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The Potential For Corporate Immunisation Of Commercial Projects

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THE POTENTIAL FOR CORPORATE IMMUNISATION OF COMMERCIAL PROJECTS

by

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The Potential for Corporate Hedging of Commercial Projects

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Abstract

Firms are continually investing resources in risky projects which involve uncertain outcomes. The need for firms to protect the net asset backing of their project portfolios and to immunise against unacceptable cash flow streams is evident in a number of contemporary practices such as factoring, sub-leasing and joint ventures. But the ad hoc farming out of projects does not provide a means of systemically deriving optimal strategies which provide adequate protection at minimum cost. The options based hedging model used here illustrates why firms use factoring, joint-ventures and similar strategies as a form of risk sharing and shows how optimal risk sharing strategies can be derived and manipulated over a project's life in response changes in key parameter values to maintain adequate protection.

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The Potential for Corporate Hedging of Commercial Projects

1. Introduction

Uncertain outcomes make decision making difficult. Whilst a firm would not wish to forgo a potentially profitable project - one which adds significantly to its profitability and strengthens its balance sheet position - they may wish to protect themselves against the downside risks attached to cash flow volatility. This is especially so when firms have a disproportionate share of their asset portfolios invested in one project, since in this instance, an unfavourable outcome may even threaten the firm's very existence. Such risks may be reduced if the firm, after deciding on a minimum acceptable expected cash flow from the project, could then insure against receiving less than this minimum amount.

Attempts to insure against the undesirable outcomes attached to risky projects are already evident in the commercial world. Obvious examples are various factoring practices such as joint ventures and the "farming out" procedures used in the oil and mining industry. Under joint ventures for example, two or more firms may agree to fund a project. The firms then participate in the outcomes in some agreed proportion, whether they be positive or negative. In this way, no one firm contributes all the required capital nor bears the full costs should the project prove to be unprofitable. Each firm gains a means of limiting its exposure in terms of funds committed to the project and its liability should the project prove unprofitable. Hence, such procedures permit participation in projects which might be beyond the resources or risk exposure of any one firm.

Similar considerations apply to the factoring procedures utilised in the oil exploration industry. In this instance, the holder of an exploration permit agrees to apportion a share of the proceeds from a productive well to a second party, if that party bears some or all of the costs associated with drilling the well. Benefits and costs are then shared in accordance with the drilling or "farming out" agreement. Again, the result is to limit both firms' exposure in terms of funding or liability for costs.

However, insurance arrangements such as these may be far from optimal. For example, they may provide too much or too little protection against unfavourable outcomes. Or where the protection is adequate, it may have been obtained by sacrificing a greater equity in the project than is necessary. This paper derives a project immunisation model which formalises the process of providing the desired protection. Section two introduces the process of forming a hedge security under a single period binomial model. This is then expanded in section three to a multi-period continuous cash flow context. Section four discusses the potential usefulness of the model in hedging situations, Productive extensions of the analysis are summarised in section five.

The analysis presented here is an extension of an options pricing approach to the valuation of projects with uncertain outcomes. Valuation methods for such derivative securities are well established in financial markets [Cox and Rubenstein 1985] and are now being applied in a variety of other contexts, including capital budgeting and projects with operational options (e.g., Winsen 1994, Brenan and Schwarz, 1985, 1985b, Trigeorgis, 1988, Sick 1989). They are often theoretically superior to the traditional NPV approach to project valuation. The deficiencies of NPV analysis in evaluating commercial projects with a range of sequential future options in situations where outcomes are uncertain and dynamic are well documented in the literature (e.g., Kulatilaka and Marcus 1992). In addition, after project commencement, the approach does not provide a means of deriving insurance strategies to immunise firms against adverse outcomes through the "farming out" procedures typically used in commercial practice.

The focus of the analysis presented here is quite specific. The paper presents a model for deriving optimal immunisation strategies under the current risk sharing arrangements mentioned above. This approach extends project decision analysis to incorporate insurance with the added advantage of avoiding the arbitrary specification of decision tree probabilities and unique discount rates by using risk free discount rates and (in the discrete context at least) by working with specified levels of minimum desired cash flow rather than risky outcome probabilities [1].

