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Abstract

Locust swarms hit subsistence-staple-crop-growing households at random and are not privately controllable. An aerial-spraying optimal control model that supports the said households' livelihood at least expected cost is therefore developed. The qualitative properties of the model are analysed under economically plausible but mild assumptions. The steady state comparative statics reveal that the locust swarm size and the probability of a household's crop being destroyed by a swarm decrease with the number of households, yield per household, and the staple crop's replacement price, and increase with the marginal cost of spraying and the planner's discount rate. A local comparative dynamics analysis is also conducted, as it provides the necessary economic intuition behind other ostensibly anomalous steady state comparative statics results.

Keywords: Optimal control, local stability, steady state comparative statics, local comparative dynamics

1. Introduction

The growing of essential crops in large parts of North and West Africa, as well as in other less developed areas inhabited by indigenous people pursuing a traditional way of life, has often been impeded by swarms of locusts. Typically, locust swarms have not been effectively dealt with by small staple-crop-growing households due to the swarms' large size, high mobility, and the random nature of the timing and target of their raids. Also compounding the problem is the lack of private capital, skill, coordination, and cooperation among the indigenous people, along with their unwillingness to share common locust-control costs. Indeed, the severe outbreak of this plague in North and West Africa in the second half of the 1980s is due, in part, to the public complacency over a period of thirty years that followed the high initial control obtained with the application of the highly toxic and environmentally persistent dieldrin in the 1950s.

The locusts' lifecycle is approximately six weeks, during which they are transformed from crawlers to walkers and hopers, to flyers and, finally, to mating insects. They dwell in equal life-phase groups clearly identifiable by their colours—white, green, pink, and yellow, respectively. The principal crop-damaging group is the flying pink locusts, known as the swarm. Unlike many other groups of smaller and better-camouflaged agricultural pests, the pink-locust swarms are highly visible and their impact is immediately evident.

A swarm's survival and regeneration depend on cultivation size and type. Consequently, drastic changes in the scale and types of farming activities can reduce the number and size of locust swarms. However, the said changes can also adversely affect the well being of indigenous farming households which, for ethnic, cultural, and human capital reasons, are not willing to give up their traditional way of life and relocate. In North and West Africa the means for supporting crop production and the traditional way of life of the indigenous population in regions affected by locusts is a regionally and internationally coordinated and financed effort, which consists of aerial spraying of organophosphate pesticides with relatively short environmental persistence [Cowan and Gunby (1996) provide an explanation as to why chemical control of agricultural pests remains the dominant technology].

The optimal control model developed in this paper focuses on a locust-alley, such as those in North and West Africa. These areas are dominated by a perpetual locust swarm and house a large and stable number of similar, traditional farming households, which are endowed with small fields for self-sustaining staple-crop production. The proposed model differs from the agricultural and environmental economic pest-control models extant for technologically advanced cash-crop farms by its design to stabilise the production of staple crops in less developed areas, and thus support the native inhabitants' traditional way of life at the least expected cost to the planner [cf. Shoemaker (1973) and Saphores (2000)].

The model takes into account the fact that a swarm moves quickly and erratically, and thus hits clusters of staple-crop-growing households at random. It also takes into account that, due to an immediate and highly noticeable presence and adverse impact, a swarm's location and density are accurately assessed and reported by the affected farming households. In view of the standard practice of scheduling the aerial spraying at a time when a swarm is most vulnerable—the dawn that follows a reporting of a swarm's location and density by the affected households—the efficacy of the aerial spraying is also taken to be certain, in contrast to Feder (1979). However, the large size of the swarm and the convexity of the spraying cost function prevents the eradication of the swarm. As is commonly the case, it is assumed that the crop of the raided households is completely devoured. As a result, previously sprayed fields are not revisited by a swarm during a crop-growing season and, in turn, the number and timing of pesticide applications [cf. Hall and Norgaard (1973) and Saphores (2000)] and re-entry [cf. Lichtenberg, Spear and Zilberman (1993)] are not relevant issues in this locust-alley environment.

2. A Locust-Control Model

Consider a topographically and climatically homogeneous locust-alley, with $N(t) \in \mathbb{R}_+$ indicating the size of the locust swarm at time t, i.e., the number of pink locusts or stock of locusts at time t, and let $F\left(N(t)\right)$ be the swarm's natural growth rate given the swarm size $N(t)$. Define $s(t) \in \mathbb{R}_+$ as the rate of pesticide aerially sprayed on the swarm at time t, and let $C\left(s(t);\alpha\right)$ be

the total cost of spraying at the rate $s(t)$. The parameter $\alpha \in \mathbb{R}_{++}$ represents a price index of the involved inputs—pesticides, planes' fuel, user cost, piloting and ground crew, coordination, and management. This price index takes into account the full price of pesticides, which includes environmental and health damages from spraying. The higher the inputs' market prices and the harsher the pesticides' impact on the environment and health, the larger is the value of α. The eradication production function $E(\cdot)$ is assumed to be a function of the size of the swarm and the rate of pesticide spraying, and thus its value may be expressed as $E\left(s(t), N(t)\right)$. Given this information, the instantaneous change in the size of the locust swarm is given by the ordinary differential equation $\dot{N}(t) = F\left(N(t)\right) - E\left(s(t), N(t)\right)$, which serves as the state equation in the optimal control model.

The following assumptions are placed on the functions $C(\cdot)$, $E(\cdot)$, and $F(\cdot)$, and explained subsequently.

(A.1) $C(\cdot): \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$, $C(\cdot) \in C^{(2)}$, $C_s(s;\alpha) > 0$, $C_\alpha(s;\alpha) > 0$, $C_{s\alpha}(s;\alpha) > 0$, and $C_{ss}(s;\alpha) > 0$ for all $(s,\alpha) \in \mathbb{R}^2_{++}$.

(A.2) $E(\cdot): \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$, $E(\cdot) \in C^{(2)}$, $E_s(s,N) > 0$, $E_N(s,N) > 0$, $E_{sN}(s,N) > 0$, $E_{ss}(s,N) < 0$, $E_{NN}(s,N) < 0$, for all $(s,N) \in \mathbb{R}^2_{++}$.

(A.3) $F(\cdot): \mathbb{R}_+ \to \mathbb{R}$, $F(\cdot) \in C^{(2)}$ for all $N \in \mathbb{R}_{++}$, $F(0) = F(K) = 0$, $F'(N) > 0$ for all $N \in [0, \bar{N})$, $F'(\bar{N}) = 0$, $F'(N) < 0$ for all $N \in (\bar{N}, K]$, and $F''(N) < 0$ for all $N \in \mathbb{R}_{++}$.

The assumption that all of the underlying functions are $C^{(2)}$ is driven by the use of general functional forms and the desire to obtain a differential qualitative characterization of the solution of the ensuing optimal control problem. Assumption (A.1) asserts that the spraying cost function is a strictly increasing and strongly convex function of the spraying rate, and that an increase in the input price index α increases the total and marginal cost of spraying. Similarly, supposition (A.2) states that the eradication production function is a strictly increasing and strongly concave function of the spraying rate and swarm size, individually, and that the marginal product of spraying is an increasing function of the swarm size—the latter reflecting the fact that a given spraying rate is more effective the larger is the size of the swarm. Finally, assumption (A.3)

states that the growth function is strongly concave, increasing for stock sizes less than the growth maximizing stock size $\bar{N} \in \mathbb{R}_{++}$, decreasing for stock sizes greater than the growth maximizing stock size, and that growth is zero when the locust population is nil or when it is at the locust-alley's environmental carrying capacity $K > \bar{N}$. The logistic growth function is an example of a specific functional form that satisfies the aforementioned assumptions on the locust growth function $F(\cdot)$.

Because of the long-range mobility of the pink locusts and the variation in the air temperature and currents, household-fields are hit at random by the swarm. Carlson and Wetzstein (1993) have argued that the likelihood of a crop getting eaten by pests is a function of cultivation size and that it is given by the entropy distribution. In the present case, however, farms are small and numerous, and similar household-fields share a similar climate and topography. Moreover, the farmers typically grow a common staple-crop. Accordingly, it is assumed that each household-field in the locust-alley has an equal probability of being hit by the swarm at any time t. Furthermore, it is assumed that the probability of a household-field being hit satisfies

(A.4) $\varphi(\cdot): \mathbb{R}_{+} \to [0,1]$, $\varphi(\cdot) \in C^{(2)}$, $\varphi'(NL^{-1}) > 0$ and $\varphi''(NL^{-1}) < 0$ for all $NL^{-1} \in \mathbb{R}_{++}$, $\lim_{NL^{-1} \downarrow 0} \varphi(NL^{-1}) = 0$, and $\lim_{NL^{-1} \uparrow +\infty} \varphi(NL^{-1}) = 1$.

