University of Wollongong
Research Online

Faculty of Business - Economics Working Papers

Faculty of Business

2007

A theoretical analysis of rational diet of healthy and junk foods

A. Levy
University of Wollongong

Publication Details

Levy, A, A theoretical analysis of rational diet of healthy and junk foods, Working Paper 07-01, Department of Economics, University of Wollongong, 2007.

Research Online is the open access institutional repository for the University of Wollongong. For further information contact the UOW Library: research-pubs@uow.edu.au

University of Wollongong

Economics Working Paper Series 2007

http://www.uow.edu.au/commerce/econ/wpapers.html

A theoretical analysis of rational diet of healthy and junk foods

Amnon Levy

WP 07-01

April 2007

A theoretical analysis of rational diet of healthy and junk foods*

Amnon Levy

School of Economics, University of Wollongong, Wollongong, NSW 2522, Australia

Abstract

Junk-food consumption, health and productivity are analysed within an expected-lifetime-utility-maximising framework in which longevity and productivity rise with health, and health deteriorates with the consumption of junk food. As long as the junk food's relative taste-price differential is positive, rational diets deviate from the physiologically optimal ones and generate lower than maximal levels of health and productivity. Taxing junk food can eliminate this discrepancy, but the outcome is not Pareto-superior. The value of health and the steady-state levels of rational junk-food consumption, health and productivity depend on the consumer's tastes, prices, endurance, appetite and time preferences.

JEL classification: I12, D91

Keywords: Taste; Price; Endurance; Appetite; Time-preferences

* The paper benefited from comments made by Gustav Feichtinger, Frank Neri and anonymous referees.

1. Introduction

Attempts to explain the prevalence of overweightness, obesity and their associated health problems have usually considered a time-value-induced reliance on commercially supplied meals, technological developments that made meals cheaper and work more sedentary, and an inverse relationship between eating and smoking (e.g., Philipson and Posner, 1999; Ruhm, 2000; Lakdawalla and Philipson, 2002; Cutler et al., 2003; Cawley et al., 2003; Chou et al., 2004). Some other attempts have explored the role of behavioural factors. Dynan (2000) has concluded that food consumption is neither addictive nor a formed habit. Komlos et al. (2002) have argued that a rise in the rate of time preference may help explain the prevalence and intensity of overweightness and obesity. O'Donoghue and Rabin's (1999, 2000) argument of time-inconsistent preferences and immediate gratification has added significance to the role of strong time preferences in explaining excessive eating and, in turn, overweightness and obesity. Levy (2002) has argued that overweightness is a rationally optimal steady-state, increasing in the individual's rate of time preference, elasticity of utility from food and degree of sedentary behaviour, decreasing in mortality risk, and responsive to cultural norms.

Complementing the above list of possible causes of overweightness, obesity and poor health is the sort of food eaten, which is the subject of the present paper. The significance of this factor is reflected in the large positive effect of the per capita number of restaurants, of which many specialize in fast food and snack food, on the body mass index and the probability of being obese (Chou et al., 2004). Food is classified in this study as junk food or healthy food. The distinction between these two categories depends on the concentration of ingredients such as fat, salt, sugar and food additives, whose presence in the human body beyond a critical level is harmful. Junk food contains large, while healthy food contains recommended, quantities of these ingredients. Junk food is also lacking vital nutrients such as fibres, proteins and vitamins.[1] Previous research has shown that central regulatory mechanisms favour foods containing sugar and fat over other nutrients (e.g., Levine et al., 2003). Due to a high concentration of these ingredients, junk food is tastier than its healthy substitute for some consumers. Furthermore, several studies, including Drenowski (2003), Lakdawalla and Philipson (2002) and Philipson and Posner (1999), have determined that junk food is often less expensive than healthy food due to cheaper ingredients, easier preparation process and storage, and value of time. Recalling O'Donghue and Rabin's (1999 and 2000) immediate gratification hypothesis, these short-term advantages of junk food might be paramount for many consumers with time-

[1] My search for a definition and the origin of the term junk food has led to the following result. Junk food is "a slang term describing food that is perceived to be unhealthy or having poor nutritional value. The term is believed to have been coined by Michael Jacobson, director of the Center for Science in the Public Interest, in 1972. Junk food typically contains high levels of fat, salt, or sugar and numerous food-additives such as monosodium glutamate and tartrazine; at the same time, it is lacking in proteins, vitamins and fiber, among others. It is popular with suppliers because it is relatively cheap to manufacture, has a long shelf life and may not require refrigeration. It is popular with consumers because it is easy to purchase, requires little or no preparation, is convenient to consume and has lots of flavor. Consumption of junk food is associated with obesity, heart disease, type 2 diabetes, and dental cavities." (http://en.wikipedia. org/wiki/Junk_food, Wikipedia, The Free Encyclopedia). A strategy known as "nutrient profiling" that sets limits on the quantities of the aforementioned ingredients helps regulatory agencies identify junk food. Recommended Dietary Allowances (RDAs) and Adequate Intakes (AIs) organized by age and gender can be found in the Dietary Recommended Intakes tables for vitamins, minerals and macronutrients, Food and Nutrients Board, Institute of Medicine, National Academy of Sciences.

inconsistent preferences. As reflected by the size of the fast-food and snack-food industries and by the prevalence of overweightness and obesity, there are substantial deviations from the physiologically optimal diet.[2] In the case of non-myopic consumers, the effects of the short-term taste and price advantages of junk food on its level of consumption are moderated by the implications of its harmful ingredients for health. A number of studies have further demonstrated the link between chronic malnutrition and loss of productivity (e.g., Deolalikar, 1988, and Alderman et al., 2005). Unlike Mialon and Mialon (2005),[3] the junk food's substitute in the present analysis contains insignificant quantities of harmful substances. Furthermore, its high price and/or less appealing flavour, as well as the higher level of activity of its healthier consumers, prevent excessive intake and accumulation of calories.

This paper analyses the consumption of junk food and healthy food and its implications for the individual consumer's health and productivity, and, in turn, for the aggregate economic growth, from an economically rational perspective. It does so by incorporating the possible short-term taste and price advantages and the long-term risk disadvantage of junk food into an expected-lifetime-utility-maximization model of consumer behaviour. The analysis explores the effects of junk-healthy foods' price, taste and risk differentials and the effects of time preferences, appetite and endurance on rational dietary choices. Rational food consumers are assumed to have self-control and time-consistent preferences. They are further assumed to maximize their expected lifetime utility from consumption of junk food and healthy food subject to the evolution of their health and its implications for their productivity and longevity.

The dynamic model of rational dietary choice developed and studied in this paper distinguishes between a physiologically optimal dietary path and a rational dietary path. The physiologically optimal dietary path excludes junk food and leads to maximum health, productivity and life-expectancy but not necessarily to maximum lifetime utility. A rational dietary path is the continuum of combinations of healthy food and junk-food that maximises the individual's lifetime expected utility from eating subject to the individual's health-transition equation. Its derivation takes into account the taste and price differentials between the two types of food and the adverse effect of junk-food eating on health and, in turn, on productivity and longevity.

In the case of consumers with a negative relative taste-price differential between junk food and healthy food, abstinence from junk food is rational and the physiologically optimal dietary path is also optimal from the perspective of expected lifetime-utility. In the case of consumers with a positive relative taste-price differential, some indulgence in junk food maximises expected lifetime-utility, but leads to lower than maximal health, productivity and longevity. In the latter case, the consumers' steady-state health and productivity levels depend on the size of the consumers' relative taste-price differential, on the rate (strength) of their time-preferences relative to their intrinsic endurance-appetite ratio, and on the health-

[2] Smith (2004) has analyzed the manipulative nature of advertisements of fast and snack foods. There are claims that the taste and price aspects have also been exploited by the least expected organisations — schools. Anderson and Butcher (2005) have argued that, due to budgetary reasons, the availability of junk foods in schools has been increased, and that the greater availability explains about one-fifth of the increase in average body mass index among adolescents in the United States over the last decade.

[3] The substitute to the harmful good in Mialon and Mialon's (2005) analysis is a less harmful good (e.g., light cigarettes and light beverages). The availability of a less harmful substitute does not necessarily improve the consumer's health. It reduces the risk for people with high taste for the more harmful good. However, it increases the risk for people with a sufficiently low taste for the more harmful good due to a large consumption of the less harmful good, which still contains significant quantities of harmful substances.

depreciating effect of junk food. As long as the consumer's time-preference rate is sufficiently low, the consumer's health and junk food consumption oscillates around the steady state. The health and productivity oscillations diminish (increase) with the passage of time if current junk-food consumption decelerates (accelerates) future junk-food consumption. A junk-food consumption-tax that is greater than the relative taste-price differential can steer health and productivity of rational consumers to the maximal levels. However, the outcome is not Pareto-superior to the free-market one that includes cases of partial and total indulgence.

