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**UNIVERSITY OF WOLLONGONG**

**DEPARTMENT OF ACCOUNTANCY**

**AUDIT RISK IN TERMS OF PROBABILITIES: THE  
AUP24 MODEL**

**by**

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## **ABSTRACT**

The AUP24 audit risk model defines audit risk implicitly as the *joint* probability of three independent events: (i) a material error occurring in an account balance, (ii) that error not being corrected by internal control procedures, and (iii) the uncorrected balance being accepted by the auditor. A more apposite risk measure, relating to these same possible events, is the *conditional* probability of a material error given that the stated balance has been subject to internal control procedures and accepted by the auditor. The two risk measures so defined are related by the laws of probability, through Bayes' theorem specifically, but are not the same and exhibit no necessary correlation. Calculation of the conditional ('Bayesian') risk measure requires consideration of both the type I (alpha) and type II (beta) error probabilities of the auditor's substantive test procedure. Unless both error characteristics are taken into account, it is not possible to interpret a test result (acceptance or rejection) in terms of the probability of the stated account balance being materially correct.

## 1. Introduction

AUP24 describes *audit risk* as '...the risk (complement of assurance) that the conclusions drawn from the audit process might be invalid' (para. 16). This risk, or lack of assurance, exists through the conjunction of (para. 10):

- (i) the 'inherent risk' that error may occur in the auditee's accounts,
- (ii) the 'control risk' that the auditee's internal control system may not reveal (and correct) an error occurring inherently
- (iii) the 'detection risk' that the auditor's test procedure may not reveal an error not corrected by the internal control system.

For an error to arise and remain in the accounts, the events described in (i), (ii) and (iii) must occur jointly (i.e. in conjunction). If no error occurs inherently, or any error occurring is corrected by internal control procedures, or the auditor detects and corrects any error not already corrected, the chain is broken and no error will 'get through' into the audited accounts. Note, however, that this does not preclude the possibility that the audit process itself might introduce a falsity into the accounts through a type I ('alpha') error (rejection of a materially correct balance) and consequent material 'correction' of that incorrectly rejected balance.

## 2. Notation

To model the relationship between audit risk and its underlying sources, inherent risk (IR), control risk (CR) and detection risk (DR), each of these risk terms is defined as a probability. The notation used is as follows:

- letter  $e$  denotes the occurrence of a material error in the stated account balance.
- letter  $i$  means that an account balance withstands internal control procedures without being seen to require correction.
- letter  $a$  means that following a well specified substantive test procedure the auditor accepts the stated balance.

We can now write

(i)  $IR=p(e)$ =the probability of an error occurring inherently in the balance stated.

(ii)  $CR=p(i|e)$ =the probability of internal control procedures not leading to correction of an account balance given a material error in that balance.

(iii)  $DR=p(a|e,i)$ =the probability of the auditors' test procedure accepting a stated balance given that this balance is both inherently in error *and* uncorrected by internal control procedures.

### 3. **Audit Risk Defined**

The risk model adopted in AUP24 (Appendix IV) represents audit risk as the product of the three component risks IR, CR and DR; that is

$$AR = IR \times CR \times DR.$$

In terms of probabilities, AR is then

$$p(e) \times p(i|e) \times p(a|e,i) \quad \dots(1)$$

which is a mathematical expansion of the joint probability,  $p(e,i,a)$ . Thus AR is defined implicitly as the probability of an error occurring *and* no correction by the internal control system *and* acceptance by the auditor. Usually it is supposed that internal control procedures and audit testing are statistically independent, in which case  $p(a|e,i)=p(a|e)$ ; cf. Cushing and Loebbecke (1983, p.30). In the terminology of statistical hypothesis testing,  $p(a|e)$  is the probability of a type II ('beta') error (i.e. acceptance of a materially incorrect balance). Under the assumption of independence, expression (1) simplifies to

$$p(e) \times p(i | e) \times p(a | e). \quad \dots(2)$$

Although the AUP24 risk measure has a straightforward mathematical interpretation, its representation of audit risk (AR) as the joint probability  $p(e, i, a)$  is curious. The probability which represents properly the auditor's degree of assurance (confidence), after the audit is completed, in the stated balance, is

$$1 - p(e | i, a),$$

or, alternatively, audit risk (being the complement of assurance) is represented by the conditional probability

$$p(e | i, a),$$

Defined this way, audit risk is the probability of an error in the stated balance given that this balance has been subject to internal control procedures *and* has been accepted by the auditor. The mathematical specification of audit risk as the *conditional* probability  $p(e | i, a)$ , rather than as the *joint* probability  $p(e, i, a)$ , is supported by the textbook interpretation of Arens *et al.* (1984, p.244):

**Audit Risk** Risk that a given segment (balance) is materially misstated after the audit is completed and the auditor has concluded that the segment is materially correct.