This assumption states that the probability of a household-field being hit by the swarm at time t increases with the size of the swarm $N \in \mathbb{R}_{+}$ relative to the number of time-invariant feeding sites or farming households $L \in \mathbb{R}_{++}$, but does so at a diminishing rate.

In view of the small size of a household's field and the large size of the swarm, it is assumed that in the event of being hit by the swarm, the household's crop is completely destroyed and that the yield (or output) of a household-field spared by the swarm is a time-invariant (due to traditional cultivation methods) scalar $y \in \mathbb{R}_{++}$. Given these assumptions, the yield of a household-field in the locust-alley at time t is binomially distributed. Consequently, yield is zero with probability $\varphi\left(N(t)L^{-1}\right)$ and $y \in \mathbb{R}_{++}$ with probability $1-\varphi\left(N(t)L^{-1}\right)$. This implies that the expected aggregate loss of yield in the locust-alley at time t is given by $yL\varphi\left(N(t)L^{-1}\right)$.

The proposed swarm-control model is based on the following additional premises. First, there is a trade-off between the instantaneous cost of aerial spraying and the instantaneous yield salvaged. Second, the livelihood of households hit by the swarm depends on free-of-charge public aid. Third, the public aid fully compensates the affected households for the loss of yield with purchased and delivered quantities of the staple crop. Fourth, the public planner is risk-neutral and farsighted when selecting the aerial spraying trajectory that minimizes the present value of the expected cost caused by the swarm. Finally, the expected cost caused by the locust swarm at any time t is comprised of the expected cost of compensating for the aggregate loss of yield, plus the cost of spraying.

Putting all of these elements together, the public planner's problem may be formally expressed as the following infinite horizon optimal control problem:

$$C^*(\boldsymbol{\theta}, N_0) \stackrel{\text{def}}{=} \min_{s(\cdot)} \int_0^{+\infty} \left[pyL\varphi\left(N(t)L^{-1}\right) + C\left(s(t);\alpha\right)\right] e^{-\rho t} dt \tag{1}$$

$$\text{s.t. } \dot{N}(t) = F\left(N(t)\right) - E\left(s(t), N(t)\right), \;\; N(0) = N_0,$$

where $p \in \mathbb{R}_{++}$ denotes the crop-replacement price, i.e., the full price of delivering the staple crop to the affected households, $\rho \in \mathbb{R}_{++}$ is the public planner's instantaneous and time-invariant discount rate, $N_0 \in \mathbb{R}_{++}$ is the initial size of the locust swarm, $C^*(\boldsymbol{\theta}, N_0)$ is the minimum present-value expected-cost of the optimal spraying plan, and $\boldsymbol{\theta} \stackrel{\text{def}}{=} (\alpha, L, p, \rho, y) \in \mathbb{R}^5_{++}$.

The ensuing additional assumptions are introduced for problem (1) and discussed below.

(A.5) There exists a unique and interior optimal solution to problem (1) for each $\boldsymbol{\gamma} \in B(\overline{\boldsymbol{\gamma}};\delta)$, $\boldsymbol{\gamma} \stackrel{\text{def}}{=} (\boldsymbol{\theta}, N_0)$, denoted by $\left(N^*(t;\boldsymbol{\theta}, N_0), s^*(t;\boldsymbol{\theta}, N_0)\right)$, with corresponding current value co-state variable $\lambda(t;\boldsymbol{\theta}, N_0)$, where $B(\overline{\boldsymbol{\gamma}};\delta)$ is an open 6-ball centered at the point $\overline{\boldsymbol{\gamma}} \in \mathbb{R}^6_{++}$ of radius $\delta > 0$.

(A.6) For each $\boldsymbol{\gamma} \in B(\overline{\boldsymbol{\gamma}};\delta)$ the optimal pair $\left(N^*(t;\boldsymbol{\theta}, N_0), s^*(t;\boldsymbol{\theta}, N_0)\right) \to \left(\hat{N}(\boldsymbol{\theta}), \hat{s}(\boldsymbol{\theta})\right)$ as $t \to +\infty$, where $\left(\hat{N}(\boldsymbol{\theta}), \hat{s}(\boldsymbol{\theta})\right)$ is the simple, unique, and interior steady state solution of problem (1).

Given the lack of functional form assumptions, the aggregate nature of the optimal control model, and the fact that neither the Mangasarian- or Arrow-type sufficiency theorems can be applied to problem (1) to establish optimality of a solution that satisfies the necessary conditions, supposition (A.5) is a natural one to make. Even if one were to make further "favourable" curvature assumptions on the functions, e.g., that $\varphi(\cdot)$ is convex and $E(\cdot)$ is concave, the Hamiltonian is not necessarily convex in (s, N) because of the concavity of $F(\cdot)$ and the fact that the current value costate variable is positive in the optimal plan, as is shown in §3. Assumption (A.6) states that the optimal solution of problem (1) converges to the unique, interior, and simple steady state solution. Recall that a simple steady state is one in which the determinant of the Jacobian matrix of the linearization of the differential equations is nonzero when evaluated at the steady state. Take in totality, assumptions (A.1)–(A.6) permit us to use the results in Caputo (1997) or Caputo (2005) to present a thorough but compact characterization of the qualitative properties of the solution to problem (1).

3. Necessary Conditions and Local Stability of the Steady State

We begin this section with the derivation of the necessary conditions obeyed by the planner's optimal spraying policy. To this end, define the current value Hamiltonian by

$$H(N, s, \lambda; \theta) \stackrel{\text{def}}{=} pyL\varphi(NL^{-1}) + C(s;\alpha) + \lambda[F(N) - E(s, N)]. \tag{2}$$

By Theorems 14.3 and 14.9 of Caputo (2005), the triplet $\left(N^*(t;\theta, N_0), s^*(t;\theta, N_0), \lambda(t;\theta, N_0)\right)$ necessarily satisfies

$$H_s(N, s, \lambda; \theta) = C_s(s;\alpha) - \lambda E_s(s, N) = 0\,, \tag{3}$$

$$\dot{\lambda} = [\rho - F'(N) + E_N(s, N)]\lambda - py\varphi'(NL^{-1})\,, \tag{4}$$

$$\dot{N} = F(N) - E(s, N)\,,\ N(0) = N_0\,, \tag{5}$$

$$\lim_{t\to+\infty} H(N, s, \lambda; \theta)e^{-\rho t} = 0\,. \tag{6}$$

Equation (3) is the typical marginal optimality condition, and asserts that the optimal spraying rate at each point in time of the planning horizon equates the marginal social cost of spraying [$C_s(s;\alpha)$] with its marginal social benefit [$\lambda E_s(s, N)$], the latter being comprised of the product

of the current value shadow cost of the swarm (λ) and the marginal product of the spraying rate [$E_s(s,N)$]. Furthermore, using assumptions (A.1) and (A.2), Eq. (3) can be rearranged to read $\lambda = C_s(s;\alpha)/E_s(s,N)$, thereby implying that $\lambda(t;\boldsymbol{\theta},N_0) > 0$ for all $t \in [0,+\infty)$. But this was to be anticipated, seeing as the planner is interested in minimizing the costs associated with controlling the locust swarm and because the latter is a "bad". In passing, note that by making use of the fact that $\lambda = C_s(s;\alpha)/E_s(s,N)$ and using assumption (A.6), one can verify that necessary condition (6) does indeed hold along the optimal solution.

In order to determine the local stability of the steady state and draw the local phase portrait corresponding to the solution to the planner's problem, we proceed to reduce Eqs. (3)–(5) down to two necessary conditions. To begin, observe that because Eq. (3) holds as an identity for all $t \in [0,+\infty)$ along the optimal path, it may be differentiated with respect to t. Doing just that, substituting Eqs. (4) and (5) in the resulting differential equation, and then making use of the relation $\lambda = C_s(s;\alpha)/E_s(s,N)$, we arrive at the following pair of ordinary differential equations:

$$\dot{N} = F(N) - E(s,N)\,, \tag{7}$$

$$\dot{s} = \frac{C_s(s;\alpha)\left\{\dfrac{E_{sN}(s,N)}{E_s(s,N)}\left[F(N)-E(s,N)\right]+\rho-F'(N)+E_N(s,N)\right\} - py\varphi'(NL^{-1})E_s(s,N)}{C_{ss}(s;\alpha)-\left[C_s(s;\alpha)/E_s(s,N)\right]E_{ss}(s,N)}\,. \tag{8}$$

By assumption (A.5), the optimal pair $\left(N^*(t;\boldsymbol{\theta},N_0), s^*(t;\boldsymbol{\theta},N_0)\right)$ is the unique solution to the dynamical system consisting of Eqs. (7) and (8).