In a project immunisation context, the problem may be stated as follows. A firm wishes to invest in a project where, at the time of investment, the distribution of the future cash flows is uncertain. The firm is faced with two options. Firstly, it may finance the entire project from its own resources. If it pursues this option, it will be completely exposed to the possibility of undesirably low (perhaps even negative) cash flows. Alternatively, the firm may immunise against the risk of actual cash flows falling below a minimum level. This can be achieved by constructing a hedge security which guarantees a minimum return, should the project's realised cash flow fall below a prescribed amount.

The process of forming the hedge security referred to above effectively involves the creation of a put option. As a consequence, the Black and Scholes (1973) analysis may appear to be a logical starting point[2]. However, the principal assumption of the Black-Schools model is that the underlying security (in our case the project's cash flows), follows a **geometric** Brownian motion[3]. This means that project cash flows could never be negative. To overcome this, we effectively assume that project cash flows follow an **additive** Brownian motion, which does admit the possibility of negative outcomes[4].

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time t - h, should the project's cash flow be $A + \delta$ or $A - \delta$ respectively. Hence, if the project's time t - h cash flow should exceed expectations, we have:

$$W_u = \begin{cases} 0 & \text{if } (A + \delta) \geq E \\ E - (A + \delta) & \text{if } (A + \delta) < E \end{cases}$$

Alternatively, if the project's time (t - h) cash flow should fall below expectations, we have:

$$W_d = \begin{cases} 0 & \text{if } (A - \delta) \geq E \\ E - (A - \delta) & \text{if } (A - \delta) < E \end{cases}$$

The initial problem is to find the values for the proportionate investment in the project (Δ) and the dollar amount to be invested in government bonds (B) which will result in the necessary values for W_u and W_d. This, given any realised cash flow and stated minimum return which the firm is just willing to accept from the project.

To do this, we solve the equations in (1) for Δ and B as follows:

$$\Delta = \frac{W_u - W_d}{2\delta}$$

(2a)

$$B = \frac{\frac{(\delta - \Lambda) W_u}{2\delta} + \frac{(\delta + \Lambda) W_d}{2\delta}}{r^h}$$

(2b)

Using these results we can compute the value W, of the hedge security at time t, as follows:

$$W = \Delta A + B$$

$$W = \frac{\frac{[(r^h - 1)A + \delta]W_u}{2\delta} + \frac{[(r^h - 1)]A - \delta]W_d}{2\delta}}{r^h} \qquad (3)$$

As an example, suppose A = 1 and that this is the firm's initial capital. We further suppose that $\delta = 0.20$, h = 1 and the pure annual rate of interest is (r - 1) = 0.125. If we want to at least guarantee return of the firm's initial capital, we have that E = 1. To form the hedge security which guarantees this result we first note that the above information implies W_u = 0 and W_d = 0.20. Substituting these values in equation (3), it then follows that Δ = -0.5 and B = 24/45 ≈ \$0.533333. In other words, we invest approximately \$0.53 in government bonds and short the project to the extent of \$0.50. The payoff schedule associated with this scenario is as follows:

	Time t	Time t - h	
		$[A_{t-h} = A_t - \delta]$	$[A_{t-h} = A_t + \delta]$
Sell short 50% share in project	0.500000	-0.40	-0.60
Buy bonds	-0.533333	0.60	0.60
Cost (-) / Payoff (+)	-0.033333	0.20	0.00

If we create the above hedge security and go long in the underlying project, we obtain the following cost and payoff schedule:

	Time t	Time t - h	
		$[A_{t-h} = A_t - \delta]$	$[A_{t-h} = A_t + \delta]$
Purchase 50% share in project	-0.500000	0.40	0.60
Buy bonds	-0.533333	0.60	0.60
Cost (-) / Payoff (+)	-1.033333	1.00	1.20

Note that this asset portfolio has the required property - namely that the firm's end of period or time t - h capital, will be at least equal to its initial or time t capital.