#### 4. Relationship Between $p(e | i, a)$ and $p(e, i, a)$

By Bayes' theorem, the relationship between the advocated measure of conditional audit risk,  $p(e | i, a)$ , and the AUP24 measure,  $p(e, i, a)$ , is

$$p(e | i, a) = \frac{p(e, i, a)}{p(e, i, a) + p(-e, i, a)} \quad \dots(3)$$

$$= \frac{p(e,i,a)}{p(e,i,a) + p(\sim e) p(i|\sim e) p(a|\sim e,i)}$$

where  $\sim$  is the negation operator, and thus  $\sim e$  denotes not- $e$  (i.e., no material error in the stated balance). From this equation, it is seen that although  $p(e|i,a)$  is a function of the AUP24 risk measure,  $p(e,i,a)$ , this latter probability is not an appropriate end in itself when evaluating audit risk. Moreover, to produce the requisite conditional probability of material error, it is necessary to allow for two further probability factors:

(iv)  $p(i|\sim e)$  = the probability of an account balance which is materially correct withstanding internal control procedures without (material) 'correction', and

(v)  $p(a|\sim e,i) = p(a|\sim e)$  = the probability of an account balance which is materially correct being accepted by the auditor's test procedure.

The first of these probabilities,  $p(i|\sim e)$  is assumed to be one, since ideally the internal control system will correct, but not introduce, errors. That is, a materially correct balance will not be altered materially. The second probability,  $p(a|\sim e)$  is equal to  $1 - p(\sim a|\sim e)$ , where  $p(\sim a|\sim e)$  represents the probability of the auditor's test procedure not accepting (i.e. rejecting) a correct balance. In hypothesis testing terminology,  $p(\sim a|\sim e)$  is the probability of a type I error (i.e., rejection of a materially correct balance). It is seen, therefore, that the advocated conditional risk measure,  $p(e|i,a)$ , takes account implicitly of both the type I and type II error probabilities of the auditor's test procedure.

The AUP24 risk measure builds in consideration of  $p(a|e)$ , the type II (beta) error probability (see expression (2)), but not the other of the two operating characteristics of the auditor's test procedure,  $p(\sim a|\sim e)$ . Although this widely

accepted measure may often yield a figure for audit risk fairly close to the conditional probability  $p(e|i,a)$ , it will sometimes understate that more appropriate measure by a factor of four or more. This will tend to be the case when inherent risk  $p(e)$  is high and/or the type I error probability,  $p(\sim a|\sim e)$ , is high (see Figure 2 below).

To demonstrate the possible divergence between the conditional or 'Bayesian' risk measure,  $p(e|i,a)$ , and the AUP24 construct  $p(e,i,a)$ , consider the example calculations provided in AUP24 (Appendix IV, para. 8). Here, it is taken for the purpose of exposition that inherent risk  $p(e)$  equals .75, control risk  $p(i|e)$  equals .3 and detection risk  $p(a|e)$  equals .22. These assumed values lead to an AUP24 audit risk measure of  $p(e,i,a)=.75 \times .3 \times .22 = .05$ . Now consider the more relevant, conditional risk measure  $p(e|i,a)$ , which equals, from (3)

$$.05 / \{.05 + (1-p(e)) (1-p(\sim a|\sim e))\} = .05 / \{.3 - .25 p(\sim a|\sim e)\}.$$

Values of this function of the type I error probability,  $p(\sim a|\sim e)$ , are shown in Figure 1.

#### FIGURE 1 ABOUT HERE

Note, for example, that for a type I error probability of .4, the conditional audit risk  $p(e|i,a)$  equals  $.05 / \{.3 - (.25)(.4)\} = .25$  or 25%, which is five times the AUP24 measure. At the very least, when  $p(\sim a|\sim e) = 0$  (see section 6 below),  $p(e|i,a) = .05 / .3 = .17$  is more than three times the conventional measure. In the extreme, the type I error probability  $p(\sim a|\sim e)$  equals one and  $p(e|i,a)$  is  $.05 / .05 = 1$  or 100%, regardless of the value of inherent risk  $p(e)$ . These disparities demonstrate clearly the importance of considering both type I and type II error probabilities, rather than the latter only as per AUP24. Figure 2 shows a contour plot or "topographical map" of the conditional risk  $p(e|i,a)$



as a function of both inherent risk,  $p(e)$ , and type I error probability,  $p(\sim a | \sim e)$ . The contours ("iso-risks") shown have values .05, .1, .2, .3, .4, .... .8, .9, going from left to right (or from light to dark). For example, it can be seen from Figure 2 that for  $p(e)=.6$  and  $p(\sim a | \sim e)=.8$ ,  $p(e | i, a)$  equals about .3 or 30%.

