_c region. Relevent parameters used in the analysis are given in Table 1. According to Berry and Wilkinson (1969), the typical values of C_c/C_k for soil in the range of 0.5-2.0 are used in the analysis. The load increment ratio is either 1or 2. Figure 3 shows the comparison of the degree of consolidation based on excess pore pressure (Fig.3a) and strain (Fig.3b) approaches both employing compression index and the variation of horizontal permeability (Eqns. 29 and 30). The maximum difference between Up and Us is around 10%. It is evident that the values of C_c/C_k and $\Delta p/\sigma'_i$ with the same initial coefficient of consolidation for horizontal drainage (c_{hi}) control the rate of consolidation process. For a given $\Delta p/\sigma'_i$, when $C_c/C_k <$ 1 (i.e. the rate of decrease in void ratio with increase in effective stress is smaller than the decrease of void ratio with decrease in permeability), the actual consolidation process is faster than the corresponding result from Hansbo's theory. In contrast, when

$C_c/C_k > 1$ for a given $\Delta p/\sigma'_i$, the actual consolidation process is slower than that predicted by Hansbo's solution. For $C_c/C_k < 1$, when the load increment ratio increases, the rate of settlement increases (Fig. 3). However, for $C_c/C_k > 1$, the reverse is true as when the load increment ratio increases, the rate of settlement decreases. It is important to note that the degree of consolidation based on strain (U_s) is greater than that based on the excess pore pressure (U_p). The magnitude of the difference between them depends on the soil properties and the applied stress levels. In order to correctly evaluate the actual consolidation, the variations in permeability and compressibility, stress history, and the magnitude of preloading pressure should all be considered. More significantly, the roles of C_c/C_k and $\Delta p/\sigma'_i$ are found to be important as demonstrated here.

Verification of the Proposed Model

The proposed model was validated by comparing the settlement predictions with laboratory data. The laboratory testing was conducted using a large-scale 450 mm-diameter large-scale consolidation apparatus having a height of 950 mm. Reconstituted alluvial clay from Moruya (New South Wales) was used in the apparatus. In order to reduce friction between the side wall of the cylinder and the soil, a Teflon sheet was laid around the inner periphery of the cell. Table 2 summarizes the properties of this reconstituted clay. The e-logσ' and e-logk_h relationships are obtained by the four conventional oedometer tests (Fig.4). From e-logσ' and e-logk_h plots, the slope of the e-logσ' line (C_c) and the slope of e-logk_h line (C_k) are found as 0.29 and 0.45, respectively. Therefore, the corresponding C_c/C_k can be calculated as 0.64. The detailed testing procedure is explained elsewhere by Indraratna and Redana (1998). Following

Burland (1990), the clay specimen was prepared with a water content slightly greater than the liquid limit. The clay was placed and compacted in layers in the apparatus. In two different tests, an initial preconsolidation pressure of 20 kPa and 50 kPa, respectively, was applied for 5 days before the installation of the vertical drain. The 100 mm × 4 mm band drain was then installed vertically in the center of the cell using a steel mandrel. After drain installation, the mandrel was withdrawn by the hoist system, and subsequently, the preconsolidation pressures of 20 kPa and 50 kPa were maintained in the two tests. The two large clay samples were then further subjected to different loading sequences i.e., loading increments (Δp) of 30 kPa and 50 kPa, to give the final total pressures of 50 kPa and 100 kPa, respectively. Both samples were then loaded in the normally consolidated range. The corresponding settlement behaviour was recorded, as shown in Fig. 5.

The initial coefficient of horizontal permeability in the undisturbed zone, k_{hi} was determined from e-logk_h plots (Fig. 4) and the initial void ratio (e_0). The diameter of the smear zone (d_s) and the initial ratio of k_h/k_s were evaluated to be 200 mm and 1.5, respectively, based on previous testing (Indraratna and Redana 1998). Table 3 shows the parameters used in the analytical model for settlement prediction.

Figures 5a and 5b illustrate the degree of consolidation based on strain from the writers' model (Eq. 30) in comparison with the laboratory results and Hansbo's solution. The initial coefficient of horizontal consolidation (c_{hi}), which is related to time factor (T_h) can be calculated by:

[31] $$c_{hi} = \frac{k_{hi}}{\gamma_w m_{vi}}$$

where, k_{hi} = initial horizontal permeability coefficient, e_0 = initial void ratio, m_{vi} = initial coefficient of volume compressibility.

As shown in Fig. 5, the settlement predictions from Hansbo's solution slightly underestimate the laboratory results, whereas the predictions incorporating the compression indices and the variation of soil lateral permeability agree very well with the laboratory results. It is noted that the rate of consolidation from the modified solution is greater than the conventional analysis for $C_c/C_k < 1$ as explained earlier in the previous section. This verifies that apart from the permeability and drain configuration, the consolidation behavior depends on the magnitude of the loading increment ratio and the values of C_c/C_k.