A significant difficulty however, is that the required initial investment of \$1.033333 exceeds the firm's endowed capital of \$1. This problem can be overcome by a partial liquidation of the hedge security, the proceeds being invested in government bonds. We thus define the scaling factor g, such that:

$$g(W + A) + D = 1 \tag{4}$$

where g(W + A) is the firm's combined investment in the hedge security and the underlying project, and D is the required extra investment in government bonds. Note that this equation imposes the requirement that the firm cannot invest more than its initial capital of \$1. However, since we want the time t - h proceeds from this investment portfolio to be at least equal to the firm's initial capital, we impose the further condition:

$$g + r^h D = 1 \tag{5}$$

This condition applies since, in the very least, the firm's investment portfolio must return the firm's opening capital of \$1, at the end of the period. Note that the firm's combined investment in the hedge security and the underlying project g(W + A) will return, as a minimum g, at time t - h. Similarly, the investment in government bonds will have accumulated to r^hD, with the effect of interest by time t - h. Hence, we can determine the parameters necessary to insure that the initial investment will not exceed the firm's endowed capital, by solving equations (4) and (5) as follows:

$$g = \frac{1 - r^{-h}}{(W + \Lambda) - r^{-h}}$$

$$D = \frac{(W + \Lambda) - 1}{(W + \Lambda) r^{h} - 1} \tag{6}$$

For the example presently under consideration, W = 0.033333. A = 1. r = 1.125 and h = 1, so that from the equations (6), we have g = 10/13 whilst D = 8/39. Previously, the firm's asset portfolio was made up of a 50% share in the project and a \$24/45 ≈ \$0.533333 investment in government bonds. Hence, the new or "scaled" portfolio is composed of a 0.5*g = 5/13 ≈ 0.384615 share in the project, and a 24/45*g + 8/39 = \$8/13 ≈ \$0.615384 investment in government bonds. This portfolio has the following cost and payoff characteristics:

	Time t	Time t - h	
		$[A_{t-h} = A_t - \delta]$	$[A_{t-h} = A_t + \delta]$
Purchase 5/13 share in project	-0.384615	0.307692	0.461538
Buy bonds	-0.615385	0.692398	0.692308
Cost (-) / Payoff (+) 1.153846		-1.000000	1.000000

Note that the firm's investment portfolio is now self financing, but at the cost of a lower cash flow when the project's returns are higher than expected.

3. The Multi period Case[5]

The above analysis may be easily extended to a discrete time analysis of multiple periods. But in the multi-period context it is more realistic to assume that project cash flows accrue continuously in time. To generalise the analysis in this way, it is merely necessary to restate equation (3) in simpler form and use Taylor series approximations for certain key expressions. These substitutions then enable us to derive expressions for forming a hedge security in a continuous time context. (see appendix 1 for details of the derivation).

As in the case of single period immunisation, the firm's asset portfolio consists of the hedge security and a long investment in the underlying project. The sum result of the above analysis is that in the multiperiod, continuous time case, the hedge security is formed, at any time t, by investing

$$e^{-it}[EN(d) + \sigma\sqrt{\frac{e^{2it}-1}{4\pi i}}\, e^{-\frac{1}{2}d^2}] \qquad (7)$$

in government bonds and selling short the proportionate investment N(d), in the underlying project. The firm's investment in the underlying project is thus equal to AN(-d).

As an example, suppose a multi-period project of 5 periods duration (t=5) at time of proposed investment. As in the case of the binomial model, A = 1, and that this is the firm's initial capital. We further suppose that σ = 0.20, whilst the risk free (continuously compounded) annual rate of interest is i = 11.78%. If we want to at least guarantee return of the firm's initial capital, we have that E = 1. From the appendix (equation 17), this information implies d =1.30. Using equation (7) we then have at the beginning of the period, that the firm's asset portfolio consists of a $0.112 investment in government bonds and a long proportionate investment of 90.30% in the underlying project.

As with the discrete example, the total cost of this asset portfolio, which is 0.9030 + 0.112 = $1.015, exceeds the firm's initial capital. However, by rebalancing the portfolio in the same manner as the discrete case, this problem is easily overcome.