2. A model of a rational choice of diet

For simplicity, the model includes only two goods, healthy food and junk food, traded in perfectly competitive markets at time-invariant prices. The presentation of the assumed direct and indirect effects of a diet of junk food and healthy food on the individual's satisfaction level, health, productivity, budget and prospects of endurance employs the following definitions and notation:

j - a junk-food component of a meal;

h – a healthy-food component of a meal;

p - a positive scalar indicating a time-invariant price-ratio of the junk-food component and the healthy-food component (hereafter, relative price);

α - a positive scalar denoting the individual's taste-ratio of the junk-food component and the healthy-food component (hereafter, relative taste) and reflecting time-consistent tastes;

c_h^o - the number of healthy-food components required, while avoiding junk food, for maintaining perfect health (i.e., the physiologically optimal diet of a perfectly healthy person) – the *utmost* physiologically optimal diet;

$c_h(t)$ - the individual's healthy-food consumption (i.e., the number of healthy-food components consumed) at t, $0 \le c_h(t) \le c_h{}^o$;

$c_j(t)$ - the individual's junk-food consumption (i.e., the number of junk-food components consumed) at t;

$c_j(t) + c_h(t)$ - the individual's diet at t;

$x(t)$ - the individual's health condition at t, a unit-interval index $0 \le x(t) \le 1$ with $x = 0$ representing terminal sickness and $x = 1$ perfect health;

$\hat{y}$ - a positive scalar indicating the individual's full-capacity income;

$y(t)$ - the individual's income at t;

$\phi(t)$ - the probability density of dying at t;

$u(t)$ - the individual's utility from consumption at t;

ρ - the individual's time-consistent rate of time preference, $0 < \rho < 1$; and

V - the individual's lifetime utility.

Productivity and income: Skill and employment opportunities determine the individual's full-capacity income, $\hat{y}$. The individual's health determines the individual productivity — the extent to which the individual realizes her/his full-capacity income. Productivity is maximal, 1, when the individual is perfectly healthy and converges to 0 as the individual becomes terminally sick. Namely, the individual's instantaneous income is given by

$$y(t) = x(t)\hat{y}. \tag{1}$$

Budget: Taking the price of healthy food to be a numeraire, the individual's instantaneous budget constraint is[4]

$$pc_j(t) + c_h(t) = x(t)\hat{y}\,. \tag{2}$$

The right-hand side of the budget constraint reflects the assumption that the healthier the person the greater her/his spending on food. Health is associated with a lower consumption of junk food and a greater consumption of the more expensive healthy food. Once reached, perfect health ($x = 1$) is maintained by adhering to the physiologically optimal diet: $c_j = 0$ and $c_h = c_h^o$. Correspondingly, and in recalling Eq. (1), the balanced-budget equation requires that the full-capacity income earned by a perfectly healthy person is equal to the cost of her/his utmost physiologically optimal diet. That is, $\hat{y} = c_h{}^o$. To let perfect health be attainable, this equality is assumed. With this assumption Eq. (2) can be rewritten as

$$c_h(t) = x(t)c_h^o - pc_j(t)\,. \tag{3}$$

Instantaneous utility: In agreement with Dynan's (2000) empirical findings of insignificant addiction, the instantaneous utility derived from consuming the two types of food is assumed to be independent of past consumption. It is represented by a function $u(c_j(t), c_h(t))$ having the following properties. Food is essential: $u(0,0) = 0$. Yet neither junk food nor healthy food is essential by itself: $u(0, c_h), u(c_j, 0) > 0$. The marginal instantaneous satisfaction with respect to each type of food is positive and diminishing: $u_j, u_h > 0$, $u_{jj}, u_{hh} < 0$. Healthy food and junk food are substitutes: $u_{jh} < 0$. Consistently, the following explicit instantaneous-utility function is used:

$$u_t = [\alpha c_j(t) + c_h(t)]^{\beta}\,. \tag{4}$$

The scalar α indicates the taste of junk food relative to healthy food. The sum $\alpha c_j + c_h$ displays the taste-adjusted meal. The scalar $0 < \beta < 1$ is the eating-satisfaction elasticity with respect to the taste-adjusted meal.[5] It reflects the consumer's ability to enjoy eating. A person who considers eating a mere physiological necessity has a small β, whereas a person who regards eating also as a pleasurable activity has a large β. We refer to β as the individual's *intrinsic appetite*. As appetite may diminish with eating, the adjective *intrinsic* is added to the description of the constant β.[6] Recalling Eq. (3), the instantaneous utility function can be further expressed as

$$u_t = [(\alpha - p)c_j(t) + x(t)c_h^o]^{\beta}\,. \tag{5}$$

---

[4] The presentation of the more general case of intertemporal-budget constraint with borrowing and lending requires the inclusion of an extra state variable (outstanding debt or credit) and interest rate. The consideration of such intertemporal budget constraint complicates the analysis tremendously while not being a major issue.

[5] The ratio of the satisfaction-elasticities with respect to junk-food and healthy food is equal to the product of the relative taste and quantities, $\alpha(c_j / c_h)$.

[6] This constant can also be interpreted as the ratio of the eating-satisfaction elasticity with respect to healthy food (ξ_h) and the share of healthy food in the taste-adjusted meal (i.e., $\beta = \xi_h / [c_h / (\alpha c_j + c_h)]$.

Health: Health is deteriorated by eating junk food and improved by a natural recovery process. The instantaneous change in the individual's health is represented by a logistic function displaying a diminishing relative health-improvement rate in junk-food consumption, a diminishing health-improvement rate (r) in the level of health, and a unit upper-bound and a zero lower-bound on the individual's health. Using $c_j(t)/c_h^o$ as an index of the excessive physiological inadequacy of the current diet, the evolution of the individual's health is given by

$$\dot{x}(t) = \underbrace{\{1-\delta[c_j(t)/c_h^o]\}[1-x(t)]}_{r}x(t)\,, \tag{6}$$

where δ is a positive scalar indicating the marginal adverse effect of the physiologically inadequate diet on the relative rate of improvement of the individual's health. When junk food is avoided the current recovery rate $(\dot{x}(t)/x(t))$ is maximal and equal to the recovery rate $1-x(t)$ facilitated by the currently affordable healthiest diet. The interpretation of the health-motion equation is enhanced by noting that

$$1-\delta[c_j(t)/c_h^o] = [\dot{x}(t)/x(t)]/[1-x(t)]\,. \tag{6'}$$

Namely, $1-\delta[c_j(t)/c_h^o]$ is the current rate of health-change ($\dot{x}(t)/x(t)$) relative to the currently maximal recovery rate ($1-x(t)$). This current *relative health-change rate* is hindered by current junk-food consumption and is negative for $c_j(t) > c_h^o/\delta$.[7]

Endurance: The probability of survival to the next period rises with the individual's health. It is equal to one when the individual is perfectly healthy ($x=1$), converges to zero as the individual's health diminishes, and is, for tractability, taken to be isoelastic. In formal terms, let $F(t)$ be the cumulative distribution function associated with the probability density of dying ($\phi(t)$) and, consequently, $\Phi(t) = 1-F(t)$ be the probability of living beyond t. It is assumed that $\Phi(t) = \Phi(x(t))$ with $\Phi_x > 0$ (a positive health effect), $\lim_{x\to 1}\Phi = 1$ and $\lim_{x\to 0}\Phi = 0$. It is further assumed that the endurance elasticity ($\Phi_x \frac{x}{\Phi}$) is equal to a positive scalar η. Namely,

$$\Phi(t) = x(t)^{\eta}\,. \tag{7}$$

Since $0 \le x \le 1$, $0 \le \Phi \le 1$ for any $\eta > 0$. Consequently, the rate of change of the endurance probability is proportional to the rate of change of health: $\frac{\dot{\Phi}}{\Phi} = \eta\frac{\dot{x}}{x}$. We refer to the coefficient η as the individual's *intrinsic endurance*. As the prospects of

[7] The case of a negative relative health-improvement rate does not violate the assumption that x lies within the (positive) unit interval as long as the initial value of x is smaller than 1. Furthermore, when x is close to zero and the consumption of junk-food is lower than c_h^o/δ, $1-\delta[c_j/c_h^o]$ can be interpreted as the junk-food weakened recovery rate from a near-death situation. Had the healthy food contained significant quantities of harmful ingredients, the Mialon-Mialon proposition indicated in footnote 3 could be reproduced by specifying the health-motion equation as $\dot{x}(t) = \{1-\delta[c_j(t)+\mu c_h(t)]\}[1-x(t)]x(t)$, where $0<\mu<1$ indicates the harm caused by consuming a unit of healthy food relative to that caused by a unit of junk-food.

endurance increase with health, the adjective *intrinsic* is added to the description of the constant η.

