Measuring the feedback parameter of a semiconductor laser with external optical feedback

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Publication Details

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Disciplines
Physical Sciences and Mathematics

Publication Details

This journal article is available at Research Online: http://ro.uow.edu.au/infopapers/3594
Measuring the feedback parameter of a semiconductor laser with external optical feedback

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Abstract: Feedback parameter (the C factor) is an important parameter for a semiconductor laser operating in the regime of external optical feedback. Self-mixing interferometry (SMI) has been proposed for the measurement of the parameter, based on the time-domain analysis of the output power waveforms (called SMI signals) in presence of feedback. However, the existing approaches only work for a limited range of C, below about 3.5. This paper presents a new method to measure C based on analysis of the phase signal of SMI signals in the frequency domain. The proposed method covers a large range of C values, up to about 10. Simulations and experimental results are presented for verification of the proposed method.

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OCIS codes: (120.3180) Interferometry; (280.3420) Laser sensors; (140.5960) Semiconductor lasers.

References and links


1. Introduction

As an active area of research, semiconductor lasers (SLs) with external optical feedback has attracted extensive theoretical and experimental work during the past decades. A significant research outcome in this area is an emerging technology for sensing and measurement called optical feedback self-mixing interferometry (SMI) technique. The basic structure of the SMI is shown in Fig. 1, consisting of an SL, a lens and a target. The optical feedback from the external cavity modulates the SL power, which then can be used to measure the movement of the external cavity or retrieve system parameters of an SMI [1–10].

![Fig. 1. Schematic SMI.](image)

The scenario behind SMI sensing is the theoretical model developed from Lang and Kobayashi equations [11]. The model consists of the following equations [1–9]:

\[ \phi_f = \phi_0 - C \cdot \sin[\phi_f + \arctan(\alpha)], \]  
(1)

\[ g(\phi_0) = \cos(\phi_0), \]  
(2)

\[ P(\phi_0) = P_0 [1 + m \times g(\phi_0)]. \]  
(3)

Definitions of the parameters and variables in Eqs. (1)-(3) are listed in Table 1.
Table 1. Definition of the variables in Eq. (1)-(3)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_0$</td>
<td>External light phase of an SL without optical feedback, $\phi_0 = 2\pi n_0 \tau$.</td>
</tr>
<tr>
<td>$C$</td>
<td>Optical feedback parameter.</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Linewidth enhancement factor of an SL.</td>
</tr>
<tr>
<td>$\phi_e$</td>
<td>External light phase of an SL with optical feedback, $\phi_e = 2\pi n_e \tau$.</td>
</tr>
<tr>
<td>$\tau$</td>
<td>External roundtrip delay. It is determined by the external cavity length $L$ and light speed $c$ as $\tau = 2L/c$.</td>
</tr>
<tr>
<td>$\nu_0$</td>
<td>Emitting laser frequency of an SL without optical feedback.</td>
</tr>
<tr>
<td>$\nu_F$</td>
<td>Emitting laser frequency of an SL under optical feedback.</td>
</tr>
<tr>
<td>$g(\phi_0)$</td>
<td>Interferometric function.</td>
</tr>
<tr>
<td>$m$</td>
<td>Modulation index of the modulated SL power.</td>
</tr>
<tr>
<td>$P_0$</td>
<td>Emitting laser power of an SL without optical feedback.</td>
</tr>
<tr>
<td>$P(\phi_0)$</td>
<td>Modulated SL power due to optical feedback, it is called an SMI signal.</td>
</tr>
</tbody>
</table>

From Eqs. (1)-(3), we see that $\phi_0$ affects the emitted power $P(\phi_0)$, which can be easily measured with a photo detector placed on the rear mirror of the SL. As $g(\phi_0) = \frac{P(\phi_0) - P_0}{mP_0}$, we can obtain $g(\phi_0)$ through data processing on $P(\phi_0)$. Without loss of generality, in this paper we use $g(\phi_0)$ to study the features of the SMI signal. Physical quantities associated with the external cavity (e.g. velocity, vibration or displacement, etc.) can be extracted from an SMI sensing signal through the phase $\phi_0 = 4\pi n_0 L / c$.