In the steady state $\dot{N} = 0$ and $\dot{s} = 0$, in which case Eqs. (7) and (8) simplify to

$$F(N) - E(s,N) = 0\,, \tag{9}$$

$$C_s(s;\alpha)\left[\rho - F'(N) + E_N(s,N)\right] - py\varphi'(NL^{-1})E_s(s,N) = 0\,. \tag{10}$$

By assumption (A.6), the pair of values $\left(\hat{N}(\boldsymbol{\theta}), \hat{s}(\boldsymbol{\theta})\right)$ is the unique solution to Eqs. (9) and (10). Furthermore, upon using assumptions (A.1), (A.2), and (A.4) in Eq. (10), it follows that

$$\rho - F'\left(\hat{N}(\boldsymbol{\theta})\right) + E_N\left(\hat{s}(\boldsymbol{\theta}), \hat{N}(\boldsymbol{\theta})\right) > 0\,. \tag{11}$$

This result will be of value when the qualitative properties of the model are derived in §4.

In order to determine the local stability of the steady state, compute the Jacobian matrix of Eqs. (7) and (8), evaluate the result at the steady state solution $\left(\hat{N}(\boldsymbol{\theta}),\hat{s}(\boldsymbol{\theta})\right)$, and then use assumptions (A.1)–(A.4). Carrying out these three steps yields

$$J^d \stackrel{\text{def}}{=} \left.\begin{bmatrix} \dfrac{\partial \dot{N}}{\partial N} & \underset{(-)}{\dfrac{\partial \dot{N}}{\partial s}} \\ \dfrac{\partial \dot{s}}{\partial N} & \underset{(+)}{\dfrac{\partial \dot{s}}{\partial s}} \end{bmatrix}\right|_{\substack{\dot{N}=0\\ \dot{s}=0}} = \begin{bmatrix} F'\left(\hat{N}(\boldsymbol{\theta})\right)-E_N\left(\hat{s}(\boldsymbol{\theta}),\hat{N}(\boldsymbol{\theta})\right) & \underset{(-)}{-E_s\left(\hat{s}(\boldsymbol{\theta}),\hat{N}(\boldsymbol{\theta})\right)} \\ j_{21}^d\left(\hat{N}(\boldsymbol{\theta}),\hat{s}(\boldsymbol{\theta})\right) & \underset{(+)}{j_{22}^d\left(\hat{N}(\boldsymbol{\theta}),\hat{s}(\boldsymbol{\theta})\right)} \end{bmatrix}, \tag{12}$$

where the sign of an element of J^d is given below it when it is unambiguous,

$$j_{21}^d\left(\hat{N}(\boldsymbol{\theta}),\hat{s}(\boldsymbol{\theta})\right) \stackrel{\text{def}}{=} \frac{\dfrac{\hat{C}_s\hat{E}_{sN}}{\hat{E}_s}\left[\hat{F}'-\hat{E}_N\right]+\hat{C}_s\left[\hat{E}_{NN}-\hat{F}''\right]-py\left[\hat{\varphi}'\hat{E}_{sN}+\hat{\varphi}''\hat{E}_sL^{-1}\right]}{\hat{C}_{ss}-\left[\hat{C}_s/\hat{E}_s\right]\hat{E}_{ss}}, \tag{13}$$

$$j_{22}^d\left(\hat{N}(\boldsymbol{\theta}),\hat{s}(\boldsymbol{\theta})\right) \stackrel{\text{def}}{=} \frac{\hat{C}_{ss}\left[\rho-\hat{F}'+\hat{E}_N\right]-py\hat{\varphi}'\hat{E}_{ss}}{\hat{C}_{ss}-\left[\hat{C}_s/\hat{E}_s\right]\hat{E}_{ss}} > 0, \tag{14}$$

and where, for example, the notation $\hat{C}_s \stackrel{\text{def}}{=} C_s\left(\hat{s}(\boldsymbol{\theta});\alpha\right)$ signifies that the function is evaluated at the steady state solution $\left(\hat{N}(\boldsymbol{\theta}),\hat{s}(\boldsymbol{\theta})\right)$.

Because problem (1) falls under the class of optimal control problems studied by Caputo (1997, 2005), we may apply Eq. (10a) of Caputo (1997) to conclude that $\operatorname{tr} J^d = \rho > 0$, thereby implying that the simple steady state is also hyperbolic. Consequently, we may also invoke Stability Lemma of Caputo (1997) or Theorem 18.1 of Caputo (2005) to conclude that the steady state $\left(\hat{N}(\boldsymbol{\theta}),\hat{s}(\boldsymbol{\theta})\right)$ is a local saddle point, or equivalently, that $\left|J^d\right| < 0$.

Given that $\left|J^d\right| < 0$, an equivalent statement of this condition is that

$$\left.\frac{-\partial\dot{N}/\partial N}{\underset{(-)}{\partial\dot{N}/\partial s}}\right|_{\substack{\dot{N}=0\\ \dot{s}=0}} < \left.\frac{-\partial\dot{s}/\partial N}{\underset{(+)}{\partial\dot{s}/\partial s}}\right|_{\substack{\dot{N}=0\\ \dot{s}=0}}, \tag{15}$$

as may be readily verified. By the implicit function theorem, the left-hand side of Eq. (15) is the slope of the $\dot{N}=0$ isocline in a neighbourhood of the steady state, while the right-hand side is the slope of the $\dot{s}=0$ isocline in a neighbourhood of the steady state. In view of the fact that only the signs of the denominators in Eq. (15) are known in general, as indicated, Eq. (15) points

to three different configurations of the isoclines that are consistent with the steady state being a local saddle point. We examine each possibility in what follows.

First consider the case in which the slope of the $\dot{N}=0$ isocline is positive in a neighbourhood of the steady state, or equivalently, that $\partial\dot{N}/\partial N\big|_{\dot{N}=0\wedge\dot{s}=0}=\hat{F}'-\hat{E}_N>0$. In this case Eq. (15) implies that in a neighbourhood of the steady state, the slope of the $\dot{s}=0$ isocline is positive and greater than the slope of the $\dot{N}=0$ isocline, and that $\partial\dot{s}/\partial N\big|_{\dot{N}=0\wedge\dot{s}=0}<0$. Figure 1 presents the phase portrait in this situation. The optimal trajectory is the stable manifold of the saddle point steady state and is depicted by the two "thick" trajectories in Figure 1. The phase diagram shows that the optimal spraying rate and swarm size increase monotonically over time if the initial swarm size is less than the steady state swarm size, while they both decline monotonically over time if the initial swarm size is greater than the steady state swarm size. Because $\partial\dot{N}/\partial N\big|_{\dot{N}=0\wedge\dot{s}=0}=\hat{F}'-\hat{E}_N>0$ in this case, the steady state swarm size is less than the growth-rate maximizing swarm size $\bar{N}$, which obeys $F'(\bar{N})=0$ from assumption (A.3).

The second case consistent with the local saddle point nature of the steady state occurs when the slope of the $\dot{s}=0$ isocline is negative in a neighbourhood of the steady state, or equivalently, when $\partial\dot{s}/\partial N\big|_{\dot{N}=0\wedge\dot{s}=0}>0$. In this situation Eq. (15) implies that in a neighbourhood of the steady state, the slope of the $\dot{N}=0$ isocline is negative and absolutely greater than the slope of the $\dot{s}=0$ isocline, and furthermore that $\partial\dot{N}/\partial N\big|_{\dot{N}=0\wedge\dot{s}=0}=\hat{F}'-\hat{E}_N<0$. The phase portrait for this case is given in Figure 2. Note that the inequality $\partial\dot{N}/\partial N\big|_{\dot{N}=0\wedge\dot{s}=0}=\hat{F}'-\hat{E}_N<0$ implies that the steady state swarm size is greater in the present case than in the previous one, and that it may also be greater than the growth-rate maximizing swarm size $\bar{N}$. Consequently, it is not surprising in this case to find that the optimal spraying rate increases monotonically to its steady state value when the initial swarm size is greater than its steady state swarm size, in contrast to the previous case.