Application of the Model to Selected Case History

The Malaysian Highway Authority was responsible for constructing a number of test embankments on the Muar Plain, with various forms of ground improvement techniques including vertical drains. Figure 6 shows the vertical cross section of one such embankment, together with the subsoil profile and PVDs installed in a triangular pattern at a spacing of 1.3 m. The details of the test embankment are explained elsewhere by Indraratna et al. (1994). Table 4 gives the details of the drain geometry. The soil parameters, the in situ effective stress and the soil permeability for Muar clay subsoils are summarised in Table 5. The relevent soil properties including compressibility indices, soil unit weights, initial void ratios, preconsolidation pressures and permeability coeeficients were obtained from CK_oU triaxial tests (Ratnayake 1991). As suggested by Tavenas et al. (1983), the slope of e-logk_h (C_k) can be calcluated by:

[32] $C_k = 0.5e_0$

In the analysis, each subsoil layer was divided into smaller sublayers to derive a more accurate effective stress distribution with depth. In the analysis, the value of soil compressibility index (C_c or C_r) is associated with the actual stress state within a given region of the foundation, where the working stress range must be considered in relation to the preconsolidation pressure of soil at that particular depth (Indraratna et al., 1994). According to laboratory experiments conducted by Indraratna and Redana (1998), the ratio of k_h/k_s is approximately 1.5-2.0. However, this ratio in the field can vary from 1.5 to 5 depending on the type of drain and installation procedures (Saye 2003). The value of k_h/k_s for this case study was taken to be 3.

The embankment load was applied in two stages. As shown in Fig. 7a, during Stage 1 construction, the embankment was raised to a height of 2.57 m in 14 days. Following a rest period of 105 days, an additional fill layer (with compacted unit weight of 20.5 kN/m^3) was placed during Stage 2, until the embankment reached the height of 4.74m in 24 days. The settlements at the centerline were monitored for about 400 days.

The embankment loading was simulated by assuming an instantaneous loading at the upper boundary. Settlement predictions were carried out at the embankment centerline using the writers' analytical model (eg. Eqs. 26-30). At the beginning of Stage 2, the initial in-situ effective stess and initial coefficient of horizontal consolidation (c_{hi}) were calculated based on the Stage 1 final degree of consolidation. As the computation of consolidation settlement at the centerline (zero lateral displacements) is straightforward and follow the 1-D consolidation theory, the use of an EXCEL spreadsheet formulation for this purpose is most convenient. The value of soil compressibility (C_c or C_r) in association with the correct working effective stress plays a very important role for predicting settlement. For Stage 1 loading, where the effective preconsolidation pressure (p'_c) is not exceeded, the value of recompression index (C_r) may be used. In particular, the surface crust is heavily overconsolidated (upto about 3 m depth). Once p'_c is exceeded, the value of compression index (C_c) follows the normally consolidated line as indicated by the values in Table 5. The measured initial horizontal permeability coefficients of undisturbed soil (k_{hi}) are also given in Table 5. The predicted settlement agrees well with the measured values at the embankment centerline (Fig. 7b). In contrast, Hansbo's solution underpredicts before 170 days and overpredicts after 170 days. The analytically predicted and the measured excess pore pressures beneath the embankment at a depth of 11.2m below ground surface and at a location of 0.65m away from the centerline are shown in Fig. 8c. acceptable agreement between the predictions and measurments is found. It is noted that the authors' and Hansbo (1981) solutions are close to each other, because the ratio C_c/C_k is almost unity at the measurement location.

Conclusions

A system of vertical drains is an effective method for accelerating soil consolidation. A revised analytical model for soft clay stabilised by vertical drains incorporating the compressibility indices (C_c and C_r) was proposed, and the variation of horizontal permeability coefficient (k_h) was represented by the e-logk_h relationship. The parameters such as the slopes of the e-$\log\sigma'$ relationship (C_c and C_r), the slope of e-$\log k_h$ relationship (C_k) and the loading increment ratio ($\Delta p/\sigma'_i$) were explicitly included in the analytical model to predict the consolidation behaviour. The writers' analysis particularly elaborates on the important role of C_c/C_k and $\Delta p/\sigma'_i$ ratios, which are found to govern the rate of consolidation. When C_c/C_k is less than 1, the actual rate of consolidation is higher than the conventional solution of Hansbo (1981). Also, the rate of consolidation decreases with the reduction of the load increment ratio ($\Delta p/\sigma'_i$). When C_c/C_k exceeds 1, the consolidation process takes place at a slower rate compared to the conventional solution. In these circumstances ($C_c/C_k>1$), the rate of consolidation increases with the decrease with $\Delta p/\sigma'_i$. It also shows that the settlement development occurs faster the excess pore pressure dissipation.

The predictions from the writers' analytical model agree well with the laboratory results based on large-scale consolidation testing. The proposed solution gives a greater accuracy of the settlement prediction, when applied to a selected case history (Muar clay, Malaysia). In this analysis, the smear effect (due to mandrel driven prefabricated drains) was included but the well resistance was ignored (drain length less than 20 m). For a given drain pattern, the findings of this study confirm that the soil compressibility indices based on the e-$\log\sigma'$ relationship as well as the load increment ratio ($\Delta p/\sigma'_i$) are major factors influencing the embankment settlement, apart from the change in the lateral soil permeability (k_h) with the void ratio.

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Appendix A. Complete details of Equation 18 derivation

Differentiating Eq. (16) with respect to the effective stress (σ') gives:

(A1) $$\frac{\partial \bar{e}}{\partial \sigma'} = -\frac{C_c}{2.303\sigma'}$$

and

(A2) $$m_v = \left(\partial \bar{e} / \partial \sigma'\right) / (1 + e_0)$$

(A3) $$m_{vi} = \left(\partial \bar{e} / \partial \sigma'_{t=0}\right) / (1 + e_0)$$

therefore,

(A4) $$\frac{m_{vi}}{m_v} = \partial \bar{e} / \partial \sigma'_{t=0} \Big/ \partial \bar{e} / \partial \sigma'$$

Substituting Equation (A1) into (A4) gives:

(A5) $$\frac{m_{vi}}{m_v} = \frac{\sigma'}{\sigma'_{t=0}}$$

but

(A6) $$\sigma' = \sigma'_i + \Delta p - \bar{u}_t$$

Substituting Equation (A6) into (A5) gives:

(A7) $$\frac{m_{vi}}{m_v} = 1 + \Delta p / \sigma'_i - R_u \Delta p / \sigma'_i$$

where, $R_u = \bar{u}_t / \Delta p$

NOTATION

The following symbols are used in this paper:

A_{cs} = cross-sectional area corresponding to flow (m^2)

a = width of the prefabricated vertical drain (m)

b_w = half width of drain (m)

C_c = comprssion index

C_k = permeability index

C_r = recompression index

c_{hi} = initial coefficient of consolidation for horizontal drainage (m^2/s)

$c_{h,pc}$ = coefficient of consolidation for horizontal drainage at effective preconsolidation pressure(m^2/s)

d = drain spacing (m)

d_e = diameter of influenced zone (m)

d_s = diameter of smear zone (m)

d_w = diameter of drain well (m)

e_0 = average void ratio at initial in-situ effective stress

H = compressible soil thiskness

h = hydraulic head (m)

k = average permeability coefficient of soil (m/s)

k_{hi} = initial horizontal permeability coefficient of undisturb zone (m/s)

k_s = average horizontal permeability coefficient of smear zone (m/s)

m_v = coefficient of volume compressibility

m_{vi} = initial coefficient of volume compressibility

n = ratio r_e/r_w or d_e/d_w

P = modification factor for pore pressure

P_{av} = average modification factor for pore pressure

p'_c = preconsolidation pressure (kPa)

Q = flow in unit cell (m^3)

R_u = average excess pore water pressure ratio ($\bar{u}_t / \Delta p$)

r = distance from center of the drain in axisymmetry unit cell (m)

r_e = radius of influenced zone (m)

r_s = radius of smear zone (m)

r_w = radius of drain well (m)

s = ratio r_s/r_w or d_s/d_w

T_h = dimensionless time factor for horizontal drainage, $T_h = c_{hi}t / d_e^2$

T_h* = modified dimensionless time factor

$T_{h,pc}$ = dimensionless time factor at $\sigma' = p'_c$

U_p = average degree of radial consolidation based on excess pore pressure

U_s = average degree of radial consolidation based on strain

u = excess pore water pressure (kN/m^2)

u_s = excess pore water pressure in smear zone (kN/m^2)

$\bar{u}_t$ = average excess pore water pressure at a given time t (kN/m^2)

V = volume of soil mass (m^3)

v_r = velocity of flow (at radious r) (m/s)

Greek letters

γ_w = unit weight of water (kN/m^3)

Δp = preloading pressure(kN/m^2)

μ = a function of n, s, k_h, k_s

ρ = settlement at a given time

ρ_c = total primary consolidation settlement

σ = vertical total stress (kN/m^2)

σ' = vertical effective stress (kN/m^2)

σ'_ι = in-situ vertical effective stress or initial vertical effective stress (kN/m^2)

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Table 1. Parameters used to compare the effects of loading increment ratio, and C_c/C_k

Parameters	Value
$n = d_e/d_w$	15
$s = d_s/d_w$	4
Horizontal permeability, k_h ($\times 10^{-9}$ m/s)	1.0
k_h/k_s	1.5
Initial void ratio, e_0	2
In-situ effective stress, σ'_i (kPa)	20
C_c	0.25

Table 2. Soil properties of reconstituted clay sample

Soil properties	Reconstituted clay
Water content (%)	45
Plastic limit, w_p (%)	17
Plasticity Index, PI (%)	25
Unit weight, γ_t, (kN/m^3)	17
C_c	0.29
C_k	0.45

Table 3. Parameters used in the analysis

Parameters	Test 1 (Final total pressure = 50 kPa)	Test 2 (Final total pressure = 100 kPa)
Diameter of influence zone, d_e (m)	0.45	0.45
Equivalent diameter of drain, d_w (m)	0.066	0.066
Diameter of smear zone, d_s (m)	0.2	0.2
$n = d_e/d_w$	6.79	6.79
$s = d_s/d_w$	3.02	3.02
Initial horizontal permeability, k_{hi} ($\times 10^{-10}$ m/s)	4.6	4.0
k_h/k_s	1.5	1.5
Initial void ratio, e_0	1.0	0.95
c_{hi} ($\times 10^{-3}$ m^2/day)	1.58	3.02
Initial height, m	0.925	0.87
Preconsolidation pressure, p'_c (kPa)	20	50
Δp (kPa)	30	50

Table 4. Geometric parameters of the vertical drain system for Muar clay embankment

Geometric parameters	Value
Installation pattern	Triangular
drain spacing, d (m)	1.3
Diameter of influence zone, d_e (m)	1.365
Equivalent drain diameter, d_w (m)	0.07
s = d_s/d_w	4

Table 5. Soil parameters for Muar clay Embankments (data source from Ratnayake, 1991)

Depth (m)	C_r	C_c	γ_t (kN/m^3)	e_0	σ' at the middle of layer (kPa)	p'_c (kPa)	k_h (In-situ value) ($\times 10^{-9}$ m/s)
0.0-1.75	0.35	0.71	16.5	3.10	4.88	60	6.4
1.50-2.50	0.37	0.71	15.0	3.10	12.25	55	5.2
2.50-5.5		1.38	15.0	3.00	22.25	50	5.2
5.5-6.5		1.38	15.5	3.00	32.50	44	3.1
6.5-8.0		0.71	15.5	1.95	39.38	51	3.1
8.0-10.0		0.71	16.0	1.82	49.50	60	1.3
10.0-12.0		0.83	16.0	1.86	61.50	73	0.6
12.0-14.0		0.83	16.0	1.89	73.50	86	0.6
14.0-16.0		0.83	16.0	1.86	85.50	97	0.6
16.0-18.0		0.83	16.0	1.86	97.50	110	0.6