It is seen that there are two parameters in Eqs. (1)-(3), that is, optical feedback parameter denoted by $C$ [12,13], and linewidth enhancement factor (or $\alpha$-factor) of an SL. As a key parameter of an SL, $\alpha$ characterizes the linewidth, the chirp, the injection lock range in an SL, and also characterizes the response of the SL to optical feedback [14]. Measurement of $\alpha$ has attracted extensive research with various approaches proposed [14], such as linewidth methods [15,16], FM/AM methods [17–19], optical injection [20–22] and optical feedback [1,6,7,23,24]. These different approaches were compared systematically in [25,26].

The feedback parameter $C$ is also an important parameter which tells the operational modes of an SL application system. An SL is considered to operate in weak optical feedback regime when $0 < C < 1$; it runs moderate feedback regime for $1 < C < 4.6$ and in strong feedback regime when $C > 4.6$ [2,5,8]. Nonlinear dynamics, spectral behavior and chaos in an SL are also closely related to the $C$ value [12,13,27–29]. Therefore, the measurement of $C$ is very important for studying the behaviors of an SL and its applications.

In recent years, a number of SMI-based approaches are proposed to measure $\alpha$, which can also yield the $C$ value [1,6,7]. However, these methods are only applicable to limited range of $C$, as discussed below.

The first SMI based technique for $\alpha$ measurement was reported in [1] where SMI operates in moderate optical feedback regime. When $C > 1$, the optical feedback in an SL causes hysteresis phenomenon in the laser power [2,11,12]. In this case, the waveform is sawtooth like [1,2,8,12]. An example of the hysteresis is plot in Fig. 2 using Eq. (1) and Eq. (2), where $C = 2$ and $\alpha = 5$. When $\phi_0$ increases, $g(\phi_0)$ follows the path $A_1-B_1-A_1$ for decreasing $\phi_0$. The area $A_1-B_1-A$ is called hysteresis area with the width represented as $\phi_{0,AB}$. $\phi_{0,AB}$ has been found in [12] as:

$$\phi_{0,AB} = 2 \left\{ \sqrt{C^2 - 1} + \arccos\left(\frac{1}{C}\right) - \pi \right\}.$$
Φ_{AB} can be measured by Φ_{AB} = Φ_{CB} - Φ_{AD} · Φ_{CB} · Φ_{AD} are read from the SMI waveform [1]. However, when C increases to a certain value, the hysteresis area will cover Φ_{CB} and Φ_{AD}, and thus they will disappear from the SMI waveform. In this case, the approach in [1] is no longer valid. Our simulations show that the approach in [1] can only be used when 1 < C < 3.5.

An SMI signal at weak feedback regime is similar to a traditional interference signal containing sinusoidal fringe pattern. In this case a data fitting algorithm and its improved version were reported in [6,7] for the measurement of α and C. The algorithm utilizes the SMI model incorporates an optimal estimate of α and C to yield the best match for the observed signals. Obviously, the approaches presented in [6,7] are only valid for the case of C < 1. Hence the existing SMI methods for C measurement can only be used for the cases of C < 3.5.

For an SL with external optical feedback, the value of C is determined by the following [2]:

\[ C = \eta \frac{\tau}{\tau_{SL}} (1 - R_2) \sqrt{\frac{R_1}{R_2} \sqrt{1 + \alpha^2}}, \]  

(5)

where R_2, R_3, η and τ_{SL} are, respectively, the power reflectivity of the SL cavity, the power reflectivity of the external cavity, the coupling coefficient of the feedback power, and the round trip time in the SL cavity. Note that η and τ are not fixed values when a moving target is used to form the external cavity of an SL. It is obvious that the value of C varies in different SL systems and the value can be greater than 3.5. Hence measurement of C over a large range is required.