The third (and last) case that is consistent with the steady state being a local saddle point is when the $\dot{N}=0$ isocline has a negative slope in a neighbourhood of the steady state and the $\dot{s}=0$ isocline has a positive slope in a neighbourhood of the steady state, which are equivalent to

$\partial\dot{N}/\partial N\big|_{\dot{N}=0\wedge\dot{s}=0} = \hat{F}' - \hat{E}_N < 0$ and $\partial\dot{s}/\partial N\big|_{\dot{N}=0\wedge\dot{s}=0} < 0$, respectively. The phase portrait for this situation is depicted in Figure 3.

Before closing out this section, we pause to briefly mention the ramifications for the phase portrait in the special case in which the eradication production function is independent of the swarm size, i.e., when $E_N(s,N) \equiv 0$. Under this supposition, it follows from Eqs. (12) and (13) that $\partial\dot{s}/\partial N\big|_{\dot{N}=0\wedge\dot{s}=0} > 0$, thereby implying, by way of the implicit function theorem and Eq. (15), that the slope of the $\dot{s}=0$ isocline is negative in a neighbourhood of the steady state. Using Eq. (15) and the implicit function theorem again, this implies that the slope of the $\dot{N}=0$ isocline is also negative in a neighbourhood of the steady state, or equivalently, that $\partial\dot{N}/\partial N\big|_{\dot{N}=0\wedge\dot{s}=0} = \hat{F}' < 0$, and that it is absolutely larger than that of the $\dot{s}=0$ isocline. These conclusions imply that Figure 2 is the relevant phase portrait under the simplifying assumption $E_N(s,N) \equiv 0$.

In closing out this section we return to the necessary and sufficient condition for local saddle point stability of the steady state, to wit, $\left|J^d\right| < 0$. We intend to establish a relationship between this condition and the Jacobian determinant of the steady state equations evaluated at the steady state. This result plays a central role in determining the local differentiability and qualitative steady state comparative statics properties of the optimal solution to problem (1), as will be shown in §4.

To this end, compute the Jacobian matrix of Eqs. (9) and (10), evaluate the result at the steady state solution $\left(\hat{N}(\boldsymbol{\theta}), \hat{s}(\boldsymbol{\theta})\right)$, and then use assumptions (A.1)–(A.4) to get

$$J^s \stackrel{\text{def}}{=} \begin{bmatrix} F'\left(\hat{N}(\boldsymbol{\theta})\right) - E_N\left(\hat{s}(\boldsymbol{\theta}), \hat{N}(\boldsymbol{\theta})\right) & \underset{(-)}{-E_s\left(\hat{s}(\boldsymbol{\theta}), \hat{N}(\boldsymbol{\theta})\right)} \\ j_{21}^s\left(\hat{N}(\boldsymbol{\theta}), \hat{s}(\boldsymbol{\theta})\right) & \underset{(+)}{j_{22}^s\left(\hat{N}(\boldsymbol{\theta}), \hat{s}(\boldsymbol{\theta})\right)} \end{bmatrix}, \tag{16}$$

where

$$j_{21}^s\left(\hat{N}(\boldsymbol{\theta}), \hat{s}(\boldsymbol{\theta})\right) \stackrel{\text{def}}{=} \hat{C}_s\left[\hat{E}_{NN} - \hat{F}''\right] - py\left[\hat{\varphi}'\hat{E}_{sN} + \hat{\varphi}''\hat{E}_s L^{-1}\right], \tag{17}$$

$$j_{22}^s\left(\hat{N}(\boldsymbol{\theta}), \hat{s}(\boldsymbol{\theta})\right) \stackrel{\text{def}}{=} \hat{C}_s\hat{E}_{Ns} + \hat{C}_{ss}\left[\rho - \hat{F}' + \hat{E}_N\right] - py\hat{\varphi}'\hat{E}_{ss} > 0. \tag{18}$$

Next, using Eq. (10b) of Caputo (1997), or by computing the determinants of Eqs. (12) and (16), it can be shown that $\left|J^s\right| = \left[\hat{C}_{ss} - \left[\hat{C}_s / \hat{E}_s\right]\hat{E}_{ss}\right]\left|J^d\right|$. Seeing as $\hat{C}_{ss} - \left[\hat{C}_s / \hat{E}_s\right]\hat{E}_{ss} > 0$ by assumptions (A.1) and (A.2) and that $\left|J^d\right| < 0$, it follows that $\left|J^s\right| < 0$. Moreover, given that the functions $C(\cdot)$, $E(\cdot)$, $F(\cdot)$, and $\varphi(\cdot)$ are $C^{(2)}$ by assumptions (A.1)–(A.4), and the fact that $\left|J^s\right| \neq 0$, the implicit function theorem implies that $\left(\hat{N}(\cdot), \hat{s}(\cdot)\right) \in C^{(1)}$ locally. This conclusion permits the use of the differential calculus to study the comparative statics properties of the steady state solution $\left(\hat{N}(\boldsymbol{\theta}), \hat{s}(\boldsymbol{\theta})\right)$ in §4.

4. Steady State Comparative Statics and Local Comparative Dynamics

The analysis of the steady state comparative statics begins by substituting the steady state solution $\left(\hat{N}(\boldsymbol{\theta}), \hat{s}(\boldsymbol{\theta})\right)$ in Eqs. (9) and (10) to get the following identities in $\boldsymbol{\theta}$:

$$F\left(\hat{N}(\boldsymbol{\theta})\right) - E\left(\hat{s}(\boldsymbol{\theta}), \hat{N}(\boldsymbol{\theta})\right) \equiv 0, \tag{19}$$

$$C_s\left(\hat{s}(\boldsymbol{\theta}); \alpha\right)\left[\rho - F'\left(\hat{N}(\boldsymbol{\theta})\right) + E_N\left(\hat{s}(\boldsymbol{\theta}), \hat{N}(\boldsymbol{\theta})\right)\right] - py\varphi'\left(\hat{N}(\boldsymbol{\theta})L^{-1}\right)E_s\left(\hat{s}(\boldsymbol{\theta}), \hat{N}(\boldsymbol{\theta})\right) \equiv 0. \tag{20}$$

Differentiation of Eqs. (19) and (20) with respect to, say, α, gives the system of linear equations

$$\begin{bmatrix} F'\left(\hat{N}(\boldsymbol{\theta})\right) - E_N\left(\hat{s}(\boldsymbol{\theta}), \hat{N}(\boldsymbol{\theta})\right) & \underset{(-)}{-E_s\left(\hat{s}(\boldsymbol{\theta}), \hat{N}(\boldsymbol{\theta})\right)} \\ j_{21}^s\left(\hat{N}(\boldsymbol{\theta}), \hat{s}(\boldsymbol{\theta})\right) & \underset{(+)}{j_{22}^s\left(\hat{N}(\boldsymbol{\theta}), \hat{s}(\boldsymbol{\theta})\right)} \end{bmatrix} \begin{bmatrix} \dfrac{\partial \hat{N}(\boldsymbol{\theta})}{\partial \alpha} \\ \dfrac{\partial \hat{s}(\boldsymbol{\theta})}{\partial \alpha} \end{bmatrix} \equiv \begin{bmatrix} 0 \\ -\hat{C}_{s\alpha}\left[\rho - \hat{F}' + \hat{E}_N\right] \end{bmatrix}. \tag{21}$$

Solving Eq. (21) with Cramer's rule, using assumptions (A.1), (A.2), and Eq. (11), and recalling that $\left|J^s\right| < 0$, gives the steady state comparative statics results

$$\frac{\partial \hat{N}(\boldsymbol{\theta})}{\partial \alpha} \equiv \frac{-\hat{E}_s \hat{C}_{s\alpha}\left[\rho - \hat{F}' + \hat{E}_N\right]}{\left|J^s\right|} > 0, \tag{22}$$

$$\frac{\partial \hat{s}(\boldsymbol{\theta})}{\partial \alpha} \equiv \frac{-\left[\hat{F}' - \hat{E}_N\right]\hat{C}_{s\alpha}\left[\rho - \hat{F}' + \hat{E}_N\right]}{\left|J^s\right|}. \tag{23}$$

Alternatively, one may simply invoke Theorem 1 of Caputo (1997) or Theorem 18.2 of Caputo (2005) to derive these and the ensuing steady state comparative statics results.