4. Discussion

Perhaps the major attribute of this analytical approach to project immunisation is the insight it can give participants or corporate financial controllers into both systematic and behavioural aspects of risk exposure inherent in their investment decisions. The model emphasises the need for prior decisions about management's risk/return preferences and the minimum cash flow outcomes for a project that can be tolerated by the firm in the context of either overall financial position or its total portfolio of projects.

Once lower bounds to cash flows are established, the model will provide a benchmark for calculating the minimum proportion of project participation which needs to be foregone in order to achieve immunisation against less than desired cash flows. After project factoring and commencement, the model can be used at any time during the project's life to calculate the effects of changes in parameter values such as interest rates fluctuations, increased knowledge about cash flow expectations and cash flow variances. The model represents a powerful tool for gaining insights into the general behaviour of immunisation functions over time and in relation to changes in parameter values.

The behaviour of these parameter functions determines the initial optimal hedge security profiles and how those profiles change in response to changing parameter values, including interest rates, standard deviations of cash flows and desired immunisation levels.

(TABLES 1, 2 AND 3 ABOUT HERE)

The major determinant of the proportion of project participation which must be foregone is the level of immunisation required. Table 1 shows the maximum percentage participation rates in a project and the minimum dollar investment in bonds which must be made to hedge against less than indicated minimum cash flows. The minimums are expressed as a percentage of original outlay. The fall in participation rates and the simultaneous rise in bond investment which accompany rising levels of cash flow immunisation are illustrated in Figure 1

The level of volatility of the cash flows attached to a project are also a major determinant of the participatyion in a project which must be foregone to provide particular levels of protection against less than desirable cash flow outcomes. Increases in the expected volatility of cash flows (standard deviation) causes a rapid decrease in project participation rates at any desired level of immunisation decrease rapidly as the expected volatility of cash flows (standard deviation) increases cash flow volatility standard deviation of expected cash flows increases (Table 2). Interest rate increases (ceteris paribus) also increase project participation rates in the hedge security (Table 1). A counter intuitive result since interest rate increases represent higher bond returns but higher borrowing costs for project funds. Any increase in immunisation levels involves decreasing equity participation levels (Table 3). In the above five period project, immunisation of initial outlays (E=1) involves forgoing 13 percent of participation (Figure 1). This foregone proportion rises to 27 percent for 130 percent immunisation (E=1.3).

Significantly, should the desired minimum cash flow insurance level change at any time during the course of the project, the model can be used to re-calculate the optimal hedge security (Figure 2). This facility preserves the advantages over more informal approaches (and NPV analysis) where hedges are unlikely to be set initially at optimal, cost-benefit levels, unlikely to be rebalanced for future operational option choice, and unlikely to be altered for changes in parameter values (eg interest rates) or desired insurance levels over time.

(FIGURES 1 AND 2 ABOUT HERE)

Hedge project profiles are affected by inter-active effects between parameter values as well as changes in individual parameters. Thus increased knowledge about expected levels of net cash flows as the project progresses may act to reduce variances and increase project participation rates. The sum of such inter-active effects will vary between firms and projects but can be easily handled in the model.

Despite the practical possibility of less than optimal immunisation due to market friction, the analysis has the advantage of indicating the remaining exposure to unfavourable movements in project cash flows after immunisation. In other words, the project immunisation model provides a cardinal measure of how the composition of the firm's asset base varies from what it ought to be.

The objective of the analytical model presented here is to create a hedge security which offers the desired level of immunisation at minimum opportunity cost, in terms of foregone project participation. The degree of profitable participation which must be forgone in the hedge security to obtain the required level of immunisation is significant to the relative attractiveness of immunisation. The opportunity costs of immunisation are the income premiums in excess of the risk free rate which attach to the forgone project equity. The level of the income premiums forgone in any particular case are determined by the parameter values for of interest rates level, project duration length, required level of immunisation, and expected level and variance of net cash flow s over time.