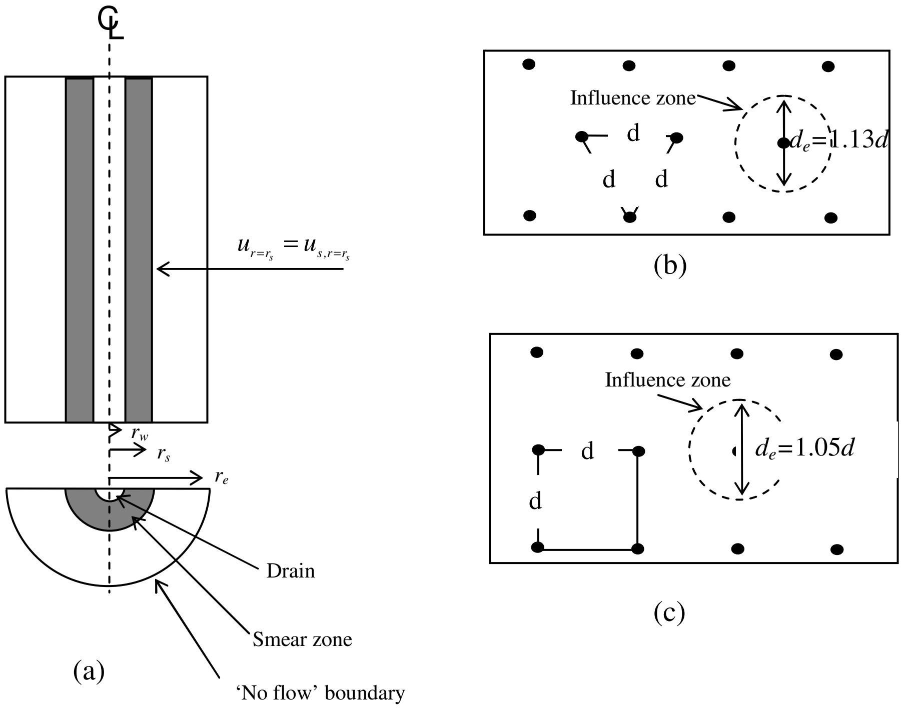


Fig. 1. Vertical drain and its layout: (a) unit cell, (b) triangular grid pattern and (c) square grid pattern