Rational consumers with self-control choose their dietary path so as to maximize expected utility from consumption over the remainder of their life, subject to their health motion equation. Since the moment of death is random, they multiply their accumulated utility from food between the starting point of their planning horizon and their possible time of death ($\int_0^t e^{-\rho\tau} u_\tau d\tau$) by the probability of dying at that instance ($\phi(t)$). The products of $\phi(t)$ and $\int_0^t e^{-\rho\tau} u_\tau d\tau$ associated with any possible life expectancy $0 \le t \le \infty$ are considered by such consumers. The sum of all these products, $\int_0^\infty \phi(t) \int_0^t e^{-\rho\tau} u_\tau d\tau dt$, is these consumers' expected lifetime-utility which, through integrating by parts and in recalling Eq. (7), can be expressed as:[8]

$$E(V) = \int_0^\infty \Phi(x(t)) e^{-\rho t} u_t dt = \int_0^\infty e^{-\rho t} x(t)^\eta u_t dt . \tag{8}$$

The right-hand-side term provides an alternative interpretation of *E*(*V*) — one that is based on the association of quality of life and health. The number of quality-adjusted life-years is used in cost-benefit analysis of health-investment projects as an index of well-being. It combines the duration of life and health condition into a single utility index. (Cf. Bleichrodt, 1995; and Bleichrodt and Quiggin, 1999.) Likewise, $0 \le x(t)^\eta \le 1$ can be alternatively viewed as a life-quality index and $\int_0^\infty e^{-\rho t} x(t)^\eta u_t dt$ as the quality-adjusted lifetime-utility from food consumption.

Substituting Eq. (5) into Eq. (8) for u_t, the rational junk-food consumption path is found by $\max_{\{c_j\}} \int_0^\infty e^{-\rho t} x(t)^\eta [(\alpha - p) c_j(t) + x(t) c_h^o]^\beta dt$ subject to the health-motion equation (6).

3. Abstinence and indulgence

When analyzing junk-food consumption the polar phenomena of abstinence and indulgence deserve attention. Abstinence is usually attributed to dogmatism and/or life-threatening physiological problem, whereas indulgence is usually due to loss of self-control. In the context of the present optimal-control problem, abstinence and indulgence may arise as corner solutions for consumers with certain pecuniary values of time and rates of time-preferences.

---

[8] See Levy (2002a, 2002b) for proof and use of $\int_0^\infty \phi(t) \int_0^t e^{-\rho\tau} u_\tau d\tau dt = \int_0^\infty \Phi(t) e^{-\rho t} u_t dt$ for analysing the prevalence of overweightness and HIV-AIDS among rational people.

PROPOSITION 1 (*Rational abstinence*): *If the relative price of junk food exceeds the relative taste of junk food ($p > \alpha$), a diet free of junk food is rationally optimal, converging to the utmost physiologically optimal diet, and maximizing health and productivity.*

The underlying rationale for this proposition is as follows. Recalling Eq. (5), $\alpha < p$ implies that the instantaneous utilities from consuming junk food are negative. Hence, a maximizer of $\int_0^\infty e^{-\rho t} x(t)^\eta u_t dt$ maintains a diet free of junk food ($c_j(t) = 0$) and $c_h(t) = y(t) = x(t)\hat{y}$ every instance *t*. Recalling Eq. (6), $\lim_{t\to\infty} x = 1$ and, in turn and recalling Eq. (1), $\lim_{t\to\infty} y = \hat{y}$. Recalling Eq. (2) and Eq. (1), $\lim_{x\to 1} c_h = \hat{y} = c_h^o$. Hence, $\lim_{t\to\infty} c_h = c_h^o$.

Proposition 1 implies that junk food does not necessarily constitute the diet of low-income earning households. Suppose that consumers have similar tastes. Since the pecuniary value of time is higher for a middle-income-earning household than for a lower-income-earning household, it is likely that the former frequents fast-food and snack-food restaurants and that the latter abstains. Indeed, casual observation reveals that junk food was significantly introduced to major cities in developing countries by foreign fast-food and snack-food companies when it became affordable to middle-income earners. The pecuniary value of time for high-income-earning households is high, but they can afford to employ a cook for home meals and/or eat in high-quality restaurants. Hence, there may be an inverted-U-shaped relationship between the probability of abstinence and income.

PROPOSITION 2 *(Rational indulgence): If the relative taste of junk food exceeds the relative price of junk food ($\alpha > p$) and the individual is myopic ($\rho \to \infty$) a diet based entirely on junk food is rationally optimal but maximizes the loss of health and productivity and ultimately leads to complete self-destruction.*

This rational-indulgence proposition is consistent with O'Donghue and Rabin's (1999 and 2000) immediate gratification stemming from time-inconsistent preferences. When $\alpha > p$ and $\rho \to \infty$ the marginal instantaneous satisfactions from the junk food are positive and only the present utility matters. The value of future health is nil and hence $c_h(t) = 0$ and $c_j(t) = x(t)\hat{y}/p$ every instance. Recalling Eq. (6), $\lim_{t\to\infty} x = 0$ and, in turn and recalling Eq. (1), $\lim_{t\to\infty} y = 0$.

4. Composite diet and the value of health

The analysis of the rational choice of junk-healthy-food composition continues under the assumptions of positive relative taste-price differential ($\alpha - p > 0$) and non-myopia. The Hamiltonian corresponding to the aforementioned constrained maximization problem is:

$$H = e^{-\rho t} x^\eta [(\alpha - p)c_j + xc_h^o]^\beta + \lambda[1 - \delta(c_j / c_h^o)](1 - x)x \qquad (9)$$

where the co-state variable λ indicates the shadow present value of the individual's health. (The time-index is omitted for tractability.) Since $0 < \beta < 1$ the Hamiltonian is concave in c_j. However, neither $e^{-\rho t} x^{\eta}[(\alpha - p)c_j + xc_h^o]^{\beta}$ nor $[1-\delta(c_j / c_h^o)](1-x)x$ is necessarily concave in the state variable (x). In turn, the Hamiltonian is not necessarily concave in x. In such a case, Mangasarian's theorem on the sufficiency of Pontryagin's maximum-principle conditions is not valid. Nonconcavity of a Hamiltonian in its state variable plays a crucial role in generating unstable steady states and, possibly, a Dechert-Nishimura-Skiba point.

In addition to the state-equation (2), maximum expected lifetime satisfaction from food requires that the change in the consumer's valuation of health is given by

$$\dot{\lambda} = -[\eta x^{\eta-1} Z^{\beta} + x^{\eta} \beta Z^{\beta-1} c_h^o] e^{-\rho t} - \lambda(1-2x)(1-\delta c_j / c_h^o) \tag{10}$$

and that along the rational food-consumption path the marginal satisfaction from eating junk food, discounted by both the consumer's time preferences and prospects of endurance, is equal to the value of the marginal loss of health caused by eating junk food:

$$x^{\eta} e^{-\rho t} \beta Z^{\beta-1}(\alpha - p) - \lambda(\delta / c_h^o) x(1-x) = 0 \tag{11}$$

where $Z \equiv (\alpha - p)c_j + xc_h^o$.

Value increases with scarcity — the poorer the health condition the greater the individual's appreciation of health. The first term on the right-hand side of the adjoint equation (10) reveals the negative combined effect of the rise in the probability of endurance and income stemming from an infinitesimal improvement in health on the evaluation of health. The second term represents the effects of an infinitesimal improvement of health on further improvement in health, $\dot{x}$. As implied by the logistic-health improvement function, up to a critical level of health (e.g., 0.5 had junk food been avoided) $\dot{x}$ increases with x due to strong natural recovery and then decreases. A large (small) health improvement decelerates (accelerates) the individual's evaluation of health. The adjoint equation (10), in conjunction with the optimality condition (11), implies further that along the rational junk-food consumption path the rate of change of the shadow value of health is given by

$$\frac{\dot{\lambda}(t)}{\lambda(t)} = -\left[(\eta / \beta)c_j + [1 + (\eta / \beta)]\frac{xc_h^o}{\alpha - p}\right](\delta / c_h^o)(1-x) - (1-2x)[1 - (\delta / c_h^o)c_j]. \tag{12}$$

In turn, the effect of junk-food consumption on the evaluation of health is

$$\frac{\partial(\dot{\lambda} / \lambda)}{\partial c_j} = [(\frac{\eta}{\beta} - 2)x - (\frac{\eta}{\beta} - 1)]\frac{\delta}{\beta c_h^o} \tag{13}$$

which leads to the following proposition.

PROPOSITION 3: *If* $\eta \geq \beta$, $\frac{\partial(\dot{\lambda} / \lambda)}{\partial c_j} < 0$ *for every x. If* $\eta < \beta$, $\frac{\partial(\dot{\lambda} / \lambda)}{\partial c_j} \gtreqless 0$ *as* $x \lesseqgtr \left(\frac{1-\eta/\beta}{2-\eta/\beta}\right)$.