Noting that the steady state probability of an individual household's crop being devoured by the locust swarm is $\varphi\left(\hat{N}(\boldsymbol{\theta})L^{-1}\right)$, and making use of assumption (A.4) and Eq. (22), it follows that

$$\frac{\partial\varphi\left(\hat{N}(\boldsymbol{\theta})L^{-1}\right)}{\partial\alpha}=\varphi'\left(\hat{N}(\boldsymbol{\theta})L^{-1}\right)\frac{\partial\hat{N}(\boldsymbol{\theta})}{\partial\alpha}L^{-1}>0\,. \tag{24}$$

Finally, using Theorem 14.10 of Caputo (2005) and assumption (A.1), we arrive at the dynamic envelope result

$$\frac{\partial C^{*}(\boldsymbol{\theta},N_0)}{\partial\alpha}=\int_0^{+\infty}C_{\alpha}\left(s^{*}(t;\boldsymbol{\theta},N_0);\alpha\right)e^{-\rho t}dt>0\,. \tag{25}$$

Let us now turn to the economic interpretation of Eqs. (22)–(25).

Because an increase in α (which is due to a rise in the spraying inputs' market prices and/or in the environmental and health sensitivities to pesticides) results in an increase in the (instantaneous) total and marginal costs of spraying, it is not surprising that Eq. (25) shows that the minimum present value expected cost of the locust control program increases with α. Equations (22) and (24) show that the steady state swarm size and the probability of a household being hit by the swarm both increase with the marginal cost of spraying. These are intuitive results in that with spraying more costly at the margin, one would expect a larger swarm size and thus an increased probability of being hit by the swarm to result. On the other hand, Eq. (23) shows the steady state spraying rate may increase or decrease when the marginal cost of spraying increases. Thus, it is possible that the steady state swarm size and the spraying rate increase as a result of an increase in the marginal cost of spraying, a rather unexpected outcome. Indeed, this is precisely what occurs when the sufficient condition that produces Figure 1 prevails, as we show in the ensuing paragraph. Moreover, with the aid of a local comparative dynamics analysis we will also provide an intuitive explanation for this surprising steady state comparative statics outcome.

Recall that in §3 we proved that if $\partial\dot{N}/\partial N\big|_{\dot{N}=0\wedge\dot{s}=0}=\hat{F}'-\hat{E}_N>0$, which is equivalent to assuming that the slope of the $\dot{N}=0$ isocline is positive in a neighbourhood of the steady state, then $\partial\dot{s}/\partial N\big|_{\dot{N}=0\wedge\dot{s}=0}<0$ and the phase portrait is given by Figure 1. Using Eq. (23) it then follows

that $\partial\hat{s}(\boldsymbol{\theta})/\partial\alpha > 0$, as was to be demonstrated. In order to come to some understanding of the economic rationale behind the seemingly counterintuitive joint occurrence of $\partial\hat{N}(\boldsymbol{\theta})/\partial\alpha > 0$ and $\partial\hat{s}(\boldsymbol{\theta})/\partial\alpha > 0$, we now examine the local comparative dynamics of an increase in α. The reader may wish to consult Caputo (2005, Chapters 15–18) for more details on the construction of comparative dynamics phase diagrams that are used below.

First observe that by using Eq. (7), it follows that the $\dot{N} = 0$ isocline is independent of α and therefore does not shift when α changes. Using Eq. (8), on the other hand, shows that the $\dot{s} = 0$ isocline is a function of α, as it appears explicitly in it. By the implicit function theorem, the effect of an increase in α on the $\dot{s} = 0$ isocline in a neighbourhood of the steady state is given by $-[\partial\dot{s}/\partial\alpha]/[\partial\dot{s}/\partial s] < 0$, as may be readily verified. This result implies that the $\dot{s} = 0$ isocline shifts down in the (N, s) phase plane in a neighbourhood of the steady state when α increases. Note that these conclusions regarding the isoclines hold regardless of their slopes.

Given the conclusions of the preceding paragraph, and maintaining the basic assumption that generates Figure 1, namely, $\partial\dot{N}/\partial N\big|_{\dot{N}=0\wedge\dot{s}=0} = \hat{F}' - \hat{E}_N > 0$, the local comparative dynamics phase portrait corresponding to an increase in α may be drawn—it is given in Figure 4. The optimal path from the old steady state to the new steady state is indicated by the "thick" trajectory, and shows that at the instant the marginal cost of spraying increases, the planner reduces the spraying rate below its old steady state value. This initial decrease in the spraying rate and the subsequent period of time in which it remains below its old steady state value are responsible for the rise in the swarm size in the new steady state—this despite the fact that the spraying rate is higher in the new steady state than it is in the old. Thus, the ostensibly counterintuitive steady state results, to wit, $\partial\hat{N}(\boldsymbol{\theta})/\partial\alpha > 0$ and $\partial\hat{s}(\boldsymbol{\theta})/\partial\alpha > 0$, are found to be intuitively plausible once the local comparative dynamics of the increase in α are understood. It is left to the interested reader to derive the local comparative dynamic phase diagrams in the intuitive cases corresponding to Figures 2 and 3, i.e., when $\partial\hat{N}(\boldsymbol{\theta})/\partial\alpha > 0$ and $\partial\hat{s}(\boldsymbol{\theta})/\partial\alpha < 0$ occur.

Let us now turn to the effects of an increase in the planner's discount rate ρ. The ensuing qualitative results can be derived by following the above procedures and using assumptions (A.1)–(A.4), or by invoking Theorem 1 of Caputo (1997) or Theorem 18.2 of Caputo (2005):

$$\frac{\partial \hat{N}(\boldsymbol{\theta})}{\partial \rho} \equiv \frac{-\hat{E}_s \hat{C}_s}{\left|J^s\right|} > 0\,,\ \frac{\partial \hat{s}(\boldsymbol{\theta})}{\partial \rho} \equiv \frac{-\left[\hat{F}' - \hat{E}_N\right]\hat{C}_s}{\left|J^s\right|}\,,\ \frac{\partial \varphi\left(\hat{N}(\boldsymbol{\theta})L^{-1}\right)}{\partial \rho} = \hat{\varphi}' \frac{\partial \hat{N}(\boldsymbol{\theta})}{\partial \rho} L^{-1} > 0\,, \tag{26}$$

$$\frac{\partial C^*(\boldsymbol{\theta}, N_0)}{\partial \rho} = -\int_0^{+\infty} t\left[pyL\varphi\left(N^*(t;\boldsymbol{\theta}, N_0)L^{-1}\right) + C\left(s^*(t;\boldsymbol{\theta}, N_0);\alpha\right)\right] e^{-\rho t} dt < 0\,. \tag{27}$$

The steady state comparative statics exhibited in Eq. (26) are qualitatively identical to those for α and thus merit only a brief remark. Recalling that an increase in ρ implies a more impatient planner, it is intuitive that the optimal plan calls for a higher steady state swarm size along with an increased probability of an attack, seeing as the more impatient planner wishes to push off "bad" events into the future even more. This intuition also explains the lower present value cost of the program, as indicated by Eq. (27). As was the case for an increase in the marginal cost of spraying, the steady state spraying rate may increase or decrease with the planner's increased impatience. In passing, note that the local comparative dynamics properties of ρ are qualitatively identical to α, and so are left for the interested reader to contemplate.