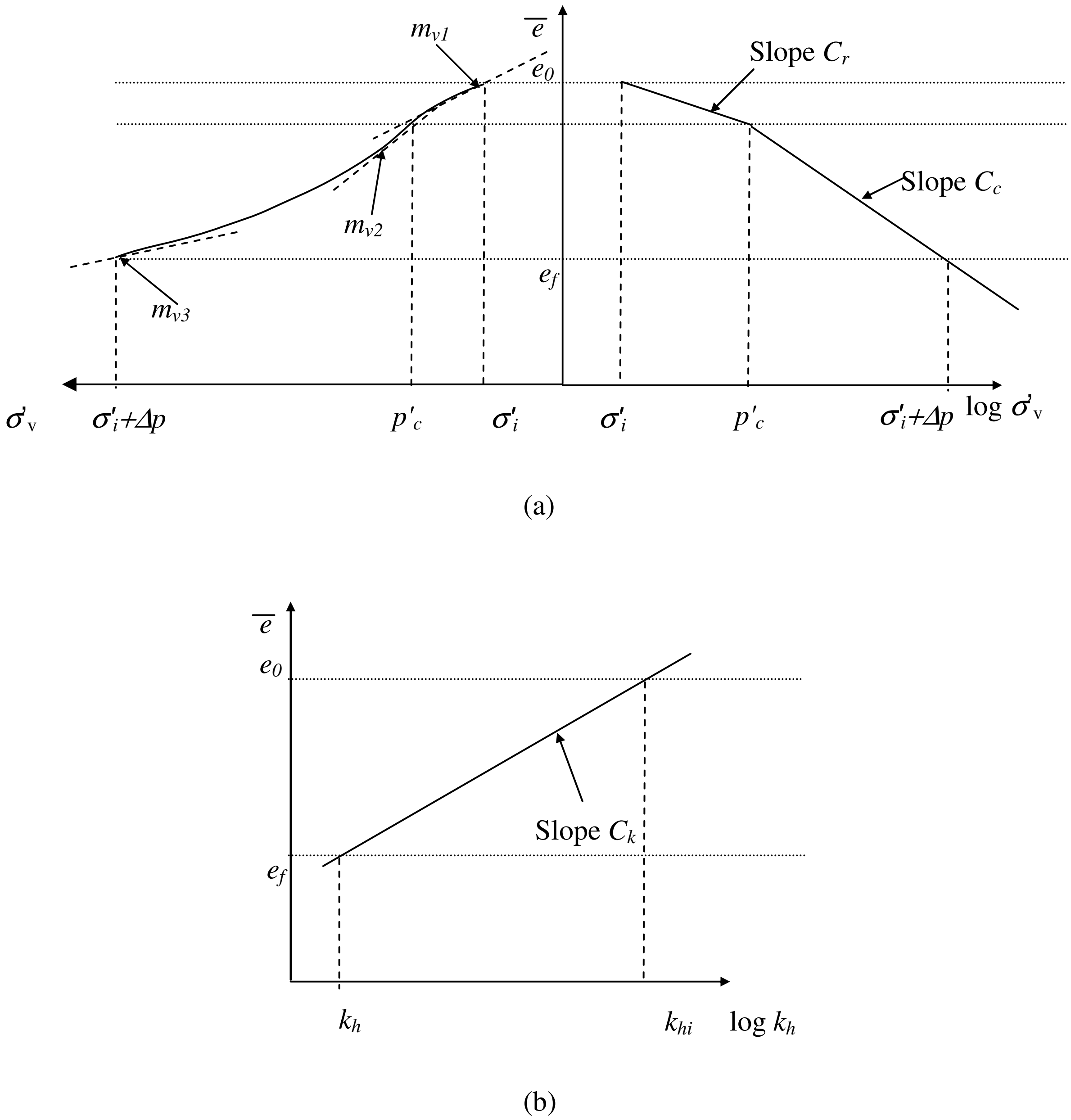
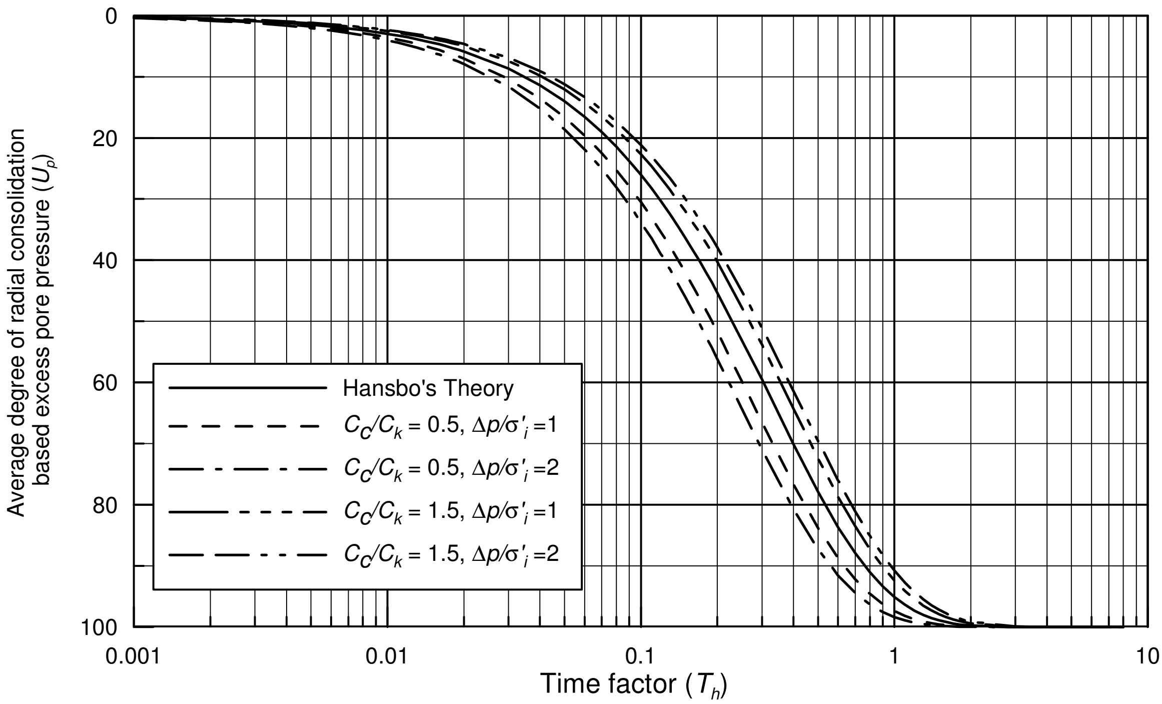