In this paper, we propose to estimate C over an extend range of its value (up to 10 and more). Compared to existing techniques which are based on waveform analysis in time domain, the proposed technique is in the frequency domain where the C value is determined by looking at the spectrum of the phase signals derived from the SMI signal g(Φ). With existing time-domain approaches [1,6,7], different algorithms must be employed in different regimes of optical feedback. In contrast, the proposed approach employs the same algorithm to determine C, which covers different optical feedback regimes.
2. The proposed method

2.1 Operation principle

Let us consider the case of an external target moving periodically, resulting in the external cavity length \( L_t \) and thus \( \phi_0 \) also periodically changing with time. Equation (2) and Eq. (1) can be rewritten as:

\[
g(t) = \cos(\phi_f(t)),
\]

\[
\phi_f(t) = \phi_0(t) - C\sin(\phi_f(t) + \arctan(\alpha)).
\]

Taking Fourier transform, denoted by \( F \{ \cdot \} \), on both sides of Eq. (7), we have

\[
\Phi_f(f) = \Phi_0(f) - C \cdot F \{ \sin[\phi_f(t) + \arctan(\alpha)] \},
\]

where \( \Phi_f(f) \) and \( \Phi_0(f) \) are the Fourier transforms of \( \phi_f(t) \) and \( \phi_0(t) \) respectively. For ease of manipulation let us introduce a function

\[
\phi_1(t) = \sin(\phi_f(t) + \arctan(\alpha)),
\]

so that Eq. (8) becomes

\[
\Phi_f(f) = \Phi_0(f) - C \cdot F \{ \phi_1(t) \} = \Phi_0(f) - C \cdot \Phi_1(f),
\]

where \( \Phi_1(f) \) is the Fourier transforms of \( \phi_1(t) \).

From the SMI setup, we are able to acquire the SMI signal \( g(t) \). Based on our previous work [9], \( \phi_f(t) \) can be obtained from \( g(t) \) by applying inverse cosine operation and phase unwrapping. Thereafter, we calculate the spectrum of \( \phi_f(t) \), that is \( \Phi_f(f) \). Meanwhile, when \( \phi_f(t) \) is available, we are able to work out \( \phi_1(t) \) using Eq. (9) and compute its spectrum \( \Phi_1(f) \).

As \( \phi_0(t) = 4\pi v_0 L(t)/c \), \( \phi_f(t) \) also describes the moving of the target. If the external target moves in a manner close to simple harmonic vibration, \( \phi_f(t) \) will be close to a sinusoidal, in which case it can be considered as of narrow band in frequency domain.

Given \( \phi_f(t) \) narrow band in nature, from Eqs. (7) and (8), \( \phi_f(t) \) should exhibit a broader spectrum than \( \phi_1(t) \) because of the nonlinear mapping from \( \phi_f(t) \) to \( \phi_1(t) \). In other words, \( \Phi_f(f) \) and \( \Phi_1(f) \) should spread over a wider frequency range than \( \Phi_0(f) \) does. Now, we can divide the frequency range of non-vanishing spectrum of \( \Phi_f(f) \) and \( \Phi_1(f) \) into two domains, denoted by \( \Omega_1 \) and \( \Omega_2 \) respectively, in which the spectrum of \( \Phi_f(f) \) is non-vanishing or vanishing, i.e.:

\[
\Omega_1 : f \subset \{ \Phi_f(f) \neq 0, \Phi_1(f) \neq 0, \Phi_0(f) \neq 0 \}, \text{ and}
\]

\[
\Omega_2 : f \subset \{ \Phi_f(f) \neq 0, \Phi_1(f) \neq 0, \Phi_0(f) = 0 \},
\]

Considering the frequency components of \( \Phi_f(f) \) and \( \Phi_1(f) \) in \( \Omega_2 \), from Eq. (10) we have

\[
\Phi_f(f) = -C \times \Phi_1(f), \quad f \subset \Omega_2.
\]