Turning to the crop-replacement price p, we have

$$\frac{\partial \hat{N}(\boldsymbol{\theta})}{\partial p} \equiv \frac{y\hat{\varphi}'[\hat{E}_s]^2}{\left|J^s\right|} < 0\,,\ \frac{\partial \hat{s}(\boldsymbol{\theta})}{\partial p} \equiv \frac{\left[\hat{F}' - \hat{E}_N\right] y\hat{\varphi}'\hat{E}_s}{\left|J^s\right|}\,,\ \frac{\partial \varphi\left(\hat{N}(\boldsymbol{\theta})L^{-1}\right)}{\partial p} = \hat{\varphi}' \frac{\partial \hat{N}(\boldsymbol{\theta})}{\partial p} L^{-1} < 0\,, \tag{28}$$

$$\frac{\partial C^*(\boldsymbol{\theta}, N_0)}{\partial p} = \int_0^{+\infty} yL\varphi\left(N^*(t;\boldsymbol{\theta}, N_0)L^{-1}\right) e^{-\rho t} dt > 0\,. \tag{29}$$

The steady state comparative statics given in Eq. (28) show that they are qualitatively the opposite of those for the marginal cost of spraying and the planner's discount rate. Because a higher crop-replacement price implies that a locust invasion is more costly, it is intuitive that the optimal policy calls for a lower steady state stock of locusts and a lower probability of an attack, and results in a higher expected present discounted cost of the optimal spraying program. On the other hand, an increase in the crop-replacement price may increase or decrease the steady state

spraying rate. If it is the case that the steady state spraying rate increases, i.e., if $\partial\hat{s}(\boldsymbol{\theta})/\partial p > 0$, then it is intuitive that there would be a lower steady state swarm size and probability of attack. If, however, the steady state spraying rate decreases, i.e., $\partial\hat{s}(\boldsymbol{\theta})/\partial p < 0$, then as was the case with an increase in the marginal cost of spraying, it seems counterintuitive that the steady state swarm size and probability of attack are lower in the new steady state too. As was the case before, investigating the local comparative dynamics of an increase in the crop-replacement price will shed light on the underlying economic intuition of this ostensibly counterintuitive case.

As before, the $\dot{N} = 0$ isocline is independent of p and thus does not shift when p changes. The $\dot{s} = 0$ isocline, however, is a function of p and thus does shift as p changes. By the implicit function theorem, the effect of an increase in p on the $\dot{s} = 0$ isocline in a neighbourhood of the steady state is given by $-[\partial\dot{s}/\partial p]/[\partial\dot{s}/\partial s] > 0$, thereby implying that the $\dot{s} = 0$ isocline shifts up when p increases in a neighbourhood of the steady state. Because these two conclusions hold regardless of the slope of the isoclines, Figures 2 and 3 are seen to generate the intuitive outcome, to wit, $\partial\hat{s}(\boldsymbol{\theta})/\partial p > 0$ and $\partial\hat{N}(\boldsymbol{\theta})/\partial p < 0$, just as they did for an increase in α and ρ. Consequently, it is again Figure 1 that yields the seemingly counterintuitive steady state comparative statics results $\partial\hat{s}(\boldsymbol{\theta})/\partial p < 0$ and $\partial\hat{N}(\boldsymbol{\theta})/\partial p < 0$. We thus turn to the local comparative dynamics associated with Figure 1 for an increase in p in order to fully understand this "puzzling" result.

Maintaining the essential assumption that generates Figure 1, that is to say, $\partial\dot{N}/\partial N\big|_{\dot{N}=0\wedge\dot{s}=0} = \hat{F}' - \hat{E}_N > 0$, it follows from Eq. (28) that $\partial\hat{s}(\boldsymbol{\theta})/\partial p < 0$ and $\partial\hat{N}(\boldsymbol{\theta})/\partial p < 0$. Given the shift in the $\dot{s} = 0$ isocline noted in the preceding paragraph, the local comparative dynamics phase portrait corresponding to an increase in p may be drawn, and is given by Figure 5. The optimal path from the old to the new steady state is again indicated by the "thick" trajectory, and shows that at the instant the crop-replacement price increases, the planner increases the spraying rate above its old steady state rate. By continuity, the spraying rate remains above its old steady state rate for a finite period of time, and as a result, the swarm size falls over time. Even though the spraying rate in the new steady state is lower than it was in the old, the initial

(but temporary) increase in the spraying rate brought about by the increase in the crop-replacement price is wholly responsible for the lower swarm size in the new steady state. Thus, even though $\partial\hat{s}(\boldsymbol{\theta})/\partial p < 0$ and $\partial\hat{N}(\boldsymbol{\theta})/\partial p < 0$ occur in this case and appear counterintuitive, the local comparative dynamics analysis has revealed the mechanism by which this qualitative result is indeed a rational outcome, to wit, there is an initial increase in the spraying rate that drives the swarm size and the probability of an attack down to their new steady state values.

The steady state comparative statics and envelope properties that result from an increase in the number of households and yield per household are qualitatively identical to that for an increase in the crop-replacement price, as shown by the ensuing formulae:

$$\frac{\partial\hat{N}(\boldsymbol{\theta})}{\partial L} \equiv \frac{-py[\hat{E}_s]^2\hat{\varphi}''\hat{N}(\boldsymbol{\theta})L^{-2}}{\left|J^s\right|} < 0\,,\ \frac{\partial\hat{s}(\boldsymbol{\theta})}{\partial L} \equiv \frac{-\left[\hat{F}' - \hat{E}_N\right]py\hat{E}_s\hat{\varphi}''\hat{N}(\boldsymbol{\theta})L^{-2}}{\left|J^s\right|}\,,$$

$$\frac{\partial\varphi\left(\hat{N}(\boldsymbol{\theta})L^{-1}\right)}{\partial L} = \hat{\varphi}'\left[\frac{\partial\hat{N}(\boldsymbol{\theta})}{\partial L}L^{-1} - \hat{N}(\boldsymbol{\theta})L^{-2}\right] < 0\,,\ \frac{\partial C^*(\boldsymbol{\theta}, N_0)}{\partial L} = \int_0^{+\infty} py\varphi\left(N^*(t;\boldsymbol{\theta}, N_0)L^{-1}\right)e^{-\rho t}dt > 0\,,$$

and

$$\frac{\partial\hat{N}(\boldsymbol{\theta})}{\partial y} \equiv \frac{p\hat{\varphi}'[\hat{E}_s]^2}{\left|J^s\right|} < 0\,,\ \frac{\partial\hat{s}(\boldsymbol{\theta})}{\partial y} \equiv \frac{\left[\hat{F}' - \hat{E}_N\right]p\hat{\varphi}'\hat{E}_s}{\left|J^s\right|}\,,\ \frac{\partial\varphi\left(\hat{N}(\boldsymbol{\theta})L^{-1}\right)}{\partial y} = \hat{\varphi}'\frac{\partial\hat{N}(\boldsymbol{\theta})}{\partial y}L^{-1} < 0\,,$$

$$\frac{\partial C^*(\boldsymbol{\theta}, N_0)}{\partial y} = \int_0^{+\infty} pL\varphi\left(N^*(t;\boldsymbol{\theta}, N_0)L^{-1}\right)e^{-\rho t}dt > 0\,.$$

Given the above observation we provide but a brief economic interpretation of these results.

When the number of households in the locust-alley increase, the new steady state has a lower swarm size and a lower probability of a household being attacked by the swarm, while the expected present value cost to the planner is higher, regardless of whether or not the spraying rate is higher or lower in the new steady state. This is driven by the fact that with more households in the locust-alley, aggregated expected losses are higher, hence the optimal policy aims for a lower swarm size and probability of attack. The same seemingly counterintuitive result can occur in this case too, as it is possible for the steady state spraying rate to fall along with the swarm size and probability of an attack. But as was the case with an increase in the crop-

replacement price, this is explained by the initial increase in the spraying rate that accompanies the increase in the number of farm-households. Essentially the same economic intuition applies when the yield per household increases, seeing as it too raises the expected aggregate loss to the planner.

5. Summary and Concluding Remarks

Locust swarms in less developed areas regularly hit clusters of subsistence-staple-crop-growing small-scale farming households at random, and inflict essentially immediate and complete damage. Moreover, the said farming households typically act in an uncoordinated manner, and as a result, the locust swarm attacks often go unchecked through locust-allies. With these empirical regularities in mind, we have developed an optimal-control model designed to stabilise the supply of staple crops in such areas and to support the inhabitants' traditional way of life at least expected cost to a public planner.

Relying on plausible economic assumptions about the costs of spraying and the eradication technology, we showed that the steady state of the model is a local saddle point. Using this feature of the model we then provided a thorough characterization of the steady state comparative statics of the optimal spraying policy. In particular, we showed that the steady state swarm size and probability of an attack increase with (i) the number of alternative feeding sites or farm-households, (ii) the amount of food for the locusts, i.e., yield per farm, and (iii) the value of the crop and hence the value of spraying. On the other hand, we showed that the steady state swarm size and probability of an attack increase with the spraying's input-prices and adverse effects on the environment and population health, and with the planner's discount rate. Using the dynamic envelope theorem, we also demonstrated that the aforesaid increases in the parameters of the model generate the expected changes in the planner's minimum present discounted expected cost of locust control.