The first part of this proposition says that as long as the intrinsic endurance is not smaller than the intrinsic appetite an increase in the intake of junk food decelerates health evaluation. A strong intrinsic appetite leads to a large consumption of the tastier and cheaper junk food and thereby to poor health. Poor health gives rise to self

concerns about, and appreciation of, health. However, having at least as strong intrinsic endurance gives rise to complacency. The second part of the proposition indicates that when the intrinsic endurance is smaller than the intrinsic appetite, an increase in junk-food consumption accelerates the evaluation of health if the level of health is below a critical health status, $\left(\frac{1-\eta/\beta}{2-\eta/\beta}\right)$. The lower the intrinsic endurance-appetite ratio the higher the critical health status.

As explained in greater detail in Appendix A, the instantaneous change in the rationally self-controlled junk-food consumption is given by:

$$\dot{c}_j = -\left\{\frac{(\alpha-p)c_j + xc_h^o}{(1-\beta)(\alpha-p)}\right\}\left\{\rho - \frac{(\frac{\eta}{\beta x}Z + c_h^o)(1-x)(\delta/c_h^o)x}{(\alpha-p)} - [\frac{\eta}{x} - \frac{(1-\beta)c_h^o}{Z}]\dot{x}\right\}. \tag{14}$$

The system comprising Eq. (14) and Eq. (6) portrays the joint evolution of the rationally self-controlled junk-food consumption and health. The complexity of this system reflects that the effects of the model parameters on the transition of junk-food consumption and health are not clear. Interior steady states (SS) are analyzed in the following section for exploring the possible long-run levels of rational junk-food consumption and health.

5. The stationary junk-food consumption and health

In steady state, the junk-food consumption is $c_{j_{ss}} = c_h^o/\delta$ and, as shown in Appendix A, the health and (in view of Eq. (1)) productivity levels are:[9]

$$x_{ss_{1,2}} = 0.5\left\{\left(1-\frac{(\alpha-p)\eta}{(\eta+\beta)\delta}\right) \pm \sqrt{\left(1-\frac{(\alpha-p)\eta}{(\eta+\beta)\delta}\right)^2 + 4\frac{(\alpha-p)(\eta-\rho\beta)}{(\eta+\beta)\delta}}\right\}. \tag{15}$$

Intuitively, one may argue that the stationary levels of health and, in turn, productivity are adversely affected by the consumer's rate of time preference. Consumers endowed with high rate of time preference are strongly attracted to the short-term taste and price advantages of junk food over healthy food and attach a low weight to the long-term adverse effect of junk-food consumption on their health, productivity and longevity. Junk food comprises a large portion of their regular diet and severely hinders their health, productivity and endurance. Eq. (15) reveals a more complex relationship. The possibility of multiple steady states and stationary health status depends on the size of the consumer's rate of time preference relative to the consumer's intrinsic endurance-appetite ratio. If the rate of time preferences is lower

[9] It can be shown that

$$x_{ss_{1,2}} = 0.5\left\{\left(1-\frac{(\alpha-p)\eta}{(\eta+\beta)\delta\hat{y}/c_h^o}\right) \pm \sqrt{\left(1-\frac{(\alpha-p)\eta}{(\eta+\beta)\delta\hat{y}/c_h^o}\right)^2 + 4\frac{(\alpha-p)(\eta-\rho\beta)}{(\eta+\beta)\delta\hat{y}/c_h^o}}\right\}.$$

The exclusion of $\hat{y}$ and c_h^o from Eq. (15) is due to $\hat{y} = c_h^o$, which is implied the assumptions that there are only two goods, that the budget is instantaneously balanced, and that $\hat{y}$ and c_h^o are, respectively, the income and diet of a perfectly healthy person.

than the intrinsic endurance-appetite ratio, the steady state is unique, interior and displays a considerably high level of health. If the rate of time preference is equal to the intrinsic endurance-appetite ratio there are two possible steady states: an interior one with a considerably high level of health and a corner one with terminal sickness. If the rate of time preference is larger than the intrinsic endurance-appetite ratio there are two possible interior steady states: one with a considerably low level of health and the other with a considerably high level of health.

Furthermore, the construction of the phase-plane diagrams in Appendix B and the analysis of the steady-states' nature in Appendix C reveal that as long as the consumer's rate of time preference is lower than a critical level (θ in Eq. (C10)) the consumer's health and junk-food consumption oscillate around the steady state. The underlying rationale is as follows. An increase in junk-food consumption deteriorates health and, in turn, productivity. The decline in health appreciates the value of health and, in turn, discourages junk-food consumption. At the same time, the decrease in income further contributes to a decline in the consumption of junk food by consumers for whom this commodity is a normal good. Contrarily, the decrease in income encourages junk-food consumption by consumers for whom it is an inferior good. However, the health-value-appreciation effect is dominant and the net change in junk-food consumption is negative, leading to a health and productivity improvement and, in turn, to a decline in the value of health and to a rise in income. Subsequently, junk-food consumption is increased, and so on and so forth. Appendix C also suggests that if the current consumption of junk food accelerates the consumption of junk food ($\partial \dot{c}_j / \partial c_j (x_{ss}, c_{j_{ss}}) > 0$), the magnitude of the fluctuations of health and junk-food consumption increases with the passage of time (i.e., the steady state is an asymptotically unstable spiral). If, in contrast, the current consumption of junk food decelerates the consumption of junk food ($\partial \dot{c}_j / \partial c_j (x_{ss}, c_{j_{ss}}) < 0$), the oscillations of health and junk-food consumption are dampened over time (i.e., the steady state is an asymptotically stable spiral). If the change in the consumption of junk food is not affected by its current consumption ($\partial \dot{c}_j / \partial c_j (x_{ss}, c_{j_{ss}}) = 0$), the oscillations of health and junk-food consumption are time-invariant (i.e., the steady state is a centre).

The aforementioned arguments are more formally summarized in propositions 4, 5 and 6 and the corresponding phase-plane diagrams depicted in Figures 1, 2 and 3. The proofs of these propositions are provided in Appendix B.

PROPOSITION 4: *If the consumer's rate of time preference is equal to the intrinsic endurance-appetite ratio ($\rho = \eta / \beta$), there exists two stationary states of health — a corner one,* $x_{ss} = 0$, and an *interior one,* $x_{ss} = 1 - [(\alpha - p)\eta / (\eta + \beta)\delta]$. *In the case of the interior steady state, the greater the intrinsic appetite and the junk food's health-depreciating effect (δ) the higher the stationary health status and productivity, whereas the greater the junk and healthy foods' relative taste-price differential ($\alpha - p$) and the intrinsic endurance the lower the stationary health status and productivity.*

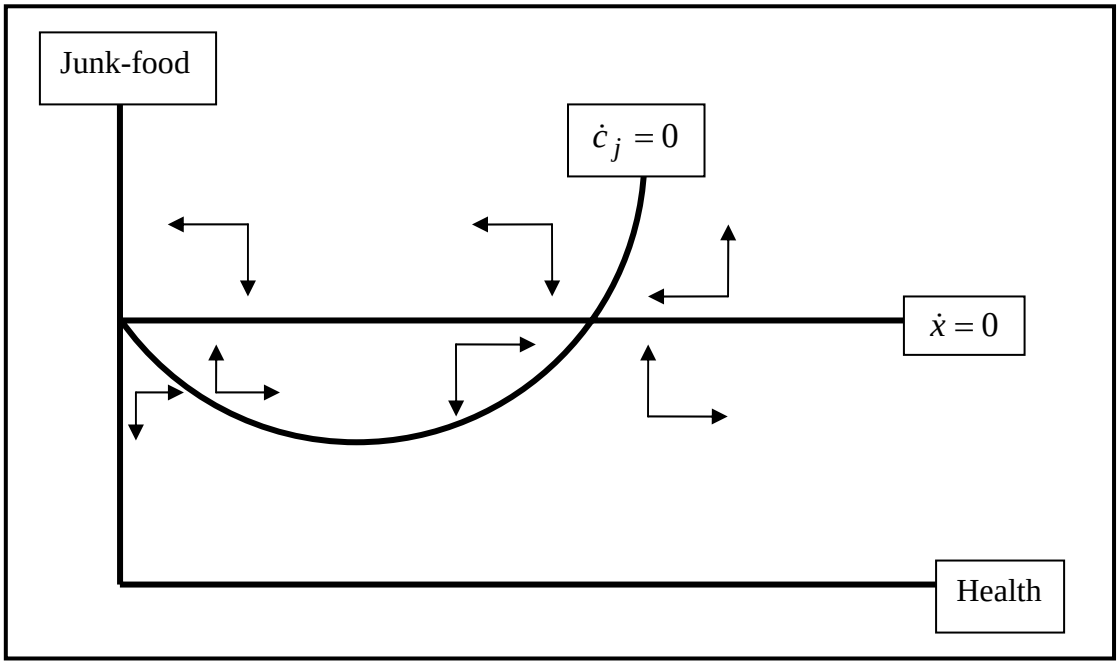

Figure 1. Phase-plane diagram with $\rho = \eta / \beta$

To ensure the existence of the interior steady state it is assumed that $(\eta + \beta)\delta$ is sufficiently large so that $(\alpha - p)\eta/(\eta + \beta)\delta < 1$. The last part of this proposition reflects a complacency effect of the intrinsic endurance. When a slight improvement in health generates significant extension of life expectancy, a higher degree of leniency toward the consumption of the less healthy, yet tastier and cheaper, type of food is entertained by a rational consumer. This complacency is manifested in a low health status and, in turn, a low level of productivity in the long run. The other comparative-static results associated with the interior steady state indicate that a large taste-price differential encourages junk-food consumption and thereby maintenance of poor health. Interestingly, poor health can also be associated with a weak intrinsic appetite. Consumers endowed with a weak intrinsic appetite might not be concerned with overweightness and obesity. Due to their overall low intake of food they might allow themselves a diet rich in the relatively tastier junk food. Yet a large marginal adverse effect of the physiologically inadequate diet on health moderates the consumption of junk food and, in turn, improves the rational consumer's stationary health.

