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Extreme contagion in emerging stock markets using extreme value copulas

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Extreme Contagion in Emerging Stock Markets Using Extreme Value Copulas

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Abstract

This paper uses multivariate extreme value theory to model the tail dependence as a measure of extreme contagion in emerging stock markets. Using the extreme value copulas, we determine the joint probabilities of simultaneous crashes between two emerging markets. The study is performed on a number of East-Asian emerging markets that have lived the well-known Asian flu. The results revealed the existence of a high contagion of crisis in the Asian emerging markets with a continuous high risk dependence structure in the aftermath of the Asian flu.

Keywords: Extreme value theory; Extreme value Copulas; Contagion.
JEL: C32, C51, C52.

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1 Introduction

During the 1990's and moving onwards, there have been many turmoil in many emerging financial markets that spread shocks throughout their geographic proximities. The most striking crisis in that period was undoubtedly the East-Asian financial tsunami. In July 1997, Thailand lived a currency crisis with a devaluation of the Thai Baht that created a transmission of shocks throughout South East Asia, South Korea, and Japan. This was known as the Asian flu and had caused the stock prices to plunge into a dramatic tandem. For instance, the Hong Kong stock index (HSI) has fallen by a 67.51% from July 1997 till June 1998, and the Thailand stock index (SET) by 66.75% during the same period. Such observation leads us to see how reacting both of these two stock markets to the crisis. On the other hand, there was a sudden withdrawal of funds and a tremendous capital outflow from the East-Asian countries. Most of the investment banks have liquidated their positions in both markets and stepped back to revise their capital allocations and reassess their capital requirements. For risk management purposes, turmoils in emerging markets are of paramount interest. Particularly, for bank's capital allocation a study on contagion of crisis provides an opportunity for analyzing the dependence structure of different portfolio's assets to generate scenarios through which the severity of a crisis could be evaluated.

In general, and since the Basle Accord of January 1996, banks are having the possibility of assessing capital requirements through the use of an internal model. It is generally the VaR methodology that supports the bank's internal models. However, the VaR methodology alone does not mean that the bank has prevented an unacceptable loss. Therefore, it has to be accompanied by simulating crisis scenarios in order for the internal model to be validated by bank regulators. For longer term control purposes, the crisis scenarios will enable the bank of managing the risk and to take into account important losses related to low-probability events. One potential tool to build these crisis scenarios is the extreme value theory. This latter, and to satisfy the regulations proposed by the Basle Accord, suggests the creation of crisis scenarios by characterizing the law of extremes of financial series that are known as the identifications of risk. In financial applications, this theory has proven to be an excellent tool for risk management (see for example Embrechts et al. 1997).

It is important to note that there is no current standard framework to build crisis scenarios. In this respect, the dependence of extremes serve to provide a way of identifying the effects of extreme changes on a bank's portfolio and hence finding the possible losses if certain scenarios happen. In a unidimensional framework, modeling the dependence structure using extreme value theory allows us to study the impacts of single risk factors on a portfolio for a fixed return time. However, the extension to multivariate case is not obvious or at least not yet fully explored. This is due to the statistical tractability of multivariate joint distributions of many assets in an existing portfolio. Nevertheless, the introduction of copulas in finance has shed light on the way to model statistical dependence in a multivariate framework. Consequently, modeling multivariate dependence with copulas could be done using the failure area concept, known also as the implied return period. This concept measures the dependence in the extremes, such as tail dependence, by using the entire conditional joint density. Dependence during extreme events has been the subject of recent studies, such as the pricing of financial options with multiple underlying assets, as in Rosenberg (2003), or in the calculation of portfolio Value-at-Risk, as in Hull and White (1998).

However, in the financial contagion literature, dependence during extreme events has not been the subject of empirical studies in testing the hypothesis of contagion effects. Most of the hypotheses are either based on checking the trade links figures or the financial links figures between different economies to find the channels of contagion in financial markets. However, the testing of these hypotheses is mostly based on causality or cointegration tests (see Kleimer and Sanders 2002 for example) and rarely on modeling the dependence structure. In fact, the nature of information and interpretations extracted from these statistical tests is different and leads to different implications. On one hand, the cointegration and causality tests are informative on the direction of contagion of a crisis or shocks from one market to another and not on the size of shocks or the severity of crisis. On the other hand, the modeling of the dependence structure could be more informative in the sense that it provides insights on the magnitude of shocks and their likelihood. Particularly, the study of tail dependence during extremes and the use of extreme value copula could be of major help in building these insights.

This paper studies the dependence of extremes in East-Asian emerging markets that have lived the well-known Asian flu. The methodology followed in this paper starts with modeling the marginal distribution of extremes using extreme value theory and continues with modeling the joint distribution of extremes using extreme value copulas. The application of this methodology is performed for bivariate case but could be extended to more general multivariate case. The main results indicate that there exists a high contagion of crisis in the Asian emerging markets with a continuous high risk dependence structure in the aftermath of the Asian flu.

This paper is structured as follows. In Section 2, the methodology of modeling the dependence during extremes is presented. In that section, a review of extreme value theory and some concepts on copulas are covered. Section 3 shows the descriptive statistics of the data and the main results. We conclude afterwards.

2 Methodology

To study the tail dependence at the extremes for different bivariate cases, we need to specify two different models. One model is for the marginal distribution of extremes and another for the joint distribution of extremes i.e. the extreme value copula. The combination of the two models will serve also to determine a measure for extreme contagion.

2.1 Marginal Distribution Model

Extreme value theory has become an essential and robust framework to evaluate extreme risks in financial markets. Generally speaking, in developing a model for risk, this approach consists of selecting a particular probability distribution for the data and estimating it through the analysis of empirical data. In this area, the extreme value theory acts in the favor of providing the best tool for estimating the tail of the distribution.

Let us consider N independent random variable $X_1, \ldots, X_n, \ldots, X_N$ having the same probability function F. In extreme value theory, the distribution of extremes[1] $\chi_N^+ = \max(X_1, \ldots, X_n, \ldots, X_N)$ is given by the Fisher-Tippet theorem (see Embrechts *et al.*, 1997). This theorem states that, If there exist some constants a_N and b_N and a non-degenerate limit distribution G such that

$$\lim_{N\to\infty} \Pr[\frac{\chi_N^+ - b_N}{a_N} \leq x] = G(x), \tag{1}$$

the distribution G can be one of the following distribution[2]:

Gumbel: $\Lambda(x) = \exp(-\exp^{-x})$, $x \in \Re$

Fréchet: $\Omega_\omega(x)= \begin{cases} 0, & x \leq 0 \\ \exp(-x^{-\omega}), & x > 0, \omega > 0 \end{cases}$

Weibull: $\Psi_\omega(x)= \begin{cases} \exp[-(-x^{-\omega})], & x \leq 0, \omega < 0 \\ 1, & x > 0. \end{cases}$

These families of distributions have only one parameter to estimate, ω, which is called the tail index. The Student-t model and the unconditional distribution of ARCH-process both fall in the domain of this type of distributions. For instance, and as represented by Gençay and Selçuk (2004), if we set $\omega = 1$ we get the density of the Weibull distribution, which is a thin-tailed distribution relative to the normal

[1]We just write the equations for the maxima as the problem identified for the minima.

[2]For instance, if we consider the exponential distribution $F(x) = 1 - e^{-x}$ and we take $a_N = 1$ and $b_N = \ln N$ then $\frac{\chi_N^+ - b_N}{a_N}$ converges asymptotically to the Gumbel distribution.

distribution.

A more general representation of these distributions is obtained by reparameterizing the tail index parameter ω to $\xi = 1/\omega$. Therefore, a unified representation with a single parameter is well known as the generalized extreme value distribution (GEV)

$$G_\xi(x) = \begin{cases} \exp[-(1+\xi\frac{x-\mu}{\sigma})^{-1/\xi}], & \text{if } \xi \neq 0, \\ \exp[\text{-}\exp(-\frac{x-\mu}{\sigma})], & \text{if } \xi = 0, \end{cases} \tag{2}$$

where $\xi = 1/\omega$ is known as the shape parameter. The case where $\xi = 0$ has to be interpreted as $\xi \to 0$ (ξ tends to zero), resulting in the Gumbel distribution. The case $\xi < 0$ corresponds to the Weibull distribution, and $\xi > 0$ to the Fréchet distribution. For application in insurance and finance, the Gumbel and the Fréchet family turn out to be the most important models for extremal events. In fact, the domain of attraction of the Weibull distribution are the thin-tailed distributions such as uniform and beta distribution which do not have much power in explaining financial time series. Whereas for the Gumbel distribution, the domain of attraction include the normal, exponential, gamma and lognormal distributions where only the lognormal distribution has a moderately heavy-tail.