In contrast to the above unambiguous and intuitive qualitative results, we found that in general, an increase in any of the parameters of the model generates an ambiguous change in the

steady state rate of spraying. This implies that the steady state rate of spraying and swarm size may both decrease when, say, the crop-replacement price increases. By examining the local comparative dynamics of a crop-replacement price increase, we showed that this ostensibly counterintuitive result is easily understood. Specifically, we showed that the result stems from the fact that at the instant the crop-replacement price increases, the spraying rate initially increases above its old steady state value. This feature provides the impetus for driving down the swarm size to its new, lower, steady state level, in spite of the fact that the spraying rate is lower in the new steady state.

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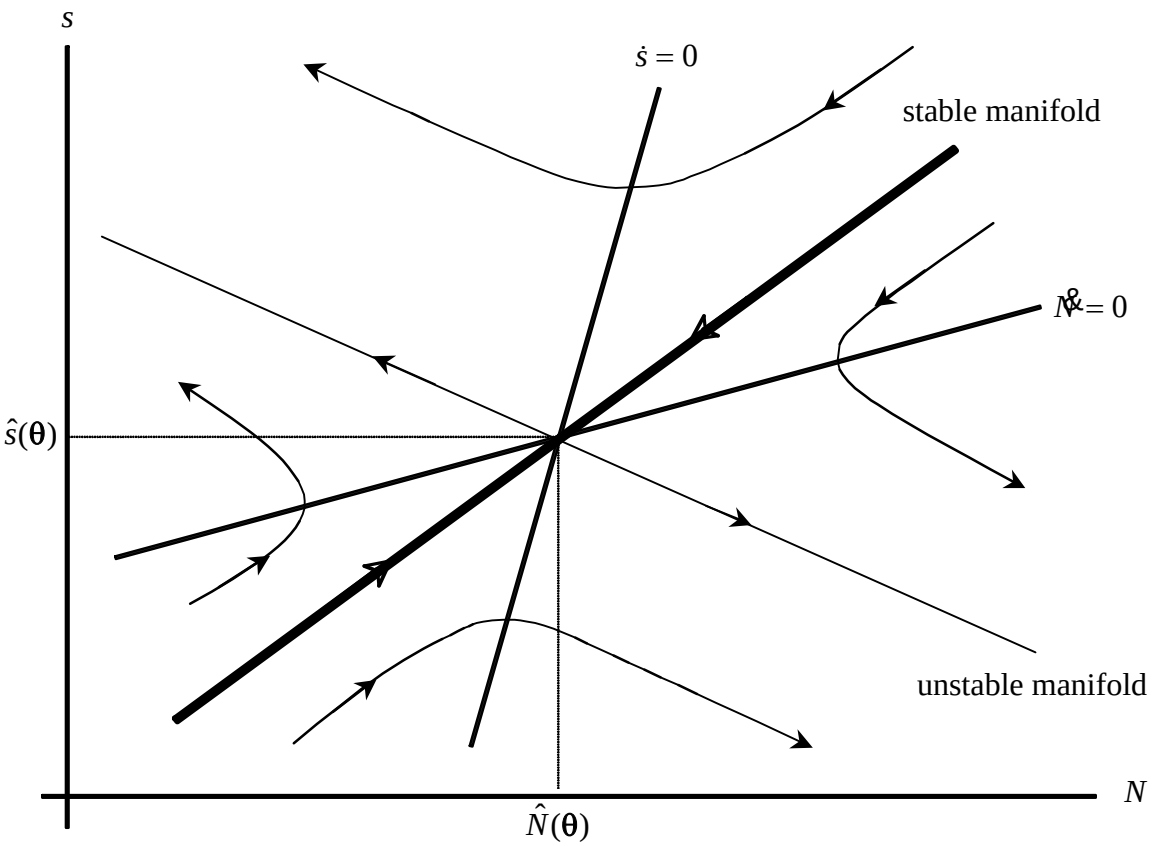


Figure 1

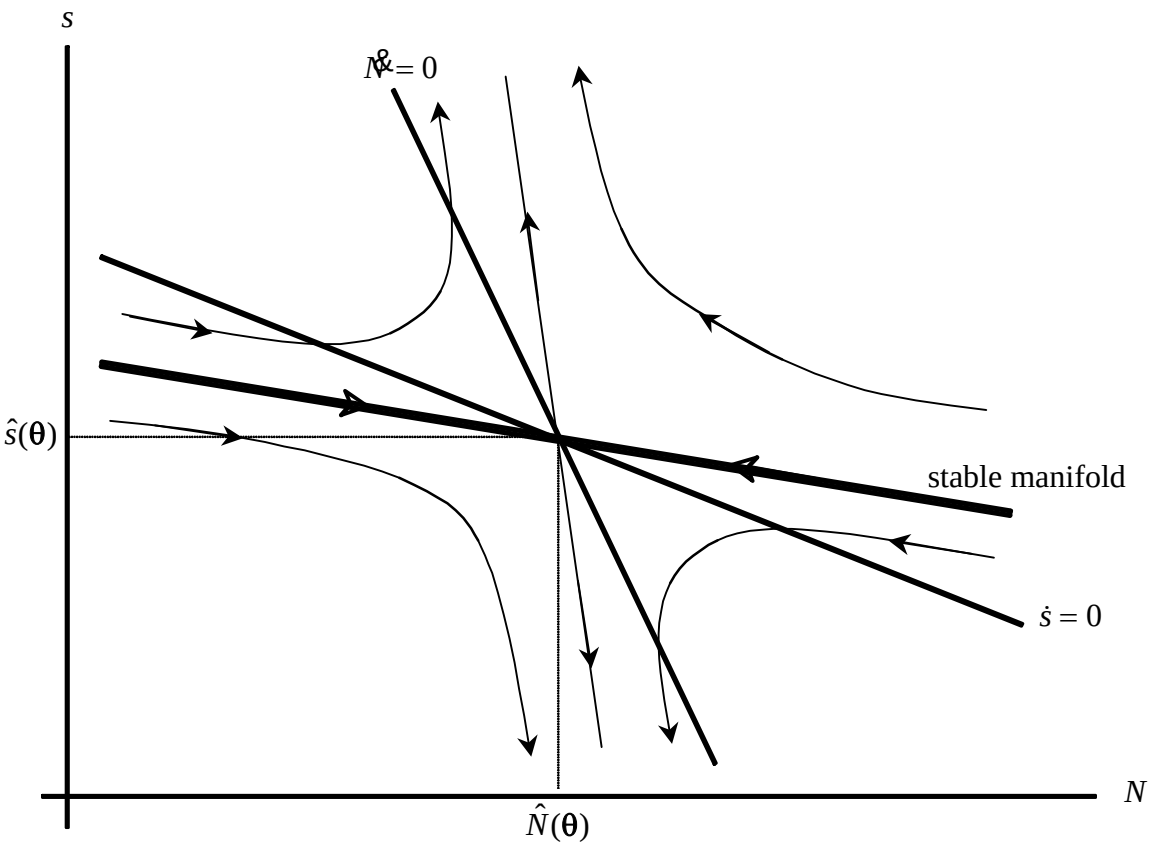
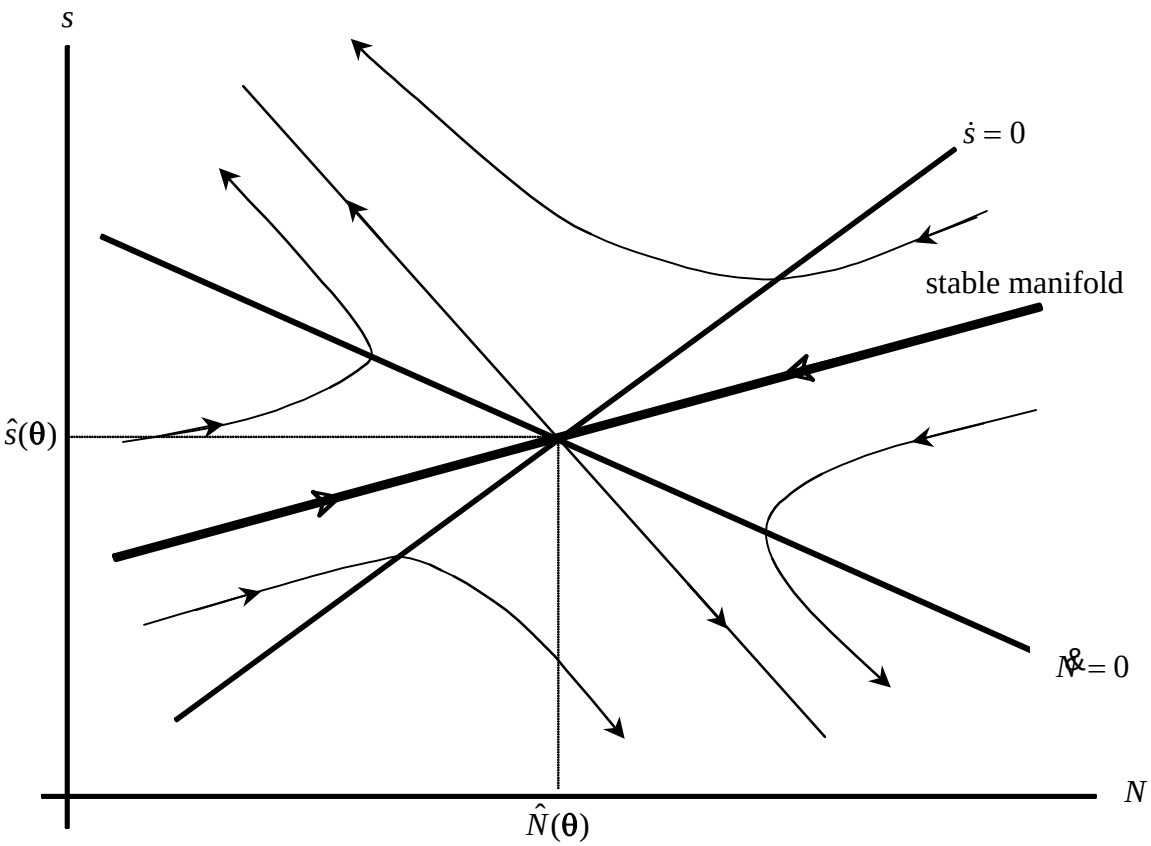