PROPOSITION 5: *If* $\rho < \eta / \beta$ *, there exists a single interior steady state with* $c_{j_{ss}} = c_h^o / \delta$ *and health and productivity level*

$$x_{ss} = 0.5\left\{\left(1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta}\right) + \sqrt{\left(1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta}\right)^2 + 4\frac{(\alpha - p)(\eta - \rho\beta)}{(\eta + \beta)\delta}}\right\}.$$

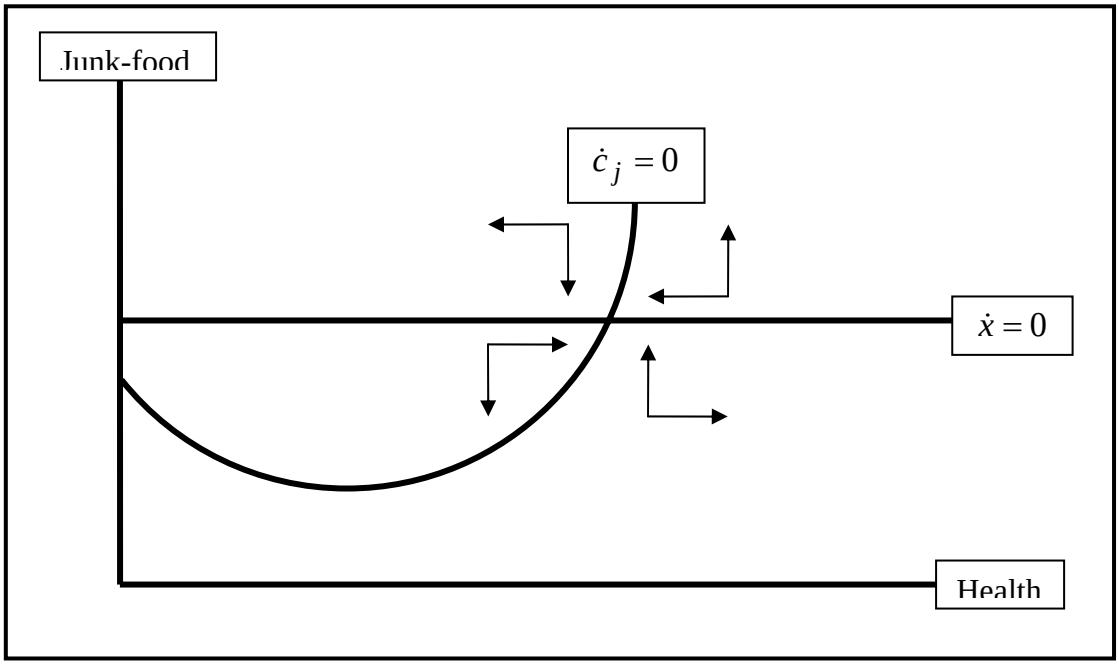

Figure 2. Phase-plane diagram with $\rho < \eta / \beta$

PROPOSITION 6: *If* $\eta / \beta < \rho < \eta / \beta + \frac{(1+\eta/\beta)\delta}{\alpha - p}\left[1 - \frac{(\alpha - p)\eta}{(\eta+\beta)\delta}\right]^2$, *there exist two interior steady states having* $c_{j_{ss}} = c_h^o / \delta$.

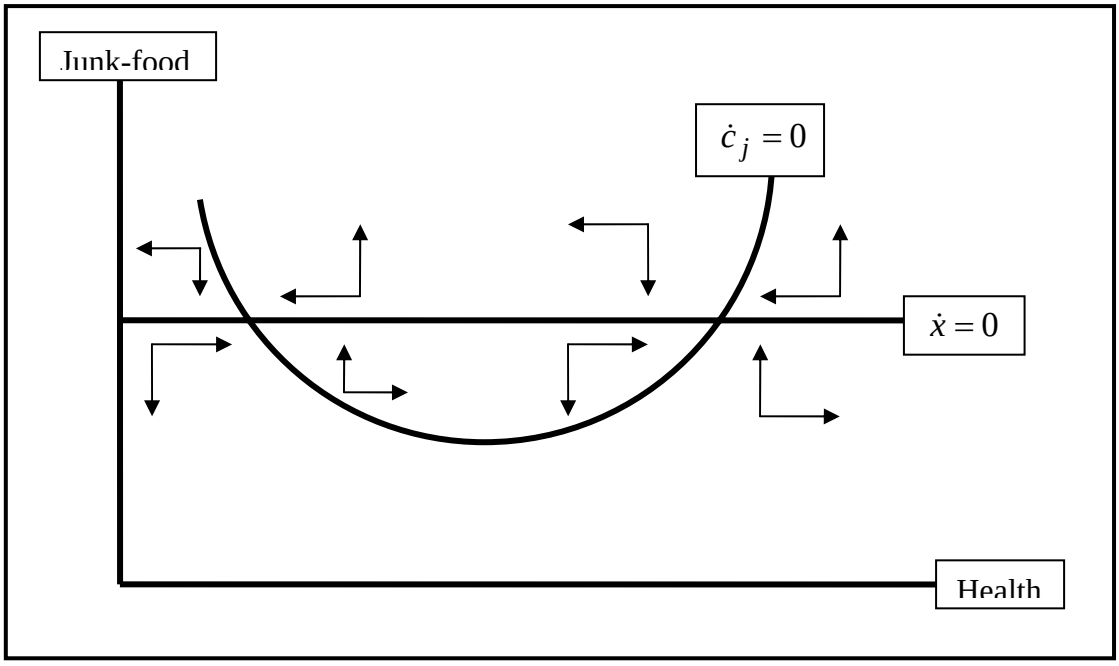

Figure 3. Phase-plane diagram with $\eta / \beta < \rho < \eta / \beta + \frac{(1+\eta/\beta)\delta}{\alpha - p}\left[1 - \frac{(\alpha - p)\eta}{(\eta+\beta)\delta}\right]^2$

6. Health and growth-maximising tax-policy

Governments can increase the personal and aggregate levels of instantaneous and lifetime health and output by taxing junk-food consumption. Consider an economy of N expected-lifetime-utility maximising agents. These agents' full-capacity incomes are equal to their physiologically optimal diets $\hat{y}_1 = c^o_{h1}, \hat{y}_2 = c^o_{h2}, \hat{y}_3 = c^o_{h3}, ..., \hat{y}_N = c^o_{hN}$. Their initial health conditions are $x_1(0), x_2(0), x_3(0), ..., x_N(0)$. Their relative tastes are $\alpha_1, \alpha_2, \alpha_3, ..., \alpha_N$. Their prices of junk food, relative to healthy food, are $p_1, p_2, p_3, ..., p_N$ and may differ from one another in accordance with local market conditions and the personal value of time. Suppose that the junk and healthy foods' price ratio is lower than the junk-healthy relative taste for some, or all, of the agents and hence stimulates junk-food consumption. Noting that the rate of change of the i-th agent's health and income is $\frac{\dot{y}_i(t)}{y_i(t)} = \frac{\dot{x}_i(t)\hat{y}_i}{x_i(t)\hat{y}_i} = \frac{\dot{x}_i(t)}{x_i(t)} = \{1 - \delta[c_{ji}(t)/c^o_{hi}]\}[1 - x_i(t)]$, the following proposition on the aggregate health and growth-maximizing tax-policy for this economy can be made.

PROPOSITION 7: A*n immediately implemented tax rate* $\tau \geq \max\{(\alpha_1 - p_1), (\alpha_2 - p_2), (\alpha_3 - p_3), ..., (\alpha_N - p_N)\}$ *on junk-food consumption maximizes the aggregate health improvement and facilitates the convergence of the actual aggregate income from* $\sum_{i=1}^{N} x_i(0)\hat{y}_i$ *to the aggregate potential income* $\sum_{i=1}^{N} \hat{y}_i$ *with the highest aggregate production growth rate,* $1 - \sum_{i=}^{N} x_i(t)^2 \hat{y}_i / \sum_{i=}^{N} x_i(t)\hat{y}_i$.