Therefore \( C \) can be calculated as:

\[
C = \left| \frac{\Phi_f(f)}{\Phi_1(f)} \right|, \quad f \subset \Omega_2.
\]
The above relation is valid for all the frequency components in $\Omega_2$. In practice, we can employ a summation over all frequency components in $\Omega_2$ to increase the accuracy of the estimate:

$$C = \frac{\sum_{f=f_0}^{f_{M}} |\phi_c(f)|}{\sum_{f=f_0}^{f_{M}} |\phi_t(f)|}.$$  \hspace{1cm} (13)

To apply the method, we shall identify $\Omega_1$ and $\Omega_2$. In the cases of harmonic vibration, $\phi_t(t)$ is a periodic function with a fundamental (vibration) frequency $f_0$. Then $\phi_t(f)$ should exhibit a large fundamental component at $f_0$, and some harmonic components. The highest harmonic component will in general depend on the actual waveform, and yet in practice we consider $15f_0$ as the upper frequency necessary to cover well the spectrum details. This choice will be confirmed by simulations discussed in the next Section.

In terms of the upper bound of $\Omega_2$, as $\phi_t(f)$ and $\phi_c(f)$ are available, we simple choose $f_M$ as high as possible while observing $\phi_t(f_M) \neq 0$ and $\phi_c(f) \neq 0$. In this paper we have taken $\Omega_2$ to be $\{15f_0, f_M\}$.

2.2 Verification of the proposed method by simulation

In order to verify the effectiveness of the formula presented in Eq. (12) and Eq. (13), let us first consider that the external target vibrates in two possible modes, simple harmonic vibration where $L(t)$ is a sinusoidal function, and a more complicated case where $L(t)$ is a triangular function. In this Section we present the results of computer simulation on these two cases.

In the first case, $\phi_c(t)$ is given as follows:

$$\phi_c(t) = \phi_0 + \Delta \phi \sin(2\pi f_0 t),$$  \hspace{1cm} (14)

where $\phi_0$ is the light phase at the equilibrium position of a vibrating target. $\Delta \phi$ is the maximum phase deviation caused by target vibration amplitude, and $f_0$ is the vibration frequency. In the numerical simulation, we set $\alpha = 3$, $C = 5$, $\phi_0 = 3.9 \times 10^6 (\text{rad})$, $\Delta \phi = 10\pi (\text{rad})$, $f_0 = 200\text{Hz}$, and take a sampling frequency $f_s = 51200\text{Hz}$. Using Eqs. (14), (9) and (7), we obtain $\phi_c(t)$, $\phi_t(t)$ and $\phi_c(t)$ as shown on the left side of Fig. 3. Applying Discrete Fourier Transform (DFT) on these three signals, we obtain their spectra (in modulus) as shown in the right side of Fig. 3. We can see that both $\phi_t(t)$ and $\phi_c(t)$ exhibit many high frequency components in contrast to signal $\phi_c(t)$. This enables us to determine the two frequency ranges $\Omega_1$ and $\Omega_2$. 

#144254 - $15.00 USD

Received 18 Mar 2011; revised 24 Apr 2011; accepted 26 Apr 2011; published 2 May 2011

(C) 2011 OSA 9 May 2011 / Vol. 19, No. 10 / OPTICS EXPRESS  9587
In order to demonstrate the process of calculating $C$, we plot details of spectra of $\phi_1(t)$, $\phi(t)$, and $\phi(t)$ on Fig. 4. It is seen that $\Phi_1(f)$ is zero but $\Phi_2(f)$ is zero when $f > 2000 \text{Hz}$. Hence we can set the lower band of $\Omega_2$ as 2000Hz, or $10f_0$. Based on Fig. 4 we can also set the upper bound of $\Omega_2$ to be 25000Hz or $125f_0$. By looking at the spectra at 5800Hz (which is on $\Omega_2$) on Fig. 4, we observe $\phi_1(5800) = 0.3701$ and $\phi_2(5800) = 0.0740$, and hence by Eq. (12) we can determine the C value as $C = 0.3701/0.0740 = 5.0001$, which is very close to the true value used in the simulations. By using Eq. (13) over $\Omega_2 : f \subset [2000 \text{Hz}, 25000 \text{Hz}]$, the C value is obtained as $C = 0.3701/0.0740 = 5.0001$, which is also very close to the true value.