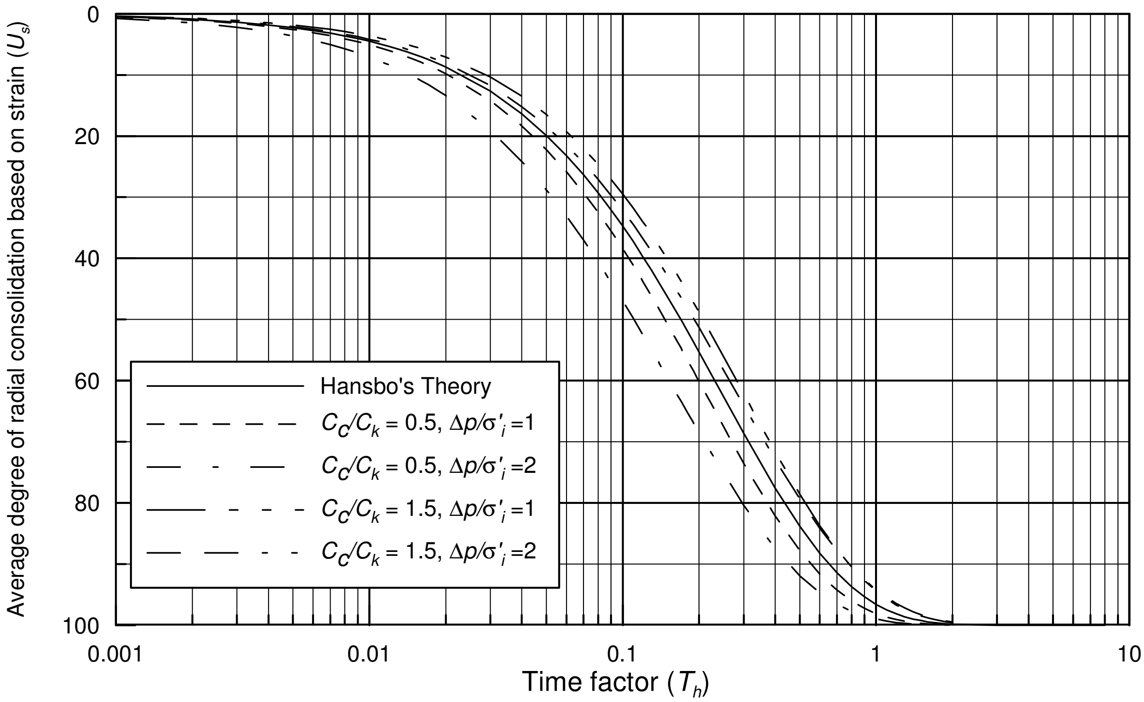