The density of the GEV distribution is

$$g(x) = \frac{1}{\sigma}[1+\xi\frac{x-\mu}{\sigma}]^{-\frac{1+\xi}{\xi}}\exp(-[1+\xi\frac{x-\mu}{\sigma}]^{-\frac{1}{\xi}}), \tag{3}$$

and the corresponding log-likelihood function is

$$L = -N\ln(\beta) - (\frac{1}{\xi}+1)\sum_{t=1}^{N}\ln(1+\frac{\xi}{\beta}x_t), \tag{4}$$

where N is the sample size. Equation 4 could be estimated using maximum likelihood estimation to get the parameters of the conditional distribution of equity returns.

2.2 Joint Distribution Model

The dependence structure between random variables is well-described by their joint distribution. In the banking sector, copulas happen to be a very useful tool especially in modeling different kinds of existing risks, such as market risk and credit risk (see for example Costinot et al. 2000). For a bank's portfolio, evaluating risks is a tedious exercise and raises difficulties as to model the portfolio's joint distribution. One major drawback is to rely on the assumption that a portfolio of multi-assets has a multivariate normal distribution. Empirically, Longin and Solnik (2001) accepted this assumption only for positive tails but rejected it for negative tails, which are essentials to model while quantifying extreme losses. However, we can split the marginal distributions of each random variable from the dependence structure. A powerful tool to achieve such operation is the use of copulas.

2.2.1 Copulas definition

Copulas are functions that link univariate marginals to their multivariate distribution. According to Nelsen (1999), a N-dimensional copula is a function C with the following properties:

1. Dom C=I^N=$[0, 1]^N$
2. C is grounded and N-increasing
3. C has margins C_n satisfying $C_n(u) = C(1, \dots, 1, u, 1, \dots, 1)$ for all u in I.

It follows from these properties that the copula C is a multivariate uniform distribution. For univariate distribution functions, $F_1, \dots, F_N$, there exist a copula function

$C(F_1(x_1), \ldots, F_n(x_n), \ldots, F_N(x_N))$ as a multivariate distribution function with margins $F_1, \ldots, F_N$ since $u_n = F_n(x_n)$ is a uniform random variable. Moreover, Sklar (1959) showed that for an N-dimensional distribution F with continuous margins $F_1, \ldots, F_N$, F has a unique copula representation such as

$$F(x_1, \ldots, x_n, \ldots, x_N) = C(F_1(x_1), \ldots, F_n(x_n), \ldots, F_N(x_N)). \tag{5}$$

This representation allows us to analyze the dependence structure of multivariate distributions without studying marginal distributions. In addition, the density c associated to the copula is given by

$$c(u_1, \ldots, u_n, \ldots, u_N) = \frac{\partial C(u_1, \ldots, u_n, \ldots, u_N)}{\partial u_1 \ldots \partial u_n \ldots \partial u_N}, \tag{6}$$

and the density f of the N-dimensional distribution F is given by

$$f(x_1, \ldots, x_n, \ldots, x_N) = c(F_1(x_1), \ldots, F_n(x_n), \ldots, F_N(x_N)) \prod_{n=1}^{N} f_n(x_n), \tag{7}$$

and where f_n is the density of the margin F_n.

As pointed out by Frees and Valdez (1998), it is not easy to identify the copula. In fact, the problem that financial applications dealt with was not the use of a given multivariate distribution but the finding of a convenient distribution to describe the relationship between different asset returns. In the finance literature, the basic assumption is that the distribution is multivariate Gaussian or log-normal distribution and it is merely proved to constitute a major drawback. In that matter, copulas provide a useful tool for finance in the sense that, in portfolio analysis, it deals first with the identification of the marginal distributions and then defining the appropriate copula to represent the dependence structure of the constituents of a portfolio.

For instance, let us suppose that we have a portfolio of two assets, X_1 and X_2, and two financial losses, x_1 and x_2. The probability of simultaneous large losses being greater for dependent variables than for independent ones, i.e. $\Pr[X_1 > x_1, X_2 > x_2] \geq \Pr[X_1 > x_1]\Pr[X_2 > x_2]$, is in terms of copulas equivalent to $C(u_1, u_2; \rho) \succ C^{\perp}$, where $C^{\perp} = u_1 u_2$ is the product copula. For example, in the bivariate gaussian copula $C(u_1, u_2; \rho) = \Phi_\rho(\Phi^{-1}(u_1), \Phi^{-1}(u_2))$, where ρ is the correlation coefficient between u_1 and u_2 and Φ_ρ is the standardized multivariate normal distribution with correlation matrix ρ.

It is worth to note that the correlation coefficient ρ is based on the covariance of the variates and it is not preserved by copulas, i.e. two pairs of correlated variates with the same copula can have different correlations. It follows that there must be a constant coefficient of the copula and it is the Kendall τ. Traditionally, dependence is always reported as a correlation coefficient. The two classical measures of association are Spearman's ρ and Kendall's τ, i.e. $\rho = 12 \int_0^1 \int_0^1 C(u,v)dudv - 3$, and $\tau = 4 \int_0^1 \int_0^1 C(u,v)dudv - 1$. However, Sperman's ρ is affected by changes of scale because it depends not only on copula but also on marginal distribution whereas Kendall's τ is a very direct and easily understood nonparametric measure of the agreement between two rankings (see Nelsen, 1999). As a conclusion, the copula is a more informative measure of dependence between two (or more) variables than the classical linear correlation.

2.2.2 Multivariate extreme value copulas

As we pointed out earlier, the choice of a copula function is not a straightforward mechanism. We generally find difficulties in choosing the right copula (see Durrleman *et al.*, 2000). In our study, we opted to work with the Gumbel copula for its appealing characteristics. First, the Gumbel copula is an extreme copula that deals with the tails of the multivariate distribution. Second, Gumbel copula is asymmetric and has more probability concentrated in the tails than does other copulas like normal copula or Student-t copula. Moreover, recent studies showed the superiority of Gumbel copula in measuring the probabilities in the tails to determine large losses (see Frees and Valdez, 1998). In studying extreme correlation of international equity markets, Longin and Solnik (2001) claimed that correlation increases in bear market but not in bull market. In follows that to analyze the dependence structure of equity markets we need a copula with less correlation in the left tail and high correlation in the right tail. Such task could be performed by assuming a Gumbel copula.

The Gumbel copula belongs to the class of Archimedean copulas and is represented as follows (see Joe, 1997):

$$C(u_1, \ldots, u_n, \ldots, u_N) = \phi^{-1}(\phi(u_1) + \ldots + \phi(u_n) + \ldots + \phi(u_N)), \tag{8}$$

with $\phi(u) = (-\ln u)^{\alpha}$. For instance, in the bivariate case, the Gumbel copula is represented as follows:

$$C(u_1, u_2) = \exp(-[(-\ln u_1)^{\alpha} + (-\ln u_2)^{\alpha}]^{\frac{1}{\alpha}}), \tag{9}$$

with α the dependence parameter ($\alpha = 1$ for independence and $\alpha = \infty$ for fully independence of extrema)[3]. The Gumbel copula is considered as an extreme value copula because it satisfies the following condition:

$$C_*(u_1^t, \ldots, u_n^t, \ldots, u_N^t) = C_*^t(u_1, \ldots, u_n, \ldots, u_N) \qquad \forall t > 0. \tag{10}$$

Applying Equation 9 to this condition, we clearly see that

$$\begin{aligned} C(u_1^t, u_2^t) &= \exp(-[(-\ln u_1^t)^\alpha + (-\ln u_2^t)^\alpha]^{\frac{1}{\alpha}}) \qquad (11) \\ &= \exp(-(t^\alpha[(-\ln u_1)^\alpha + (-\ln u_2)^\alpha])^{\frac{1}{\alpha}}) \\ &= [\exp(-[(-\ln u_1)^\alpha + (-\ln u_2)^\alpha]^{\frac{1}{\alpha}})]^t \\ &= C^t(u_1, u_2). \end{aligned}$$