Let us consider the second case, where $L(t)$ is a triangular function with fundamental frequency of 200 Hz. Figure 5 shows $\phi_1(t)$, $\phi(t)$, and $\phi(t)$ in time domain and frequency domains, respectively with $C = 5$ and $\alpha = 3$. We noticed that $\phi(t)$ only exhibit spectrum over
the frequency range from 0Hz to 3000Hz, and that \( \phi_1(t) \) and \( \phi_2(t) \) has a much broader range than \( \phi_3(t) \), that is, they still have frequency components in a wide range of \( f \geq 3000 \text{ Hz} \).

We plot detailed spectra for signals \( \phi_1(t) \), \( \phi_2(t) \) and \( \phi_3(t) \) on Fig. 6, based on which we can have \( \Omega_2 : f \in [3000 \text{Hz}, 25000 \text{Hz}] \). For example, using Eq. (12) at the frequency of 3800Hz we are able to obtain 5.0060 for \( C \). Applying Eq. (13) to all the frequency components over [3000Hz, 25000Hz], we get \( C = 5.0001 \) which is closer to the true value.

![Waveforms and spectra of signals](image1)

**Fig. 5.** Waveforms and spectra of signals \( \phi_1(t) \), \( C\phi_1(t) \), and \( \phi_3(t) \) for the triangular case.

![Detailed spectra of signals](image2)

**Fig. 6.** Detailed spectra of signals \( \phi_1(t) \), \( \phi_2(t) \), and \( \phi_3(t) \) for triangular case.

Extensive simulations have been done for verification of the proposed method. We generated SMI data using different sets of \( C \) and \( \alpha \) shown in Table 2. The estimated \( C \) values using the method in [1] and the proposed method in this paper are shown in Table 2. It is seen that the proposed method is more accurate than the one in [1].
Table 2. Estimated $C$ Values Using Method in [1] and Proposed Method in this Paper

$\hat{C}_{\sin}$: estimated $C$ with a sinusoidal vibrating target; $\hat{C}_{\tri}$: estimated $C$ with a triangular vibrating target; N/A: not applicable

<table>
<thead>
<tr>
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<td>3.0574</td>
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Note that the results shown in Table 2 are obtained using an accurate $\alpha$ value. In practice, it is hard to obtain accurate $\alpha$ for an SL. In fact $\alpha$ is such a complex parameter in that for the same SL, different $\alpha$ values can be obtained by using different measurement technique [26]. However, $\alpha$ is usually considered between 3 and 7 [2]. Hence, we may simply choose an $\alpha$ value within the possible range to measure $C$ using the proposed approach. This requires us to investigate the influence of using such an inaccurate $\alpha$ value on the performance of the proposed approach. To this end, we carried out a set of simulations with the results shown in Table 3. In the simulations, the true value of $\alpha$ is assumed to be 5 and $C$ takes different values from 0.5 to 6.5. For each $C$ values, we choose different $\alpha$ values (that is, 3, 4, 5, 6 and 7) respectively to estimate $C$ using Eq. (13). It is seen that the estimated $C$ values are always close to the true values as shown by the small relative standard deviations in Table 3.

Table 3. Influence of $\alpha$ on the Estimation of $C$

<table>
<thead>
<tr>
<th>True $C$</th>
<th>Estimated $\hat{C}$ using different $\alpha$</th>
<th>Relative standard deviation</th>
</tr>
</thead>
<tbody>
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<td>$\alpha = 4.0000$</td>
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</tr>
<tr>
<td>5.5000</td>
<td>5.5125</td>
<td>5.5125</td>
</tr>
</tbody>
</table>

3. Verification using experimental data

In order to test the measuring approach described in Section 2, we implemented an experimental SMI system, which is the same as the one described in our previous work [8]. The optical set-up consists of a laser diode (LD), a focus lens and a loudspeaker as the external target. The electrical set-up consists of LD driving devices, an optical power detection circuit and a digital oscilloscope. In the experiment, the laser diode (LD) is a HL7851G laser diode emitting at
$\lambda_0 = 785$ nm. The LD is biased with a DC current of 90 mA and operates in single longitudinal mode regime. The sinusoidal vibration is generated by a loudspeaker driven by a signal generator.