Fig. 2. (a) Compression during preloading and (b) Semi-log permeability-void ratio relationship



(a)



(b)

Fig. 3. Average degree of radial consolidation U plotted against time factor T_h for varying compressibility and permeability relationships (a) based on excess pore pressure, (b) based on strain

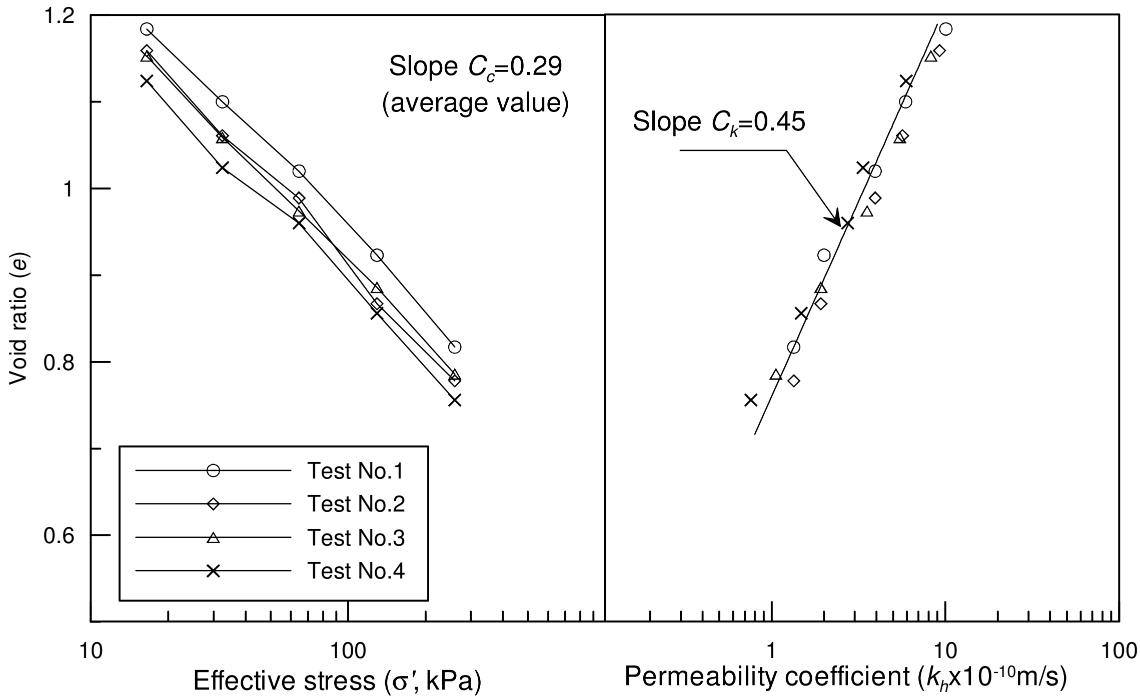
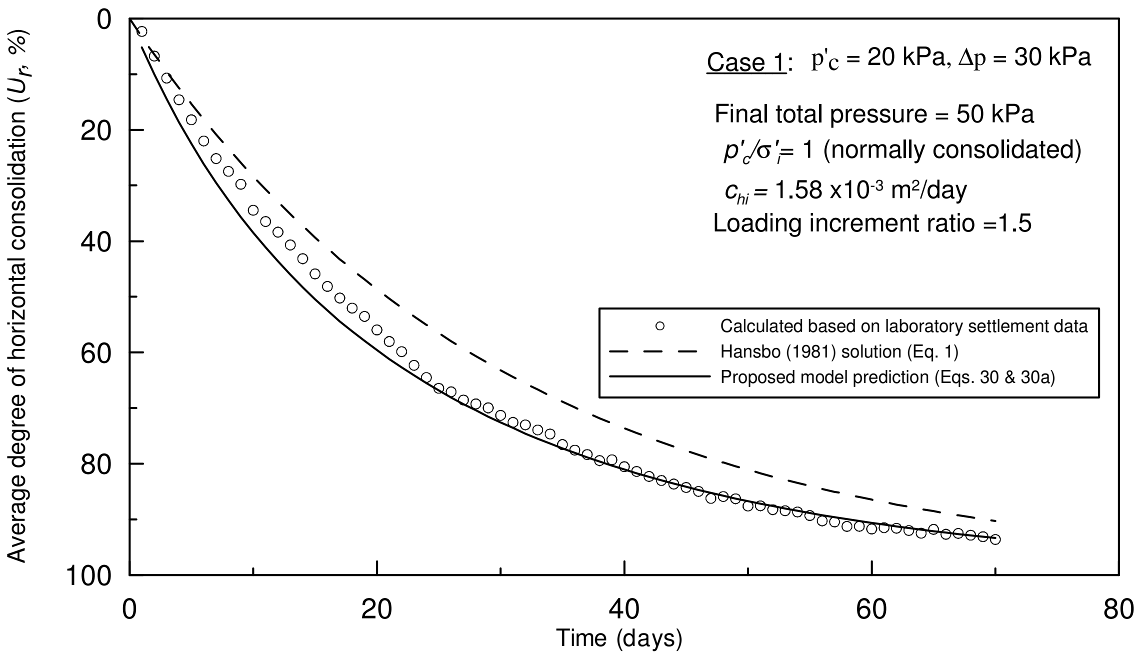
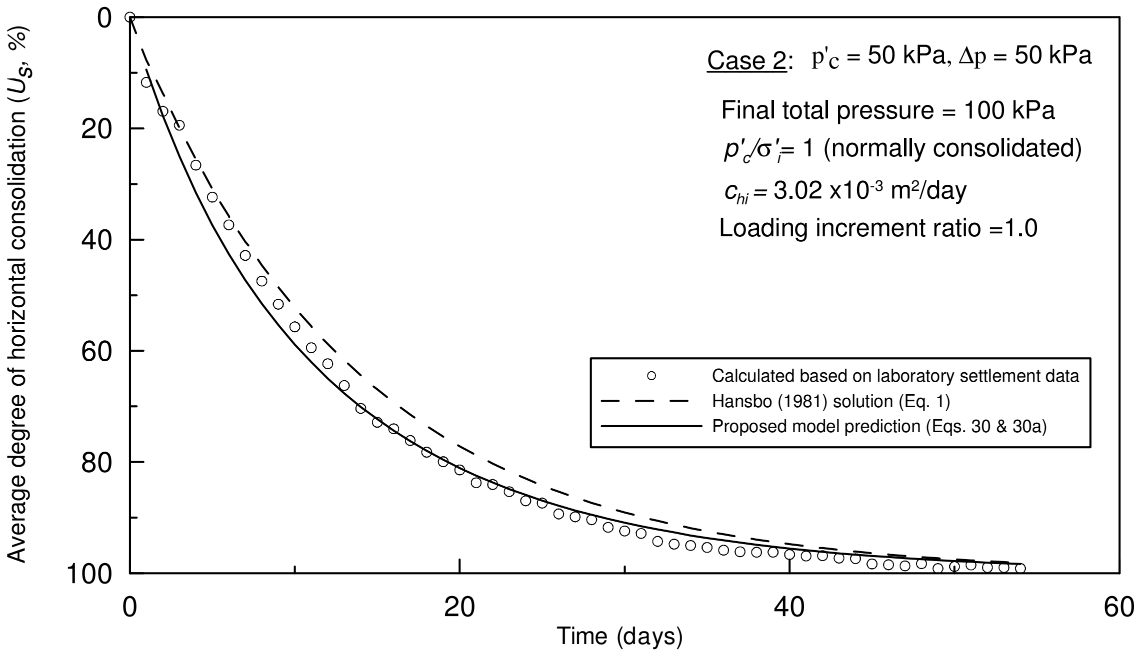