The estimation of the dependence parameter α is performed by using maximum likelihood estimation. The corresponding log-likelihood is

$$\begin{aligned} l(u_1, u_2; \alpha) &= -(u_1^\alpha + u_2^\alpha)^{\frac{1}{\alpha}} + u_1 + u_2 + (\alpha - 1)\ln(u_1 u_2) \qquad (12) \\ &+ (\alpha^{-1} - 2)\ln(u_1^\alpha + u_2^\alpha) + \ln[(u_1^\alpha + u_2^\alpha)^{\frac{1}{\alpha}} + \alpha - 1]. \end{aligned}$$

2.3 Extreme contagion and tail dependence

There is a link between extreme value copulas and the multivariate extreme value theory. Joe (1997) shows that the class of multivariate extreme value distribution is the class of extreme copulas with nondegenerate marginals. If we suppose that C is an extreme copula and that the marginals $F_1, \ldots, F_N$ are univariate extreme distributions, the N-variate distribution F is a multivariate extreme value distribution, where

[3]There is a relationship between the Kendall's τ and the coefficient α of the Gumbel model that is $\tau = 1 - \alpha^{-1}$ (see Durrleman, 2000).

$F(x_1, \ldots, x_n, \ldots, x_N) = C(F_1(x_1), \ldots, F_n(x_n), \ldots, F_N(x_N))$. We further assume that the limit distribution exists and so F belongs to the maximum domain of attraction of a distribution G (see Equation 1). Additionally, we denote $G_1, \ldots, G_N$ the margins of G that follow a GEV and C_* its corresponding extreme copula. For the maxima, we have

$$G(\chi_1^+, \ldots, \chi_n^+, \ldots, \chi_N^+) = C_*(G_1(\chi_1^+), \ldots, G_n(\chi_n^+), \ldots, G_N(\chi_N^+)), \tag{13}$$

and where G is defined as in Equation 3.

It is then possible to compute the probability of a simultaneous loss in a portfolio of two assets. For a set of values (χ_1^+, χ_2^+), the probability is given by (Bouyé *et al.*, 2000)

$$\begin{aligned} \Pr[\chi_1^+ > \chi_1, \chi_2^+ > \chi_2] &= 1 - \Pr[\chi_1^+ \leq \chi_1] - \Pr[\chi_2^+ \leq \chi_2] \\ &+ \Pr[\chi_1^+ \leq \chi_1, \chi_2^+ \leq \chi_2] \\ &= 1 - G_1(\chi_1) - G_2(\chi_2) + C_*(G_1(\chi_1), G_2(\chi_2)). \end{aligned} \tag{14}$$

This probability serves also in building multivariate stress tests with copulas and construct a failure area that corresponds to a set of extreme values. The failure area can be used to quantify the stress tests provided by the economists for the stress testing program of a bank. Moreover, it could be a good tool to measure the severity of crisis, although the strength of a crisis is generally seen as a subjective notion (Legras and Soupé, 2000). In this paper, this probability will serve as a measure of the severity of crisis in two different emerging markets.

Moreover, we can study the dependence structure during extremes by measuring the tail dependence. In fact, the interesting application of tail dependence measure is in capturing the behavior of random variables during extreme events. It measures the probability of observing an extremely large positive (negative) realization of one variable, given that we have observed that the other variable also took an extremely large positive (negative) value. Two widely used bivariate tail dependence concepts are upper and lower tail dependence between a pair of random variables. The concepts are useful when we are concerned with dependence in extreme values, such as the simultaneous crash between two stock markets.

To characterize the dependence of extremal risks for a set of extremes (χ_1^+, χ_2^+) and a given loss, l, the upper tail dependence coefficient λ (see Joe, 1997) is used:

$$\begin{aligned} \lambda &= \lim_{l \to 0} P\left\{\chi_2^+ > G_2^{-1}(l) | \chi_1^+ > G_1^{-1}(l)\right\} \\ &= \lim_{l \to 0} \frac{P\left\{G_2(\chi_2^+) > l, G_1(\chi_1^+) > l\right\}}{P\left\{G_1(\chi_1^+) > l\right\}}. \end{aligned} \tag{15}$$

We interpret the coefficient λ as the probability that one extreme observes a large loss given that the other extreme has observed the same large loss. In this paper, we adopt this coefficient as a measure of extreme contagion in the East-Asian emerging markets.

3 Empirical Application

3.1 Data

The dataset used for this study comprises daily stock index prices from Indonesia, Thailand, Taiwan, the Philippines, Malaysia, and Hong Kong. The sample period starts from January 1, 1990 to December 12, 2004. Daily data (5 days a week) in total of 3906 observations are taken from the database of Datastream. Three subperiods are used to estimate the tail dependence between two different markets. Period 1 (Pre-crisis) covered from January 1, 1990 to May 30, 1997; Period 2 (Crisis: the Asian Flu) covered from June 1, 1997 to June 16, 1998; Period 3 (Post-crisis) continued from June 17, 1998 through December 12, 2004. Descriptive statistics of daily returns are presented in Table 1. The daily returns are defined as

$$X_{i,t} = \log(p_{i,t} - p_{i,t-1}) * 100 \tag{16}$$

where $p_{i,t}$ is the daily closing value of the stock market index in country i on day t.

The highest averages of the daily returns during the pre-crisis are in Malaysia and Philippines (0.048%). During the Asian flu, the average returns in most of the markets are negative with an exception to Hong-Kong market. The standard deviations of most markets has increased during the crisis showing an increase in the volatility with exception to Taiwan and Hong Kong where the volatility has registered a decrease than before the crisis. According to the sample kurtosis estimates and the Bera and Jarque (1981) normality test statistics, the daily rate of returns are far from being normally distributed. The lowest kurtosis estimates are 6.46 (Taiwan) and 6.96 (Philippines), while the highest estimate is 22.05 (Indonesia). Based on the sample kurtosis estimates, it may be argued that the return distributions in all the markets are fat tailed. The sample skewness shows that the daily returns have an asymmetric distribution in all the markets. The returns have either positive or negative skewness throughout the three subperiods. The sample skewnesses are negative in Taiwan (-0.071 before crisis and -0.348 during crisis) indicating that the asymmetric tail extends more towards negative values than positive ones. In all other countries, positive skewness ranges from 0.011 (Hong Kong) to 1.714 (Indonesia).

3.2 Results

In a given market, a portfolio manager is not only interested in expected returns but also in extreme returns. Any financial institution is more likely to see the possible change of its balance sheet under extreme stress. For this reason, we consider a decrease ranging from 1% to 10% in the daily stock returns of each market. We use the maximum likelihood estimation procedure to estimate the parameters of the marginal and the joint distributions. Additionally, and for the implementation of the bivariate extreme value copula, the data are first transformed into uniforms with the conditional distribution of the GEV and then the Gumbel copula is used to get the bivariate distribution.

Table 2 presents the parameter estimates with corresponding standard errors of the GEV marginal distributions during the three subperiods and for all markets. From the results we notice that the shape parameter ξ is nonsignificant during the crisis period suggesting that the marginal distribution of the minimas falls into the Gumbel family. However, during the pre-crisis and post-crisis periods, the shape parameter is significantly positive (low values) at the 1% level with the exception to Indonesia during the pre-crisis and Taiwan during post-crisis. This is an indication of the high risk associated with these markets. The mean μ and the standard deviation σ are all significant at the 1% level.

Table 3 presents the estimation of the parameter α of the Gumbel copula model for all markets and during the three subperiods. Pairwisely, we observe that there is greater dependence between the markets during the crisis period than during the pre-crisis and post-crisis period. This reflects the effect of the Asian flu on the dependence structure between the markets. Moreover, we observe that this dependence structure has decreased in the post-crisis period between Malaysia and Hong Kong, and Taiwan and Philippines than during the pre-crisis period. On the other hand, and for the remaining bivariate cases, the dependence structure between markets has gone stronger in the post-crisis than the pre-crisis. Consequently, these markets remain very much affected by any economical turmoil that could take place.