Figure 2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N
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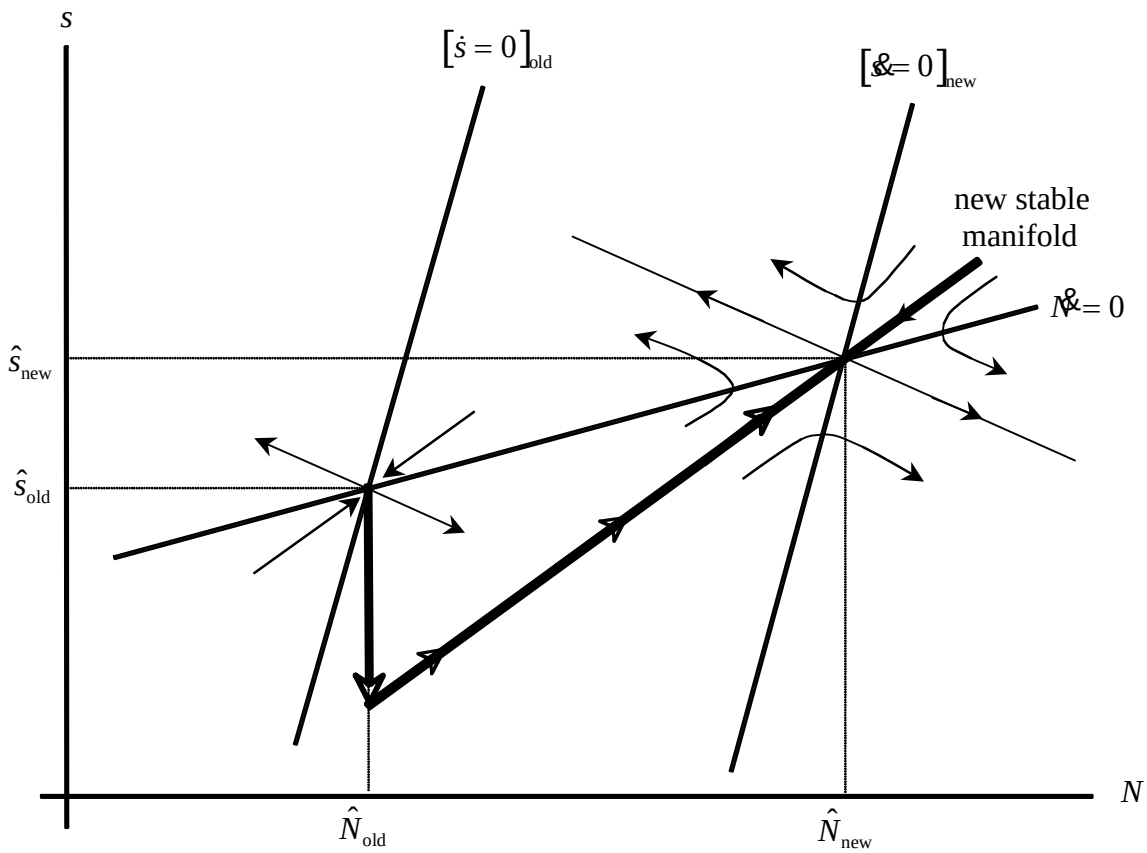
Figure 3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N
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$\hat{N}_{new}$

Figure 4

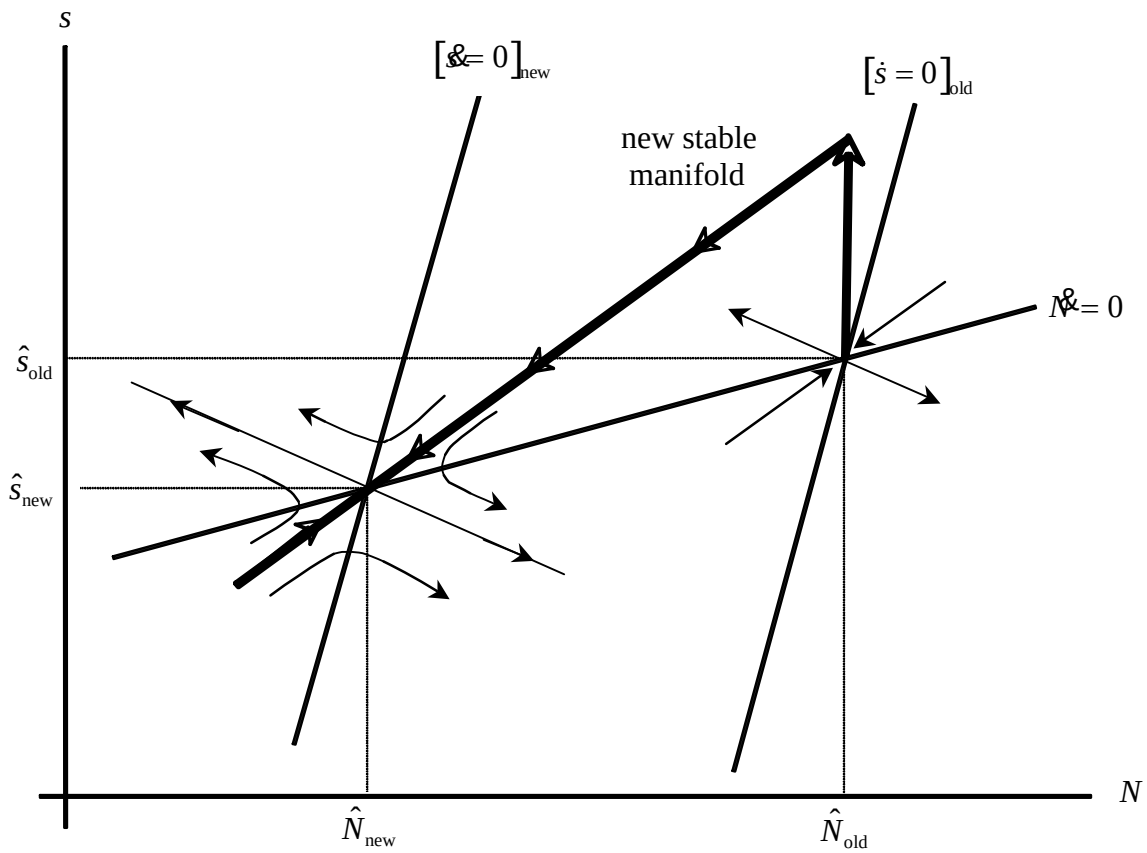

Figure 5