Fig. 4. Typical e-logσ' and e-logk_h plots for Moruya clay



(a)



(b)

Fig. 5. Comparison between measured and predicted results from proposed model and Hansbo's solution, (a) p'_c = 20 kPa, Δp = 30 kPa. and (b) p'_c = 50 kPa, Δp = 50 kPa

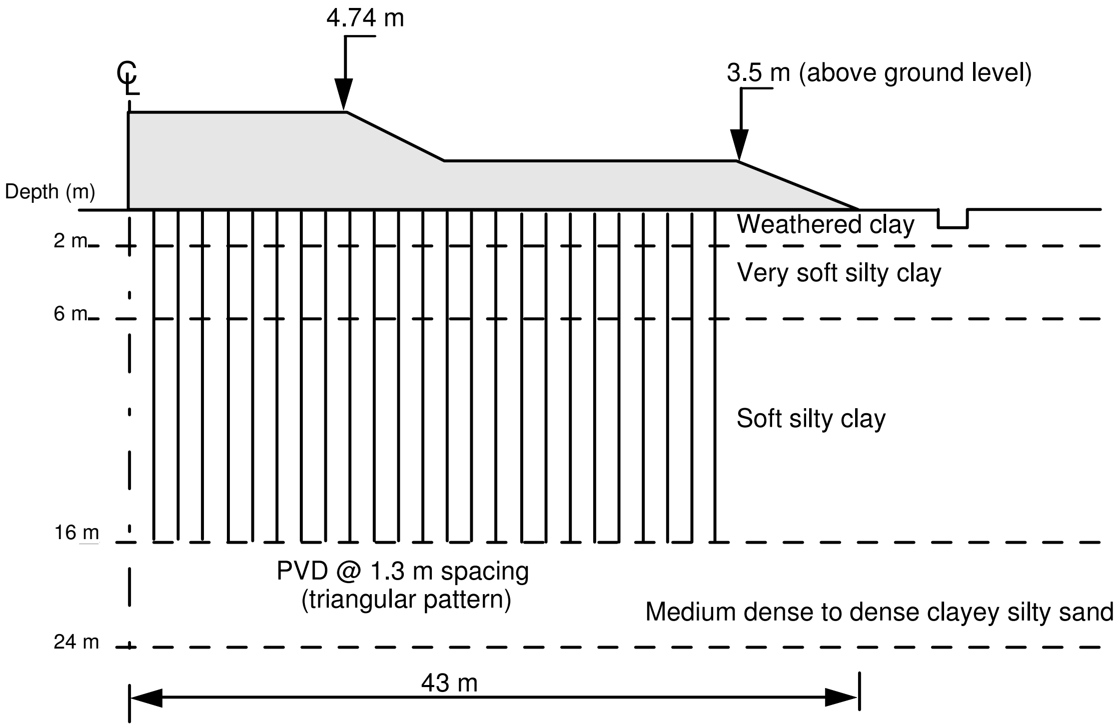
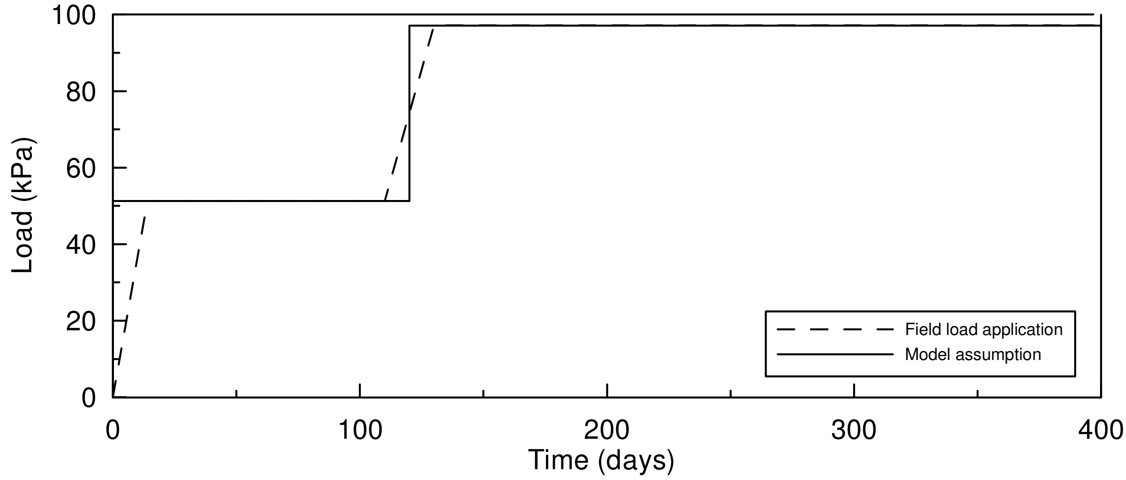
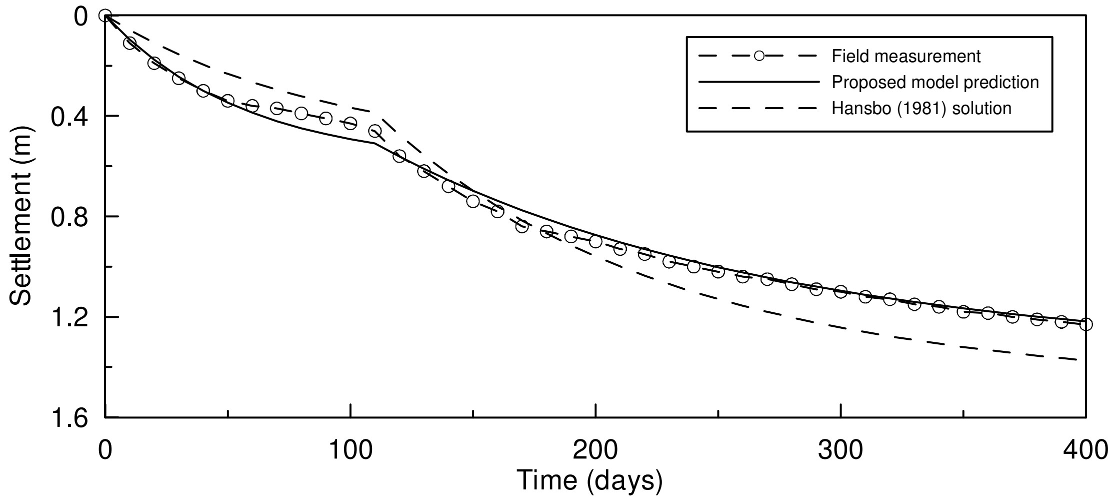


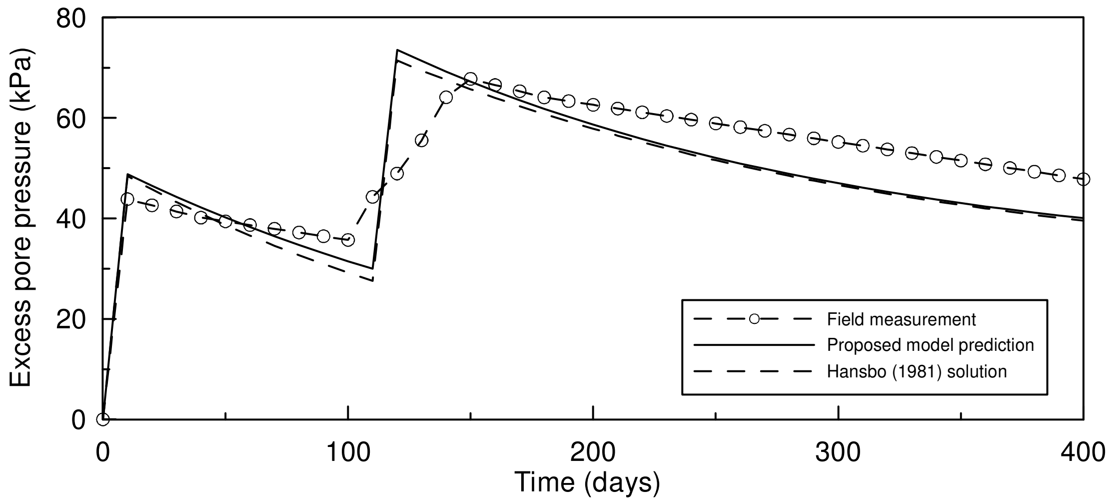
Fig. 6. Cross section of embankments with soil profile, Muar clay, Malaysia



(a)



(b)



(c)

Fig. 7. Muar clay embankment in Malaysia; (a) stages of loading, (b) surface settlements under the embankment centreline and (c) excess pore pressures at a depth of 11.2m below ground surface, 0.65m away from the centerline