In an attempt to catch the severity of crisis in the East-Asian markets, we present the probabilities of simultaneous decrease in the daily returns for the various markets as calculated in Equation 14. The severity of crisis is taken as a subjective notion and is determined for losses ranging between 1% and 5% in the daily returns. Figures 1, 2, and 3 represent the joint probabilities of a simultaneous decrease (loss) in the extremes for different bivariate cases during the three subperiods. The results converge almost to the same conclusion. There is an increase in the joint probabilities during the crisis period comparing to the other period levels. This shows that the crisis has been affecting the various markets to each other pairwisely. However, we notice that, in the bivariate cases of Indonesia-Thailand (Fig. 1.), Taiwan-Thailand and Taiwan-Philippines (Fig. 2.), the Asian crisis was not that severe comparing to what these countries have lived before and after the crisis. This is possibly due to the existence of a strong economic linkage between these countries such as being trade partners and hence living in a continuous turmoil. Moreover, It seems that there was a poor banking control and regulations. From a regulatory point of view, it is important that banks should keep enough capital to protect themselves against extreme market conditions.

The results continue with studying the upper tail dependence of extremes in the various markets pairwisely. Figures 4, 5, 6, 7, 8, and 9 display the upper tail dependence of the minimas of respectively Indonesia, Thailand, Taiwan, Philippines, Malaysia, and Hong Kong on other markets. This upper tail dependence is calculated as in Equation 15 as the conditional probability of a loss in one market such that we had a loss in another market. We have kept the same range of losses as for the severity of crisis.

If markets are historically cross-correlated then a sharp change in one market will have an expected change in magnitude in the other markets. If there is no sharp increase then the markets are simply reacting to each other. An inspection of the tail dependence during extremes in most of the figures, reveals a significant increase in the tail dependence in the various markets during the crisis period in comparison to historical period. More precisely, Indonesia seems to have the most impact on the various markets as it is displayed in Fig. 4. This shows that an increase in the dependence structure, through a decrease in the daily returns, has been tremendous during the crisis period. As pointed out by Van der Werff (1998), Indonesia has all the elements to affect the Asian contagion. There is a significant transmission of pressure, for losses ranging between 1% and 2% in the daily returns, in the various other markets (see Fig. 4.). In addition, Fig. 5. shows that the contagion of Thailand on Malaysia remains the same after the crisis where, for losses ranging between 1% and 2%, the tail dependence remains stronger (an order of 3%) after the crisis. On another hand, the Taiwan-Philippines dependence structure has been almost the same before and during the crisis, which shows that these markets are only reacting to each other (see Fig. 6.). The probability of contagion for the chosen range of losses in the extremes is relatively higher that the ones registered for Thailand, Philippines, Malaysia, or Honk Kong stock markets as they are shown in figures 5, 7, 8, and 9 respectively. There are many possible explanations related to these findings. From a trading point of view, there may be an effect of investors' behavior on stock markets, the result of which may cause a stock market crisis. Additionally, the adverse shock to a single country gets transmitted to a wider set of countries due to the illiquidity characteristic of emerging markets,

4 Conclusion

In this paper, we use bivariate extreme value theory to model the tail dependence as a measure of extreme contagion in emerging stock markets. Using the dependence in extreme values, we determine the joint probabilities of simultaneous crashes between two emerging East-Asian stock markets. Our study is performed on a number of East-Asian emerging stock markets that have lived the well-known Asian flu between 1997 and 1998. The results revealed the existence of a high contagion of crisis in the Asian emerging markets with a notable bivariate dependence structure that could persist in the coming future. We conclude that extreme value copulas may be another appealing tool for portfolio managers and risk managers in general. There may be many possible directions for future research but mainly a multivariate approach should be adopted to have a complete figure of the risk and contagion in the emerging markets.

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Table 1: Descriptive statistics of the daily returns, $\log(p_{i,t} - p_{i,t-1}) * 100$, from six emerging stock markets.

	Indonesia			Thailand		
	Pre-Crisis	Crisis	Post-Crisis	Pre-Crisis	Crisis	Post-Crisis
Mean	0.00028	-0.0019	0.00052	-0.00022	-0.0028	0.00056
Std dev	0.009	0.028	0.016	0.016	0.026	0.017
Skewness	1.504	0.160	0.128	-0.272	0.937	0.399
Kurtosis	22.05	6.90	8.53	8.42	6.12	6.32
Jarque-Bera	29978*	174*	2170*	2397*	150*	825*
	Taiwan			Philippines		
	Pre-Crisis	Crisis	Post-Crisis	Pre-Crisis	Crisis	Post-Crisis
Mean	-0.00009	-0.0028	-0.00012	0.00048	-0.0017	0.00002
Std dev	0.021	0.016	0.017	0.016	0.022	0.014
Skewness	-0.071	-0.348	0.021	0.051	0.241	1.300
Kurtosis	6.45	4.96	5.08	6.96	7.04	19.36
Jarque-Bera	963*	49*	308*	1264*	188*	19447*
	Malaysia			Hong Kong		
	Pre-Crisis	Crisis	Post-Crisis	Pre-Crisis	Crisis	Post-Crisis
Mean	0.00048	-0.0031	0.00041	0.00005	0.0014	-0.00032
Std dev	0.011	0.028	0.015	0.018	0.010	0.011
Skewness	-0.022	1.714	-0.198	0.144	0.011	-0.085
Kurtosis	11.49	15.876	47.80	12.96	4.30	4.17
Jarque-Bera	5810*	2019*	142120*	7993*	20*	19*

The sample period starts from January 1, 1990 to December 12, 2004. Daily data (5 days a week) in total of 3906 observations are taken from the database of Datastream. Three subperiods are used for empirical application: Period 1 (Pre-crisis) covered from January 1, 1990 to May 30, 1997 (1934 observations); Period 2 (Crisis: the Asian Flu) covered from June 1, 1997 to June 16, 1998 (273 observations); Period 3 (Post-crisis) continued from June 17, 1998 through December 12, 2004 (1699 observations).

Table 2: Maximum Likelihood Estimates (MLE) of the parameters of the GEV distribution[a]

Markets		Pre-Crisis	Crisis	Post-Crisis
Indonesia	μ	0.0041	0.0185	0.0095
		(0.0003)*	(0.0025)*	(0.0005)*
	σ	0.0061	0.0183	0.0095
		(0.0002)*	(0.0020)*	(0.0004)*
	ξ	0.0378	0.1888	0.1250
		(0.0266)	(0.1367)	(0.0408)*
Thailand	μ	0.0092	0.0292	0.0113
		(0.0005)*	(0.0011)*	(0.0005)*
	σ	0.0092	0.0173	0.0097
		(0.0004)*	(0.0008)*	(0.0004)*
	ξ	0.1543	0.0488	0.0432
		(0.0363)*	(0.1187)	(0.0414)
Taiwan	μ	0.0119	0.0100	0.0122
		(0.0007)*	(0.0014)*	(0.0006)*
	σ	0.0135	0.0090	0.0103
		(0.0005)*	(0.0011)*	(0.0004)*
	ξ	0.0809	0.2200	0.0457
		(0.0287)*	(0.1401)	(0.0407)
Philippines	μ	0.0085	0.0144	0.0090
		(0.0005)*	(0.0018)*	(0.0004)*
	σ	0.0091	0.0123	0.0079
		(0.0003)*	(0.0014)*	(0.0003)*
	ξ	0.0712	0.1338	0.1107
		(0.0335)*	(0.1122)	(0.0402)*
Malaysia	μ	0.0055	0.0182	0.0060
		(0.0003)*	(0.0021)*	(0.0003)*
	σ	0.0059	0.0142	0.0065
		(0.0002)*	(0.0016)*	(0.0003)*
	ξ	0.1823	0.1075	0.2689
		(0.0355)*	(0.0984)	(0.0410)*
Hong Kong	μ	0.0070	0.0172	0.0104
		(0.0004)*	(0.0024)*	(0.0005)*
	σ	0.0078	0.0159	0.0088
		(0.0003)*	(0.0019)*	(0.0004)*
	ξ	0.1020	0.1819	0.1059
		(0.0289)*	(0.1299)	(0.0471)*

[a]The parameters of the marginal distribution are estimated using Equation 4. The numbers between parenthesis are the standard errors and * corresponds to a significance at the 1% level.