For the purpose of comparison, we use the same experimental data presented in [8]. We also preprocessed the signal in order to remove unwanted noise and slow time-varying fluctuation in its amplitude. The measurement procedure is summarized below:

- **Step 1.** Preprocess an SMI signal by means of digital filtering for spike-like noise removal [10];
- **Step 2.** Normalize the SMI signal to get $g(t)$;
- **Step 3.** Determine the vibration frequency $f_0$ by auto-correlation [7];
- **Step 4.** Carry out inverse cosine function operation on $g(t)$ and then phase-unwrapping to obtain $\phi(t)$ [9];
- **Step 5.** Calculate $\phi(t)$ using Eq. (9) with assuming $\alpha = 3$ (or obtain a $\alpha$ value by the methods mentioned in [26]).
- **Step 6.** Apply DFT on $\phi(t)$ and $\phi(t)$ and obtain their spectra;
- **Step 7.** Set $\Omega_2 \subset [15 f_0, f_M]$, where $f_M$ is chosen as half of the sampling frequency. Calculate $C$ value using Eq. (13) over all the frequency components on $\Omega_2$.

The waveforms of the experimental data are shown in Fig. 7, which are taken from [8]. Each $C$ value shown on Fig. 7 was obtained in [8] by simulating waveforms with different $C$ values to match the experimental ones (which is a tedious task). We applied the above procedure to the data and obtained $C$ values for each of the waveforms depicted in Fig. 7, and the results are shown in Table 4. For illustration purposes, Fig. 8 shows the details of applying the proposed method to the waveform in Fig. 7. (c), yielding $C = 0.2901/0.0444 = 6.534$. For comparison, we also show the results of the $C$ values provided by [8] and the results obtained by the method in [1]. It is seen that the $C$ values obtained by the proposed method are consistent to the values provided in [8]. However, for most cases, the method in [1] is not able to determine the value of $C$. 
Fig. 7. Experimental SMI signals with different $C$ values [8].

Table 4. Estimated $C$ Values Using Experimental Signals

<table>
<thead>
<tr>
<th>Experimental signals shown on Fig. 7</th>
<th>Proposed approach</th>
<th>Approach in [1]</th>
<th>$C$ provided in [8]</th>
</tr>
</thead>
<tbody>
<tr>
<td>signal (a)</td>
<td>1.58</td>
<td>1.18</td>
<td>1.35</td>
</tr>
<tr>
<td>signal (b)</td>
<td>3.12</td>
<td>3.05</td>
<td>3.30</td>
</tr>
<tr>
<td>signal (c)</td>
<td>6.53</td>
<td>NA</td>
<td>5.57</td>
</tr>
<tr>
<td>signal (d)</td>
<td>5.20</td>
<td>NA</td>
<td>6.00</td>
</tr>
<tr>
<td>signal (e)</td>
<td>6.12</td>
<td>NA</td>
<td>6.80</td>
</tr>
<tr>
<td>signal (f)</td>
<td>8.70</td>
<td>NA</td>
<td>7.40</td>
</tr>
</tbody>
</table>
4. Conclusion

The paper describes an SMI-based method for measuring feedback factor $C$ in a semiconductor laser with external optical feedback. In contrast to existing methods, which are usually based on time-domain analysis of SMI signals, the proposed one is of frequency domain in nature, which is based on analysis of the magnitude spectra of two phase signals derived from an SMI signal. The proposed approach is advantageous in that it can be used for any optical feedback level, and hence lifts the limitation in existing SMI based methods for measuring $C$. Effectiveness of the proposed method has been confirmed by both simulations and experimental verifications.