This proposition is justified as follows. Recalling Proposition 1, an immediately implemented tax rate $\tau > \max\{(\alpha_1 - p_1), (\alpha_2 - p_2), (\alpha_3 - p_3), ..., (\alpha_N - p_N)\}$ on junk food ensures that every member of the society immediately chooses a diet free of junk food ($c_{hi}(t) = y_i(t) = x_i(t)\hat{y}_i \ \forall i = 1,2,3,...,N$). Recalling equations (6) and (1) and that $\hat{y}_i = c^o_{hi}$, the health-growth rate is, in turn, maximal and the convergence of the actual aggregate product, $Y(t) = \sum_{i=1}^{N} x_i(t)\hat{y}_i$, to the potential aggregate product, $\sum_{i=1}^{N} \hat{y}_i$, is feasible and most rapid:

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\sum_{i=1}^{N} \dot{y}_i(t)}{\sum_{i=1}^{N} x_i(t)\hat{y}_i} = \frac{\sum_{i=1}^{N} \dot{x}_i(t)\hat{y}_i}{\sum_{i=1}^{N} x_i(t)\hat{y}_i} = \frac{\sum_{i=1}^{N} x_i(t)[1 - x_i(t)]\hat{y}_i}{\sum_{i=1}^{N} x_i(t)\hat{y}_i} = 1 - \frac{\sum_{i=1}^{N} x_i(t)^2 \hat{y}_i}{\sum_{i=1}^{N} x_i(t)\hat{y}_i}.$$

If the agents in this economy have identical skills and employment opportunities and hence identical full-capacity income, the implementation of such a tax rate on junk-food consumption leads to aggregate production growth rates that are equal to the highest aggregate improvement rates of health: $1 - \sum_{i=}^{N} x_i(t)^2 / \sum_{i=}^{N} x_i(t)$. If the agents were

identical in every respect — skill, employment opportunities, access to markets, value of time, tastes and initial health — the highest instantaneous growth rates of aggregate production and health are induced by $\tau \geq \alpha - p$ and are equal to $1 - x(t)$, where α, p and x are the common relative taste, relative price and level of health. However, consumers do have different relative prices and tastes, and obtaining information about their prices and tastes is costly. Yet it is possible to obtain maximum growth by setting a very high tax rate on junk food.

7. Conclusion

As long as the difference between the relative taste and the relative price of junk food is positive, the individual's rational diet deviates from the physiologically optimal junk-free diet and generates losses of health, income, longevity and quality of life. The extents of these losses depend on the individual's health-sensitivity to a physiologically inadequate diet, time-preferences and intrinsic endurance and appetite. The level of the stationary health and productivity depends on the strength of the consumers' time preferences relative to their intrinsic endurance-appetite ratio. When the consumers' rates of time preference are sufficiently low, health and productivity oscillates around the steady state. If current junk-food consumption decelerates (accelerates) future junk-food consumption, these oscillations diminish (increase) over time.

A tax rate that bridges the largest positive gap between the relative price and the relative taste of junk food can ensure the choice of a junk-free diet by every member of the society. The universal choice of junk-free diet generates the fastest converging path to the highest individual and aggregate levels of health and production. In addition to the health and productivity loss argument, the case for taxing junk food can be supported by other aspects, which are not included in the model, such as negative externalities and imperfect information. Junk-food consumption reduces health and hence increases the demand for health care. Greater health-care costs are paid for through higher fees and/or taxes. Hence, one's junk-food consumption adversely affects other people's welfare. It is also possible that consumers are not well informed about the health costs of eating junk food and that the costs of acquiring this information are too high. Yet a tax-induced universal abstinence is not Pareto-superior to the free-market outcome that includes cases of partial and total indulgence. People who have a high taste for, and low price of, junk food might be especially hurt by a tax on junk food. Furthermore, taxing junk food might not achieve the intended objective of increasing productivity if for many consumers junk food is a Giffen good. The tax would reduce their real income, increase their consumption of junk food and, in turn, deteriorate their health and lower their productivity.

Appendix A: The solution of the optimal-control problem and steady states

$$H(t) = \Phi(x)e^{-\rho t}\underbrace{[(\alpha - p)c_j + xc_h^o]}_{Z}{}^{\beta} + \lambda\underbrace{[1-\delta(c_j / c_h^o)][1-x(t)]x(t)}_{\dot{x}} \tag{A1}$$

$$\dot{\lambda} = -\frac{\partial H}{\partial x} = -[\Phi_x \underbrace{Z^{\beta}}_{u} + \Phi \underbrace{\beta Z^{\beta-1}c_h^o}_{u_x}]e^{-\rho t} - \lambda(1-2x)[1-\delta(c_j / c_h^o)] \tag{A2}$$

$$\frac{\partial H}{\partial c_j} = \Phi e^{-\rho t}\underbrace{\beta Z^{\beta-1}(\alpha - p)}_{u_{c_j}} - \lambda(\delta / c_h^o)x(1-x) = 0 \tag{A3}$$

Eq. (14) is obtained as follows. By differentiating the optimality condition (A3) with respect to time, substituting the right-hand sides of the adjoint equation (A2) and the optimality condition (A3) for $\dot{\lambda}$ and λ:

$$\begin{aligned}
&-\rho\Phi e^{-\rho t}\beta Z^{\beta-1}(\alpha - p) - (1-\beta)\Phi e^{-\rho t}\beta Z^{\beta-2}(\alpha - p)[(\alpha - p)\dot{c}_j + c_h^o\dot{x}] \\
&+\dot{\Phi}e^{-\rho t}\beta Z^{\beta-1}(\alpha - p) \\
&+\begin{bmatrix} [\Phi_x Z^{\beta} + \Phi\beta Z^{\beta-1}c_h^o]e^{-\rho t} \\ +(1-2x)[1-\delta(c_j / c_h^o)]\dfrac{\Phi e^{-\rho t}\beta Z^{\beta-1}(\alpha - p)}{(1-x)(\delta / c_h^o)x} \end{bmatrix}(1-x)(\delta / c_h^o)x \\
&-\frac{\Phi e^{-\rho t}\beta Z^{\beta-1}(\alpha - p)}{(1-x)x}(1-2x)\dot{x} = 0
\end{aligned} \tag{A4}$$

Multiplying both sides by $e^{\delta t} / \Phi Z^{\beta-2}\beta(1-\beta)(\alpha - p)$ and collecting terms:

$$\begin{aligned}
&\left(\frac{\dot{\Phi}}{\Phi} - \rho\right)Z - (1-\beta)[(\alpha - p)\dot{c}_j + c_h^o\dot{x}] \\
&+\left(\frac{\Phi_x}{\Phi}\frac{Z^2}{\beta(\alpha - p)} + \frac{Zc_h^o}{(\alpha - p)}\right)(1-x)(\delta / c_h^o)x \\
&+(1-2x)[1-\delta(c_j / c_h^o)]Z - \frac{(1-2x)}{(1-x)x}Z\dot{x} = 0
\end{aligned} \tag{A5}$$

Recalling that $\dot{\Phi} / \Phi = \eta(\dot{x} / x)$ and $\Phi_x / \Phi = \eta / x$,

$$\begin{aligned}
&[\eta(\dot{x} / x) - \rho]Z - (1-\beta)[(\alpha - p)\dot{c}_j + c_h^o\dot{x}] \\
&+\left(\frac{(\eta / x)Z^2}{\beta(\alpha - p)} + \frac{Zc_h^o}{(\alpha - p)}\right)(1-x)(\delta / c_h^o)x \\
&+(1-2x)[1-\delta(c_j / c_h^o)]Z - \frac{(1-2x)}{(1-x)x}Z\dot{x} = 0
\end{aligned} \tag{A6}$$

By rearranging terms,

$$-\rho Z-(1-\beta)(\alpha-p)\dot{c}_j+\left(\frac{\eta}{\beta x}Z+c_h^o\right)(1-x)(\delta/c_h^o)x\frac{1}{(\alpha-p)}Z$$
$$+(1-2x)[1-\delta(c_j/c_h^o)]Z-\{[\frac{(1-2x)}{(1-x)x}-\frac{\eta}{x}]Z+(1-\beta)c_h^o\}\dot{x}=0 \tag{A7}$$

Subsequently,

$$\dot{c}_j=-\frac{\rho}{(1-\beta)(\alpha-p)}Z+\left(\frac{\eta}{\beta x}Z+c_h^o\right)(1-x)(\delta/c_h^o)x\frac{1}{(\alpha-p)^2(1-\beta)}Z$$
$$+\frac{1}{(1-\beta)(\alpha-p)}(1-2x)[1-\delta(c_j/c_h^o)]Z \tag{A8}$$
$$-\frac{1}{(1-\beta)(\alpha-p)}\{[\frac{(1-2x)}{(1-x)x}-\frac{\eta}{x}]Z+(1-\beta)c_h^o\}\dot{x}$$

or, equivalently,

$$\dot{c}_j=-\frac{Z}{(1-\beta)(\alpha-p)}\left\{\begin{array}{l}\rho-(1-2x)[1-\delta(c_j/c_h^o)]\\ -\dfrac{(\frac{\eta}{\beta x}Z+\hat{y})(1-x)(\delta/c_h^o)x}{(\alpha-p)}\\ +\{[\dfrac{(1-2x)}{(1-x)x}-\dfrac{\eta}{x}]+\dfrac{(1-\beta)c_h^o}{Z}\}\dot{x}\end{array}\right\} \tag{A9}$$