Table 3: Maximum Likelihood Estimates (MLE) of the parameter α of the the Gumbel Copula[a]

Bivariate Markets	Pre-Crisis	Crisis	Post-Crisis
Indonesia,Thailand	1.2534	1.3563	1.2600
	(0.0098*)	(0.0618***)	(0.0103*)
Indonesia,Taiwan	1.0489	1.1863	1.383
	(0.0085*)	(0.0584***)	(0.0095*)
Indonesia,Philippines	1.1842	1.5201	1.3038
	(0.0090*)	(0.0773***)	(0.0109*)
Indonesia,Malaysia	1.2351	1.4171	1.2905
	(0.0093*)	(0.0690***)	(0.0110*)
Indonesia,Hong Kong	1.1332	1.4046	1.2984
	(0.0088*)	(0.0669***)	(0.0108*)
Thailand,Taiwan	1.1108	1.2438	1.1295
	(0.0086*)	(0.0591***)	(0.0095*)
Thailand,Philippines	1.1849	1.3245	1.2980
	(0.0090*)	(0.0534***)	(0.0108*)
Thailand,Malaysia	1.2661	1.3942	1.2818
	(0.0093*)	(0.0509**)	(0.0109*)
Thailand,Hong Kong	1.1584	1.3750	1.2865
	(0.0089*)	(0.0576***)	(0.0108*)
Taiwan,Philippines	1.1450	1.1846	1.1103
	(0.0089*)	(0.0557***)	(0.0096*)
Taiwan,Malaysia	1.0847	1.1983	1.1468
	(0.0084*)	(0.0543***)	(0.0098*)
Taiwan,Hong Kong	1.0296	1.2660	1.2054
	(0.0082*)	(0.0636***)	(0.0107*)
Philippines,Malaysia	1.1669	1.4890	1.2637
	(0.0087*)	(0.0787***)	(0.0108*)
Philippines,Hong Kong	1.0954	1.4782	1.2676
	(0.0084*)	(0.0735***)	(0.0106*)
Malaysia,Hong Kong	1.3826	1.4486	1.3233
	(0.0116*)	(0.0627***)	(0.0110*)

[a]The data are first transformed into uniforms with the conditional distribution of the GEV and then used in the Gumbel copula as in Equation 9. The dependence parameter of the Gumbel copula is estimated using Equation 12. The numbers in parenthesis are the standard errors and * corresponds to a significance at the 1% level, ** to a significance at the 5% level, and *** to a significance at the 10% level.

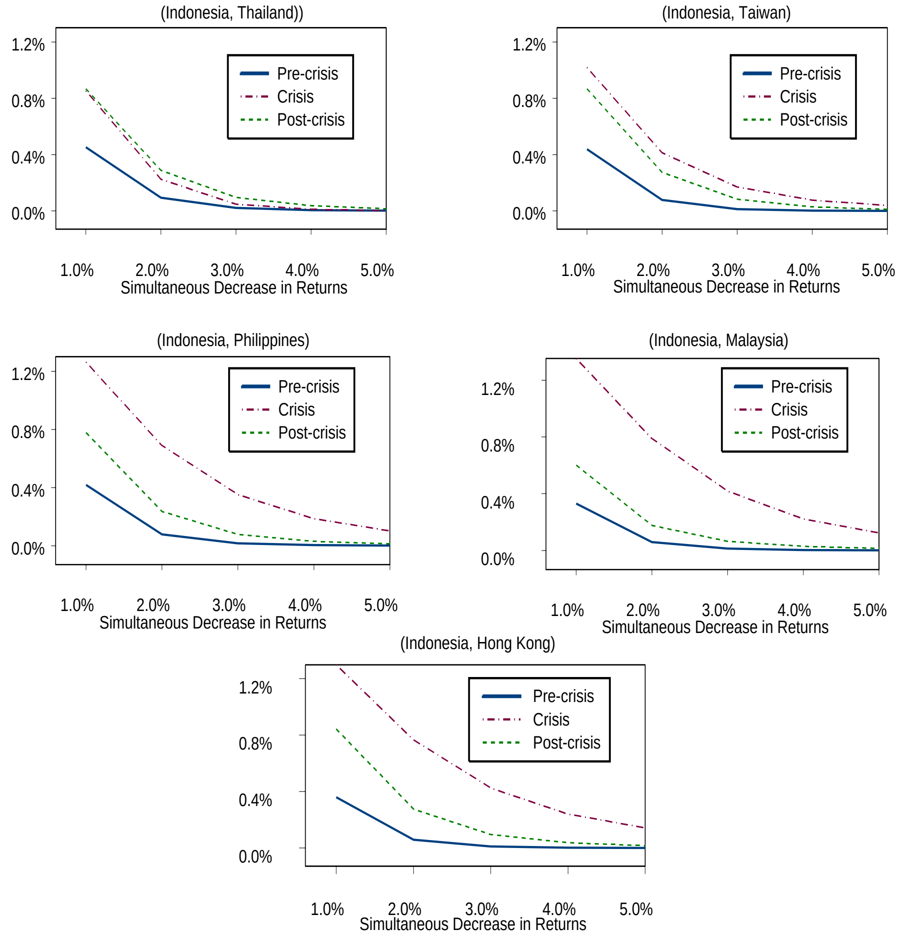

Figure 1: Bivariate probabilities of simultaneous loss in extreme returns.

The figure presents the probabilities of simultaneous decrease in the daily extreme returns for the various markets pairwisely as calculated in Equation 14. These probabilities are calculated for losses ranging between 1% and 5% during three subperiods: Period 1 (Pre-crisis) covered from January 1, 1990 to May 30, 1997; Period 2 (Crisis: the Asian Flu) covered from June 1, 1997 to June 16, 1998; Period 3 (Post-crisis) continued from June 17, 1998 through December 12, 2004..

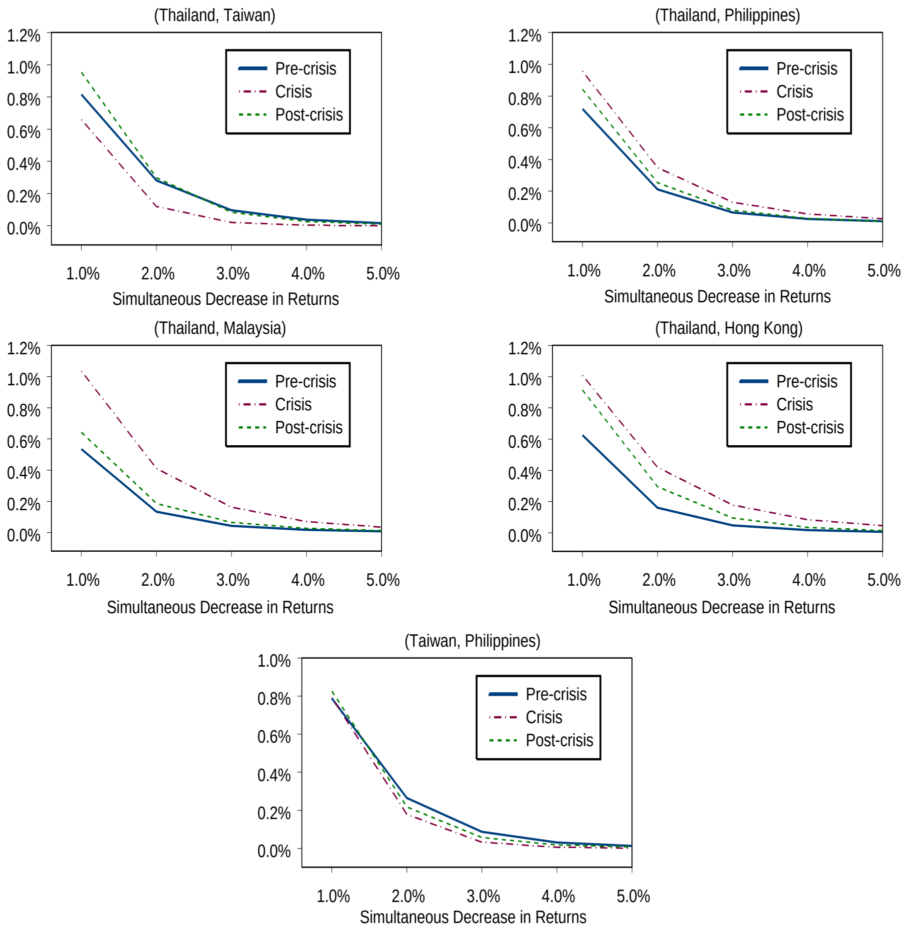

Figure 2: Bivariate probabilities of simultaneous loss in extreme returns.