## FIGURE 2 ABOUT HERE

### 5. Conclusion

The auditors assurance or level of confidence in an audited account balance is properly represented by the probability of that balance being correct conditional on it having been subject first to internal control procedures and then to substantive testing. Alternatively, the complement of audit assurance, audit risk (AR), is the probability of an error in the audited balance given the internal control procedures and tests carried out by the auditors. This probability is written mathematically as  $p(e | i, a)$ , and is a function of both the type I and type II error probabilities of the auditor's substantive test procedure.<sup>1</sup> By comparison, the AUP24 measure of audit risk is the joint probability  $p(e, i, a)$ , which although related to  $p(e | i, a)$  through the laws of probability, is not the same as that probability, and is of little interest *per se*.

If auditors require a logically and mathematically valid quantitative assessment of audit risk, the AUP24 model must be extended to measure the conditional probability  $p(e | i, a)$  rather than merely its mathematical 'component', the joint probability  $p(e, i, a)$ . Without this revision, auditors' reliance on the AUP24 model is not well based in probability theory. From a logical perspective, the AUP24 risk measure is useful only in that it is calculated as one step toward finding what is really required, namely a measure of confidence in the reported balance conditional on the internal control procedures and substantive tests (statistical or otherwise) which that balance has withstood beforehand.

## 6. *Postscript: The Canadian (CICA) Model*

In a research study of the *Canadian Institute of Chartered Accountants* (1980, p.97) 'overall audit risk' is specified mathematically as

$$\frac{\text{joint risk}}{\text{joint risk} + \text{inherent confidence}}$$

where 'joint risk' refers to the AUP24 audit risk measure - i.e. the joint probability  $p(e,i,a)$  - and 'inherent confidence' is the probability,  $p(\sim e)=1-p(e)$ , of no inherent error in the balance tested. This model is Bayesian, and is as advocated above, except for its implicit assumption that the type I error probability is zero. To see this assumption, note from (3) that the CICA model presumes that  $p(\sim e,i,a)=p(\sim e)$ , thus requiring that both  $p(i|\sim e)$  and  $p(a|\sim e)$  equal one, and by implication, that the type I error probability  $p(\sim a|\sim e)$  equals zero.

The basis for the assumption of a zero type I error probability is that in practice a materially correct balance which is initially rejected by audit testing will ultimately, perhaps after further sampling, be accepted without material alteration. On this assumption, type I errors occur but do not endure, and are therefore merely 'efficiency' errors, having only the legacy of the costs incurred in their revelation.

### Footnotes

1. Further discussion on this point, although in the context of interpreting test results in empirical research rather than in auditing, can be found in Burgstahler (1987). A more technical paper, Johnstone and Lindley (1994), extends this discussion. Similarly, on the Bayesian probabilistic interpretation of test results in audit sampling, see Johnstone (1994).

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## ***Headings for Figures***

**Figure 1:** Bayesian Audit Risk Given Inherent Risk of .75

**Figure 2:** Contour Plot of Bayesian Audit Risk as a Function of Inherent Risk and the Type I Error Probability of the Auditor's Test Procedure.

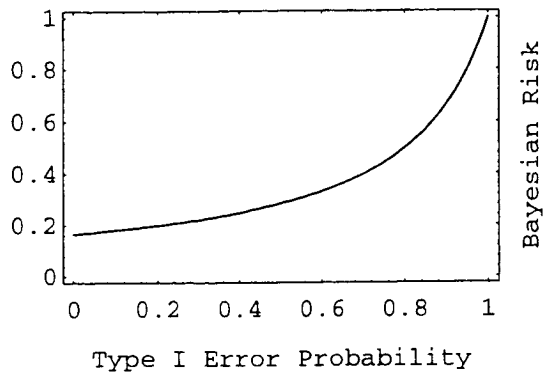


Figure 1

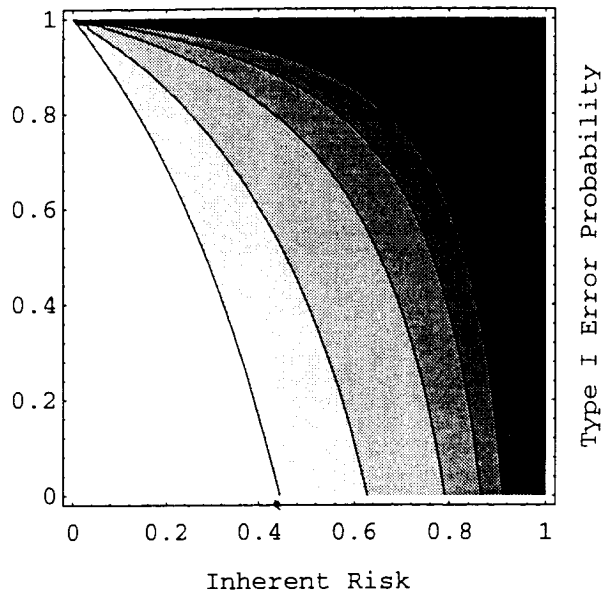


Figure 2