Recalling that $\frac{\dot{x}}{(1-x)x}=1-\delta(c_j/c_h^o)$ and $Z=(\alpha-p)c_j+xc_h^o$,

$$\dot{c}_j=-\left\{\frac{(\alpha-p)c_j+xc_h^o}{(1-\beta)(\alpha-p)}\right\}\left\{\rho-\frac{(\frac{\eta}{\beta x}Z+c_h^o)(1-x)(\delta/c_h^o)x}{(\alpha-p)}-[\frac{\eta}{x}-\frac{(1-\beta)c_h^o}{Z}]\dot{x}\right\}. \tag{A10}$$

Eq. (15) is obtained as follows. The substitution of $\dot{c}_j=\dot{\Phi}=\dot{x}=0$ into (A6) implies:

$$-\rho Z+\left(\frac{(\eta/x)Z^2}{\beta(\alpha-p)}+\frac{Zc_h^o}{(\alpha-p)}\right)(1-x)(\delta/c_h^o)x+(1-2x)[1-\delta(c_j/c_h^h)]Z=0. \tag{A11}$$

By rearranging terms,

$$-\beta(\alpha-p)\rho+(\delta/c_h^o)\eta Z(1-x)+(\delta/c_h^o)\beta\hat{y}(1-x)x$$
$$+\beta(\alpha-p)(1-2x)[1-(\delta/c_h^o)c_j]=0 \tag{A12}$$

Recalling that $c_{j_{ss}}=c_h^o/\delta$,

$$-\beta(\alpha-p)\rho+(\delta/c_h^o)\eta[(\alpha-p)/(\delta/c_h^o)+xc_h^o](1-x)$$
$$+(\delta/c_h^o)\beta c_h^o(1-x)x=0 \tag{A13}$$

Rearranging terms,

$$-\beta(\alpha-p)\rho+[(\alpha-p)\eta+\delta\eta x](1-x)+\delta\beta(1-x)x=0 \tag{A14}$$

or, equivalently,

$$(\beta+\eta)\delta x^2 - [(\beta+\eta)\delta - (\alpha-p)\eta]x - (\alpha-p)(\eta-\beta\rho) = 0 \tag{A15}$$

or, equivalently,

$$x_{ss}{}^2 - \frac{[(\beta+\eta)\delta - (\alpha-p)\eta]}{(\beta+\eta)\delta} x_{ss} - \frac{(\alpha-p)(\eta-\beta\rho)}{(\beta+\eta)\delta} = 0 \tag{A16}$$

or, equivalently,

$$x_{ss}{}^2 - \left(1 - \frac{(\alpha-p)\eta}{(\eta+\beta)\delta}\right) x_{ss} - \frac{(\alpha-p)(\eta-\rho\beta)}{(\eta+\beta)\delta} = 0\,. \tag{A17}$$

Consequently, the individual's steady-state health level(s) is (are) given by Eq. (15).

Appendix B: Phase-plane diagrams and the proofs of Propositions 4, 5 and 6

From (6), the isocline $\dot{x}=0$ is given by a horizontal line in the plane spanned by x and c_j:

$$c_j = c_h^o / \delta . \tag{B1}$$

From (14) and the definition of Z, along the isocline $\dot{c}_j = 0$

$$\rho - \frac{\{\frac{\eta}{\beta x}[(\alpha - p)c_j + xc_h^o] + c_h^o\}(1-x)(\delta / c_h^o)x}{(\alpha - p)} = 0 \tag{B2}$$

By rearranging terms the isocline $\dot{c}_j = 0$ is given by:

$$c_j = [\rho\beta / (\delta / c_h^o)\eta]\frac{1}{(1-x)} - \frac{c_h^o[1+\beta/\eta]}{(\alpha - p)}x \tag{B3}$$

The slope of the isocline $\dot{c}_j = 0$ is:

$$\frac{dc_j}{dx}\Big|_{\dot{c}_j=0} = (\rho\beta / \tilde{\delta}\eta)\frac{1}{(1-x)^2} - \frac{c_h^o[1+\beta/\eta]}{(\alpha - p)} \gtreqless 0$$

$$as\ (\rho\beta / \tilde{\delta}\eta)(\alpha - p) \gtreqless c_h^o[1+\beta/\eta](1-x)^2 \tag{B4}$$

where $\tilde{\delta} \equiv \delta / c_h^o$.

Recalling that $0 \le x \le 1$ and $\alpha - p > 0$, $\frac{dc_j}{dx}\Big|_{\dot{c}_j=0} \gtreqless 0\ as\ (1-x)^2 \lesseqgtr \frac{(\rho\beta / \tilde{\delta}\eta)(\alpha - p)}{c_h^o[1+\beta/\eta]}$

and $\frac{d^2c_j}{dx^2}\Big|_{\dot{c}_j=0} > 0$. The isocline $\dot{c}_j = 0$ is U-shaped in the plane spanned by x and c_j.

In the case of $\rho = \eta / \beta$ (considered by Proposition 4), the second term in the discriminant is equal to zero and hence $x_{ss} = 0$ or $x_{ss} = 1 - [(\alpha - p)\eta / (\eta + \beta)\delta]$. Note further that in this case $c_j\big|_{\dot{c}_j=0} = \frac{\rho}{\eta/\beta}(c_h^o / \delta) = (c_h^o / \delta)$ when $x = 0$.

In the case of $\rho < \eta / \beta$ (considered by Proposition 5),

$$\left(1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta}\right) < \sqrt{\left(1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta}\right)^2 + 4\frac{(\alpha - p)(\eta - \rho\beta)}{(\eta + \beta)\delta}} .$$

Hence, $x_{ss_2} < 0$ and only $0 < x_{ss_1} < 1$. Note further that for $x = 0$ $c_j\big|_{\dot{c}_j=0} = \frac{\rho}{\eta/\beta}(c_h^o / \delta) < (c_h^o / \delta)$ since $\rho < \eta / \beta$. The isocline $\dot{c}_j = 0$ intersects the isocline $\dot{x} = 0$ only once as displayed by Figure 2.

In the case of $\eta / \beta < \rho < \eta / \beta + \frac{(1+\eta/\beta)\delta}{\alpha - p}\left[1 - \frac{(\alpha - p)\eta}{(\eta + \beta)\delta}\right]^2$ (considered by Proposition 6), the discriminant in Eq. (15) is positive,

$$\left(1-\frac{(\alpha-p)\eta}{(\eta+\beta)\delta}\right)>\sqrt{\left(1-\frac{(\alpha-p)\eta}{(\eta+\beta)\delta}\right)^2+4\frac{(\alpha-p)(\eta-\rho\beta)}{(\eta+\beta)\delta}}\ ,$$

and $c_{j|_{\dot{c}_j=0}}=\frac{\rho}{\eta/\beta}(c_h^o/\delta)>(c_h^o/\delta)$ for $x=0$. In this case, the isocline $\dot{c}_j=0$ may intersect the isocline $\dot{x}=0$ twice as displayed by Figure 3.

Comment: The discriminant in Eq. (15) is zero, or negative, when ρ is equal to, or greater than, $\eta/\beta+\frac{(1+\eta/\beta)\delta}{\alpha-p}\left[1-\frac{(\alpha-p)\eta}{(\eta+\beta)\delta}\right]^2$.