The figure presents the probabilities of simultaneous decrease in the daily extreme returns for the various markets pairwisely as calculated in Equation 14. These probabilities are calculated for losses ranging between 1% and 5% during three subperiods: Period 1 (Pre-crisis) covered from January 1, 1990 to May 30, 1997; Period 2 (Crisis: the Asian Flu) covered from June 1, 1997 to June 16, 1998; Period 3 (Post-crisis) continued from June 17, 1998 through December 12, 2004..

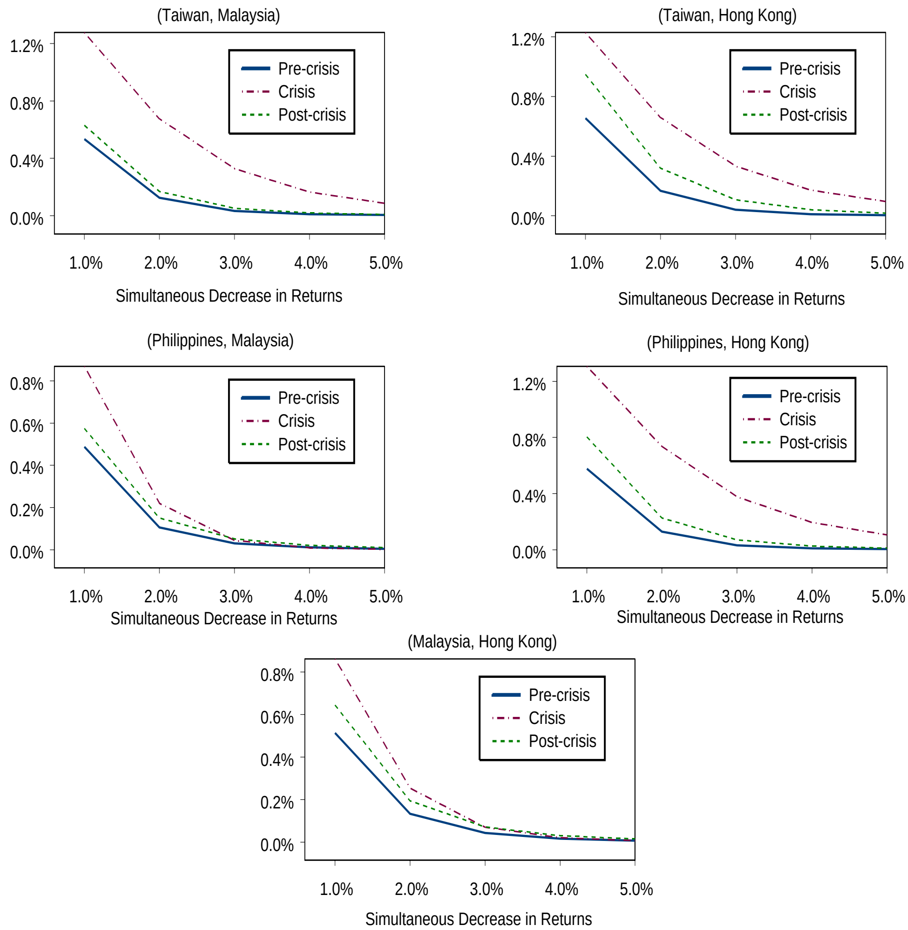

Figure 3: Bivariate probabilities of simultaneous loss in extreme returns.

The figure presents the probabilities of simultaneous decrease in the daily extreme returns for the various markets pairwisely as calculated in Equation 14. These probabilities are calculated for losses ranging between 1% and 5% during three subperiods: Period 1 (Pre-crisis) covered from January 1, 1990 to May 30, 1997; Period 2 (Crisis: the Asian Flu) covered from June 1, 1997 to June 16, 1998; Period 3 (Post-crisis) continued from June 17, 1998 through December 12, 2004.

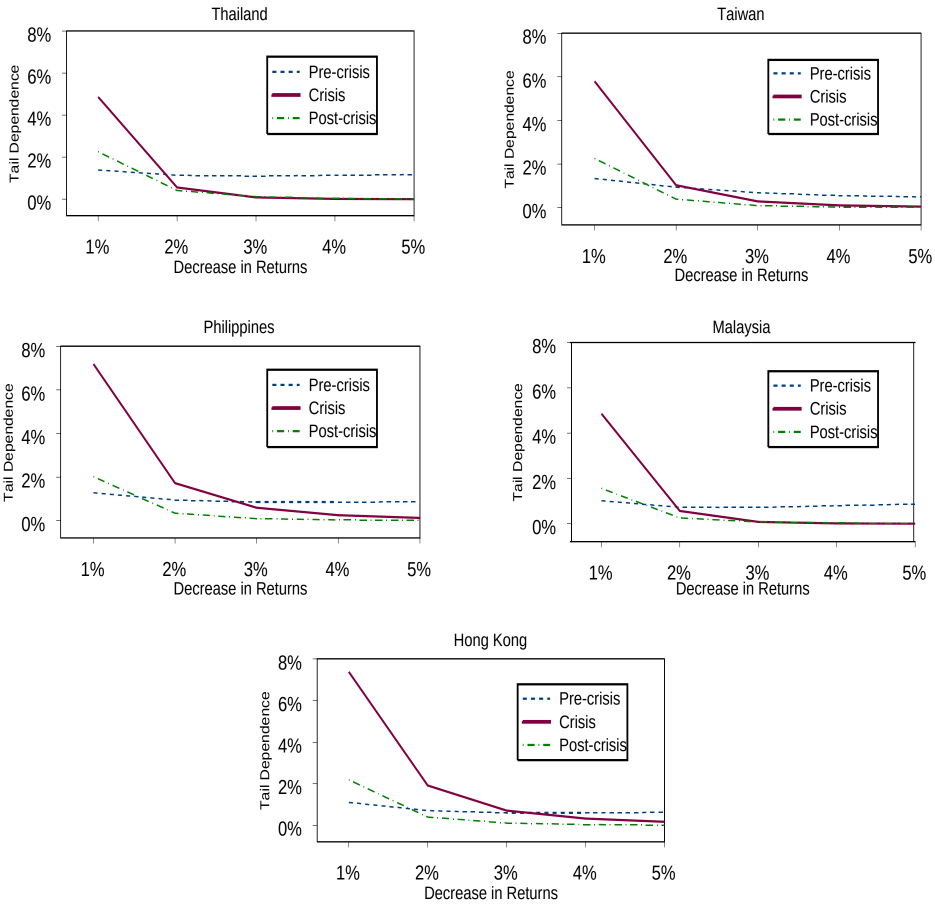

Figure 4: Upper tail dependence of Indonesia and other markets.

The figure plots the upper tail dependence of Indonesia and various stock markets during three subperiods: Period 1 (Pre-crisis) covered from January 1, 1990 to May 30, 1997; Period 2 (Crisis: the Asian Flu) covered from June 1, 1997 to June 16, 1998; Period 3 (Post-crisis) continued from June 17, 1998 through December 12, 2004. The upper tail dependence is calculated as in Equation 15, that is the conditional probability of a loss in one market such that we had a loss in another market. The losses are ranging between 1% and 5% in the daily extreme returns.

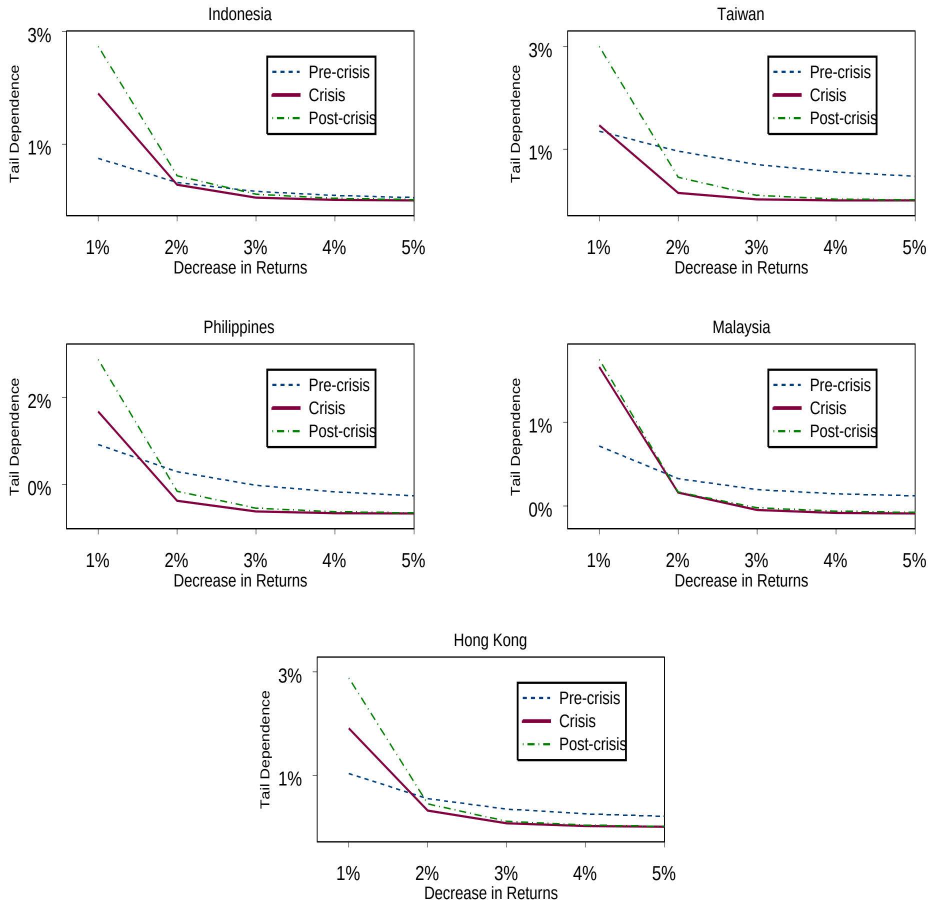

Figure 5: Upper tail dependence of Thailand and other markets.

The figure plots the upper tail dependence of Thailand and various stock markets during three subperiods: Period 1 (Pre-crisis) covered from January 1, 1990 to May 30, 1997; Period 2 (Crisis: the Asian Flu) covered from June 1, 1997 to June 16, 1998; Period 3 (Post-crisis) continued from June 17, 1998 through December 12, 2004. The upper tail dependence is calculated as in Equation 15, that is the conditional probability of a loss in one market such that we had a loss in another market. The losses are ranging between 1% and 5% in the daily extreme returns.

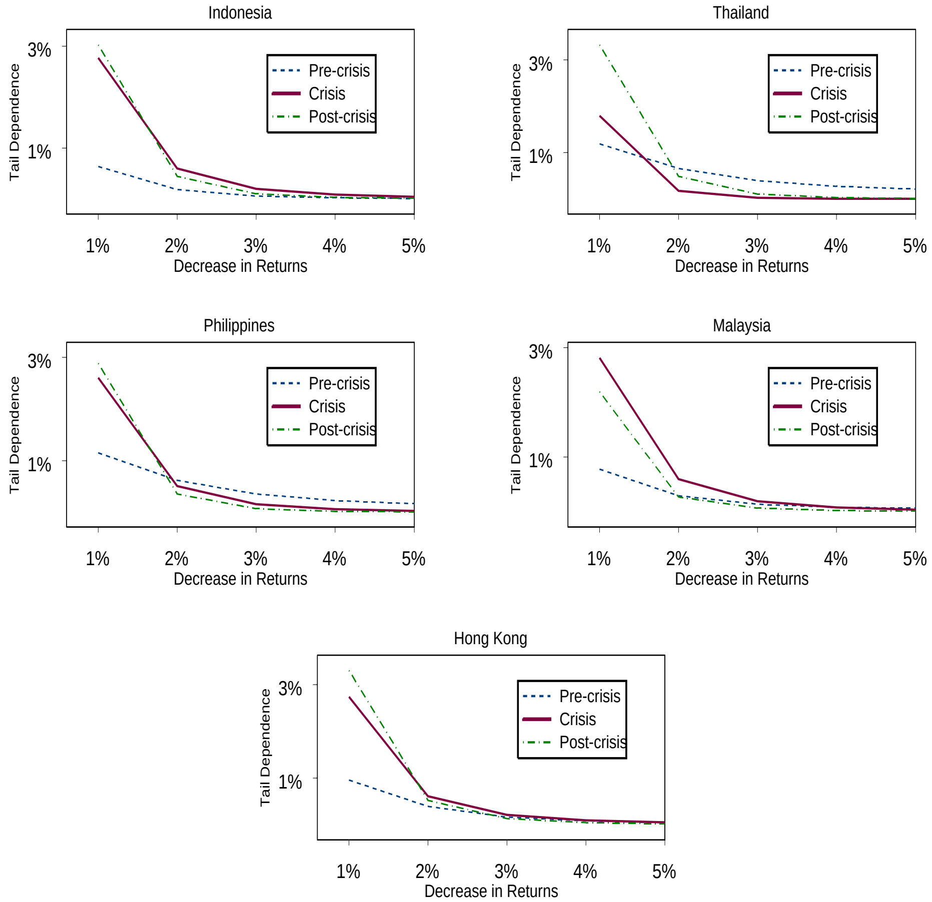

Figure 6: Upper tail dependence of Taiwan and other markets.

The figure plots the upper tail dependence of Taiwan and various stock markets during three subperiods: Period 1 (Pre-crisis) covered from January 1, 1990 to May 30, 1997; Period 2 (Crisis: the Asian Flu) covered from June 1, 1997 to June 16, 1998; Period 3 (Post-crisis) continued from June 17, 1998 through December 12, 2004. The upper tail dependence is calculated as in Equation 15, that is the conditional probability of a loss in one market such that we had a loss in another market. The losses are ranging between 1% and 5% in the daily extreme returns.

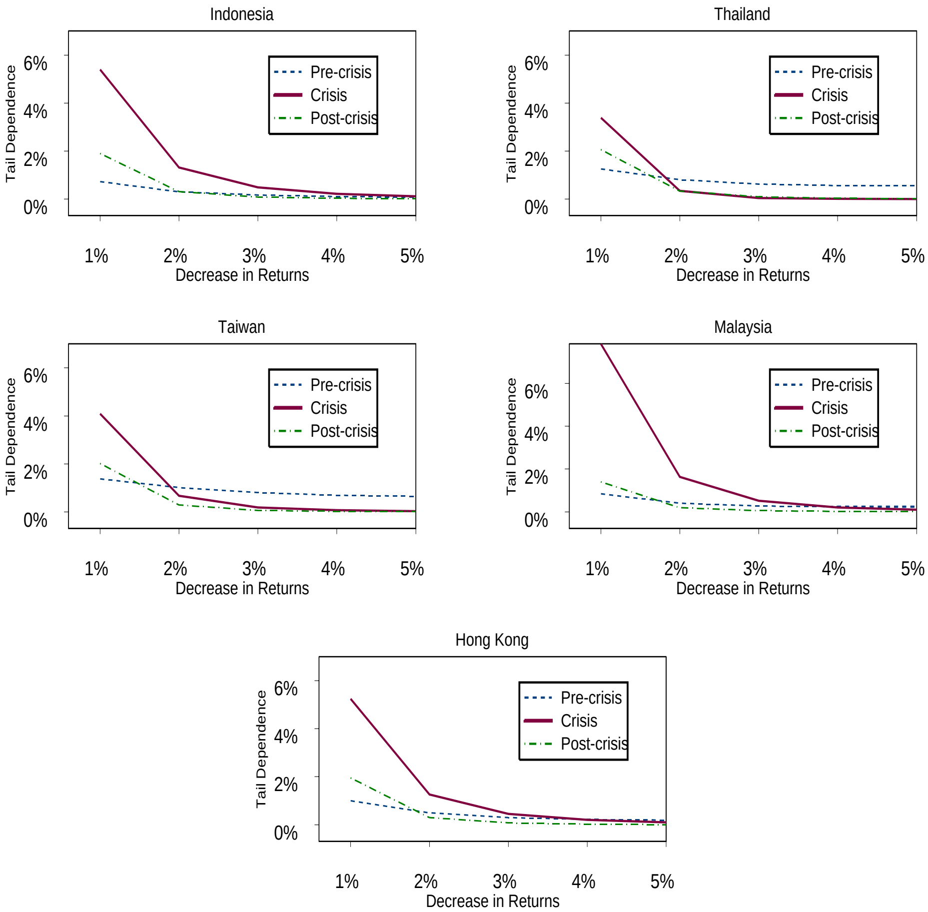

Figure 7: Upper tail dependence of Philippines and other markets.