Appendix C: The properties of the steady states

To assess the steady states' properties consider the state-transition matrix (Ω) of the linearized form of Eq. (6) and Eq. (14) in the vicinity of steady state:

$$\Omega = \begin{bmatrix} \frac{\partial \dot{x}}{\partial x}(x_{ss}, c_{j_{ss}}) & \frac{\partial \dot{x}}{\partial c_j}(x_{ss}, c_{j_{ss}}) \\ \frac{\partial \dot{c}}{\partial x}(x_{ss}, c_{j_{ss}}) & \frac{\partial \dot{c}}{\partial c_j}(x_{ss}, c_{j_{ss}}) \end{bmatrix}. \tag{C1}$$

Since

$$\frac{\partial \dot{x}}{\partial x}(x_{ss}, c_{j_{ss}}) = (1 - \tilde{\delta} c_{j_{ss}})(1 - 2x_{ss}) = (1 - \tilde{\delta}(1/\tilde{\delta}))(1 - 2x_{ss}) = 0 \tag{C2}$$

the eigenvalues of Ω are given by

$$\psi_{1,2} = 0.5\{\frac{\partial \dot{c}}{\partial c_j}(x_{ss}, c_{j_{ss}}) \pm \sqrt{[\frac{\partial \dot{c}_j}{\partial c_j}(x_{ss}, c_{j_{ss}})]^2 + 4\frac{\partial \dot{c}_j}{\partial x}(x_{ss}, c_{j_{ss}})\frac{\partial \dot{x}}{\partial c_j}(x_{ss}, c_{j_{ss}})} \tag{C3}$$

with:

$$\frac{\partial \dot{x}}{\partial c_j}(x_{ss}, c_{j_{ss}}) = -\tilde{\delta}(1 - x)x < 0 \tag{C4}$$

$$\begin{aligned} \frac{\partial \dot{c}}{\partial c_j}(x_{ss}, c_{j_{ss}}) = & -\frac{1}{(1-\beta)}(\rho - R_{2_{ss}}) + R_{1_{ss}}(\eta/\beta)\tilde{\delta}(1 - x_{ss}) \\ & - R_{1_{ss}} R_{3_{ss}} \tilde{\delta}(1 - x_{ss})x_{ss} \end{aligned} \tag{C5}$$

$$\begin{aligned} \frac{\partial \dot{c}_j}{\partial x}(x_{ss}, c_{j_{ss}}) = & -\frac{c_h^o}{(1-\beta)(\alpha - p)}(\rho - R_{2_{ss}}) \\ & + R_{1_{ss}} \frac{\{[(\eta/\beta)/(\tilde{\delta}x_{ss})] + [(\eta/\beta) + 1]c_h^o\}\tilde{\delta}(1 - 2x_{ss}) + (\eta c_h^o/\beta)(1 - x_{ss})\}}{\alpha - p} \end{aligned} \tag{C6}$$

and where,

$$R_{1_{ss}} = \frac{(\alpha - p)/\tilde{\delta} + x_{ss}c_h^o}{(1-\beta)(\alpha - p)} > 0 \tag{C7}$$

$$R_{2_{ss}} = \frac{(\frac{\eta}{\beta x_{ss}} Z_{ss} + c_h^o)(1 - x_{ss})\tilde{\delta}x_{ss}}{(\alpha - p)} = \left\{\frac{\eta}{\beta} + \frac{[(\eta/\beta) + 1]\delta}{\alpha - p}x_{ss}\right\}(1 - x_{ss}) \geq 0 \; as \; x_{ss} \geq 0 \tag{C8}$$

$$R_{3_{ss}} = [\frac{\eta}{x_{ss}} - \frac{(1-\beta)c_h^o}{Z_{ss}}]. \tag{C9}$$

From Eq. (C6), $\frac{\partial \dot{c}_j}{\partial x}(x_{ss}, c_{j_{ss}}) > 0$ for consumers endowed with

$$\rho < \theta = R_{2_{ss}} + R_{1_{ss}} \frac{\{[(\eta/\beta)/(\tilde{\delta}x_{ss})] + [(\eta/\beta) + 1]c_h^o\}\tilde{\delta}(1 - 2x_{ss}) + (\eta c_h^o/\beta)(1 - x_{ss})\}(1-\beta)}{c_h^o} > 0 \cdot \tag{C10}$$

In the vicinity of $x_{ss} = 0$, $\frac{\partial \dot{c}_j}{\partial x}(x_{ss}, c_{j_{ss}}) > 0$ since $\theta|_{x_{ss}=0} = \frac{(\alpha - p)/\tilde{\delta}}{(1-\beta)(\alpha - p)} \frac{\{[(\eta/\beta)/0] + [(\eta/\beta)+1]c_h^o\}\tilde{\delta} + (\eta c_h^o/\beta)\}(1-\beta)}{c_h^o} \to \infty$.

This explains the direction of the vertical arrows at the vicinity of $x_{ss} = 0$ in Figure 1. The vertical arrows in the vicinity of the interior steady states in the phase-plane diagrams are displayed under the assumption that $\rho < \theta$. The directions of the horizontal arrows in the phase-plane diagrams are explained by Eq. (C4). The directions of the horizontal and vertical arrows indicate that the steady states in the phase-plane diagrams are centres if $tr\Omega = \frac{\partial \dot{c}_j}{\partial c_j}(x_{ss}, c_{j_{ss}}) = 0$, asymptotically stable spirals if $tr\Omega = \frac{\partial \dot{c}_j}{\partial c_j}(x_{ss}, c_{j_{ss}}) < 0$, or asymptotically unstable spirals if $tr\Omega = \frac{\partial \dot{c}_j}{\partial c_j}(x_{ss}, c_{j_{ss}}) > 0$. In the case of $x_{ss} = 0$, $\frac{\partial \dot{c}}{\partial c_j} = \frac{1}{(1-\beta)}[(\eta/\beta) - \rho] = 0$ and hence the associated steady state in Figure 1 is a centre. In the cases of the interior steady states ($x_{ss} > 0$) the sign of $tr\Omega$ is not clear. (See Eq. (C5).)

References

Alderman, H., Behrman, J.R., Hoddinott, J., 2005. Nutrition, Malnutrition, and Economic Growth. In Lopez-Casasnovas, Guillem Rivera, Berta Currais, Luis (eds.) *Health and Economic Growth: Findings and Policy Implications*, MIT Press, Cambridge and London, 169-194.

Anderson, P. M., Butcher, K. F., 2005. Reading, Writing and Raisinets: Are School Finances Contributing to Children's Obesity? *NBER Working Papers*: 11177.

Becker, G., Murphy, K., 1988. The Theory of Rational Addiction. *Journal of Political Economy* 96, 675-700.

Bleichrodt, H., 1995. QALYs and HYEs: Under what conditions are they equivalent? *Journal of Health Economics* 14, 17-37.

Bleichrodt, H., Quiggin, J. 1999. Life-cycle preferences over consumption and health: When is cost-effectiveness analysis equivalent to cost-benefit analysis? *Journal of Health Economics* 18, 681-708.

Cawley, J., Markowitz, S., Tauras, J., 2003. Lighting up and slimming down: The effects of body weight and cigarette prices on adolescent smoking initiation. *NBER working paper 9561*.

Chou, S.Y., Grossman, M., Saffer, H., 2004. An economic analysis of adult obesity: results from the Behavioral Risk Factor Surveillance System. *Journal of Health Economics* 23(3), 565-587.

Cutler, D., E. Glaeser, Shapiro, J., 2003. Why Have Americans Become More Obese? *NBER Working Paper 9446*.

Deolalikar, A.B., 1988. Nutrition and Labor Productivity in Agriculture: Estimates for Rural South India. *Review of Economics and Statistics* 70 (3), 406-13.

Drenowski, A., 2003. Fat and Sugar: An Economic Analysis. *The Journal of Nutrition* 133, 838S-840S.

Dynan, K. E., 2000. Habit Formation in Consumer Preferences: Evidence from Panel Data. *American Economic Review* 90, 391-406.

Finkelstein, E., Fiebelkorn, I.C., Wang, G., 2003. National medical Spending Attributable to Overweight and Obesity: How Much and Who’s paying? *Health Affairs* W3, 219-226.

Komlos, J., Smith, P., Bogin, B., 2002. The Rate of Time Preference and Obesity: Is there a Connection? Mimeo, University of Michigan-Dearborn.

Lakdawalla, D., Philipson, T.J., 2002. The Economics of Obesity: A Theoretical and Empirical Examination. *NBER Working Paper* 8946.

Levine, A.S., Kotz, C.M., Gosnell, B.A., 2003. Sugars and Fats: The Neurobiology of Preference. *The Journal of Nutrition* 133, 831S-834S.

Levy, A., 2002a. Rational Eating: Can It Lead to Overweightness or Underweightness? *Journal of Health Economics* 21(5), 887-899.

Levy, A., 2002b. A Lifetime Portfolio of Risky and Risk-Free Sexual Behaviour and the Prevalence of AIDS. *Journal of Health Economics* 21 (6), 993-1007.

Mialon, H. M., Mialon, S. H., 2005. Sinful Indulgences, Soft Substitutes, and Self-Control. *Applied Economics Letters* 12(12), 719-722.

National Heart, Lung, and Blood Institute, 1998. *Clinical Guidelines on the Identification, Evaluation, and Treatment of Overweight an Obesity in Adults*, NIH publication 98-4083.

O'Donoghue, T.O., Rabin, M., 1999. Doing it Now or Later. *American Economic Review* 89, 103-124.

O'Donoghue, T., Rabin, M., 2000. The economics of immediate gratification. *Journal of Behavioral Decision Making* 13(2), 233-250.

Philipson, T.J., Posner, R.A., 1999. The Long-Run Growth in Obesity as a Function of Technological Change. *NBER Working Paper* 7423.

Ruhm, C.J., 2000. Are recessions good for your health? *Quarterly Journal of Economics* 115, 616–650.

Smith, T. G., 2004. The McDonald's Equilibrium: Advertising, Empty Calories, and the Endogenous Determination of Dietary Preferences. *Social Choice & Welfare* 23 (3), 383-413.