The figure plots the upper tail dependence of Philippines and various stock markets during three subperiods: Period 1 (Pre-crisis) covered from January 1, 1990 to May 30, 1997; Period 2 (Crisis: the Asian Flu) covered from June 1, 1997 to June 16, 1998; Period 3 (Post-crisis) continued from June 17, 1998 through December 12, 2004. The upper tail dependence is calculated as in Equation 15, that is the conditional probability of a loss in one market such that we had a loss in another market. The losses are ranging between 1% and 5% in the daily extreme returns.

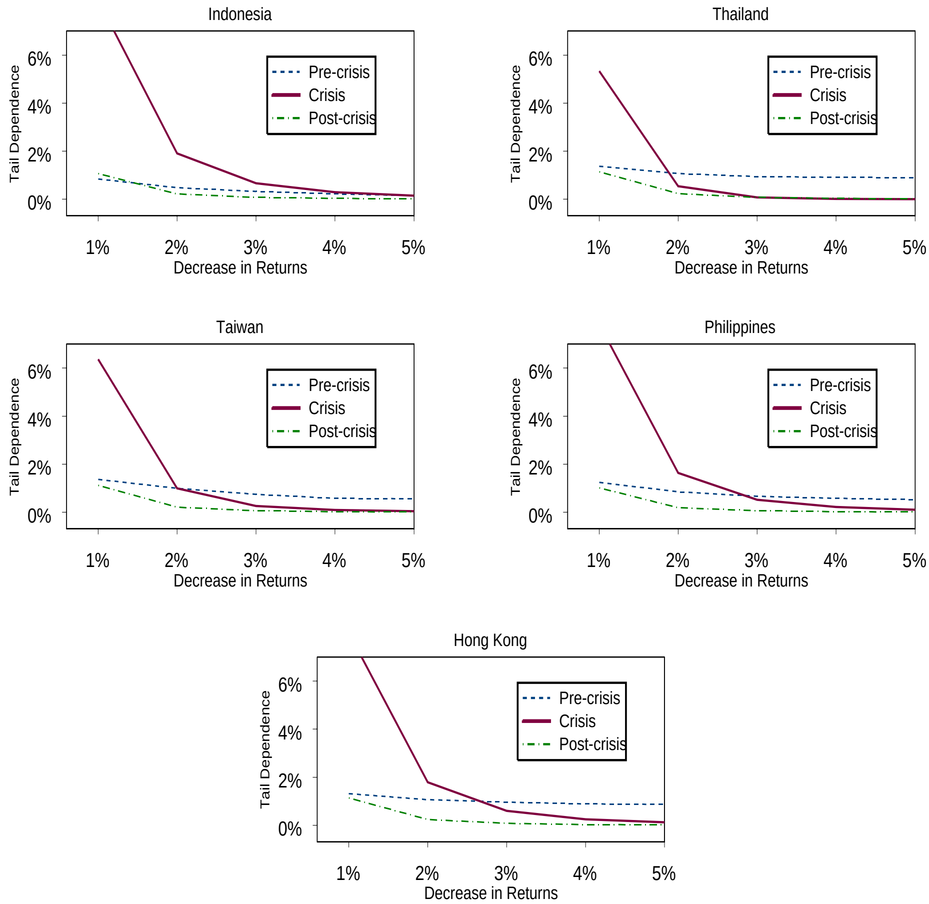

Figure 8: Upper tail dependence of Malaysia and other markets.

The figure plots the upper tail dependence of Malaysia and various stock markets during three subperiods: Period 1 (Pre-crisis) covered from January 1, 1990 to May 30, 1997; Period 2 (Crisis: the Asian Flu) covered from June 1, 1997 to June 16, 1998; Period 3 (Post-crisis) continued from June 17, 1998 through December 12, 2004. The upper tail dependence is calculated as in Equation 15, that is the conditional probability of a loss in one market such that we had a loss in another market. The losses are ranging between 1% and 5% in the daily extreme returns.

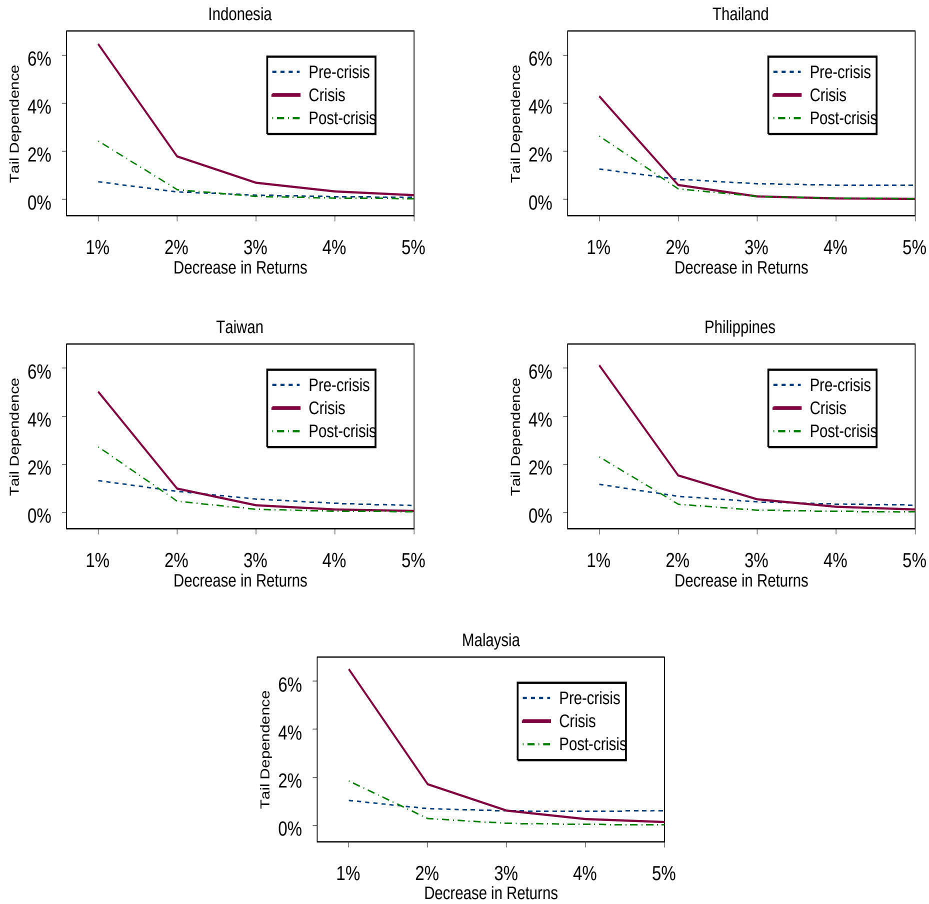

Figure 9: Upper tail dependence of Hong Kong and other markets.

The figure plots the upper tail dependence of Hong Kong and various stock markets during three subperiods: Period 1 (Pre-crisis) covered from January 1, 1990 to May 30, 1997; Period 2 (Crisis: the Asian Flu) covered from June 1, 1997 to June 16, 1998; Period 3 (Post-crisis) continued from June 17, 1998 through December 12, 2004. The upper tail dependence is calculated as in Equation 15, that is the conditional probability of a loss in one market such that we had a loss in another market. The losses are ranging between 1% and 5% in the daily extreme returns.