The technique may be particularly valuable where a firm has not one, but a number of projects making up its project portfolio. Where the firm has multiple projects, the analysis requires consideration of the covariance of returns between projects as well as the performance of individual projects. The problem is to accumulate projects which will yield an investment portfolio with the desired overall risk characteristics and cash flow requirements. To do this, the firm needs to be able to continually redefine the risk profiles of individual existing projects as compared with the desired risk profile of the total project portfolio. Over time, information on the realised risk characteristics and cash flow streams of existing projects will become known. Then, as new investment opportunities become available, the firm can select those projects which will re-balance the firm's risk and cash flow profile in the sense of moving it towards the desired base.

5. Summary and Extensions

The project immunisation model presented here has several advantages over current ad hoc attempts to insure against unacceptably low returns from risky projects. The model derives the least cost hedging solution. It indicates the the minimum project equity which must be foregone to provide desired protection against the possibility of cash flows falling below a minimum guaranteed return. The model applies equally to a firm wishing to ensure a minimum guaranteed return from only one project, or to a firm with a portfolio of projects which seeks to ensure a minimum net asset base, or a minimum cash flow stream over any period of time.

Secondly, the model emphasises the need for a firm to explicitly define the desired risk profile and minimum cash flow requirements for its project portfolio on an on-going basis. These parameters are required for the initial immunisation analysis and are the benchmark for expectation revisions and immunisation strategy changes as additional information on the variance and value of cash flows received from any project become available.

Through sensitivity analysis on changes in underlying parameter values, the model also provides a means of adjusting immunisation strategy adjustments to provide desired levels of protection while retaining maximum equity participation in risky projects.

TABLE 1

HEDGE PROFILES FOR ALTERNATIVE IMMUNISATION LEVELS
(Cash Flows as a percentage of original outlays)

Minimum CF (%)	1.00	1.05	1.10	1.15	1.20	1.25	1.30	1.35
Hedge								
Project equity(%)	0.87	0.85	0.83	0.81	0.78	0.76	0.73	.70
Bonds invest ($)	0.13	0.15	0.17	0.19	0.22	0.24	0.27	.30

SENSITIVITY ANALYSIS*

TABLE 2

HEDGE PROFILES FOR VARYING INTEREST RATE LEVELS (5-15%)

Interest Rate(%)	0.05	0.07	0.09	0.11	0.13	0.15	0.17
Hedge							
Project Equity(%	0.54	0.67	0.78	0.85	0.90	0.94	0.96
Bonds Invest($)	0.46	0.33	0.22	0.15	0.10	0.06	0.04

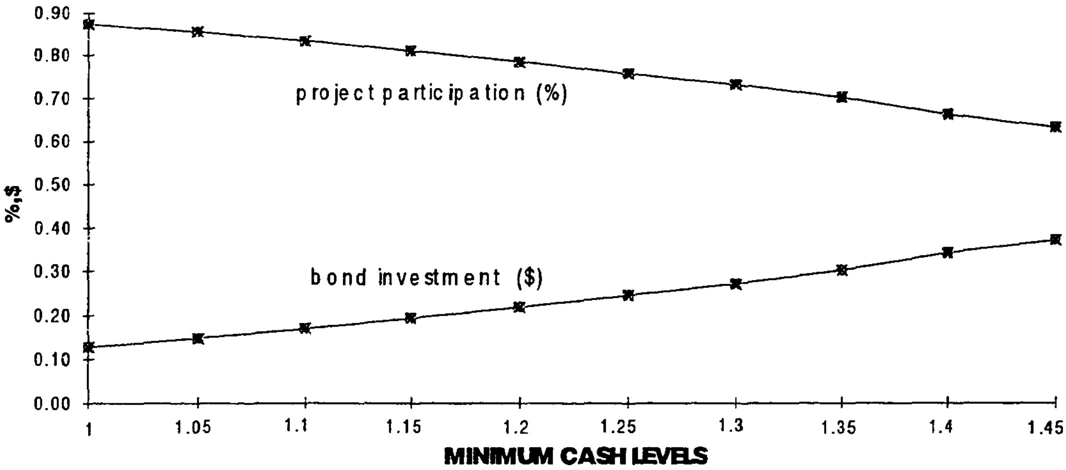
TABLE 3

HEDGE PROFILES FOR CASH FLOW VOLATILITY LEVELS
(Standard Deviation)

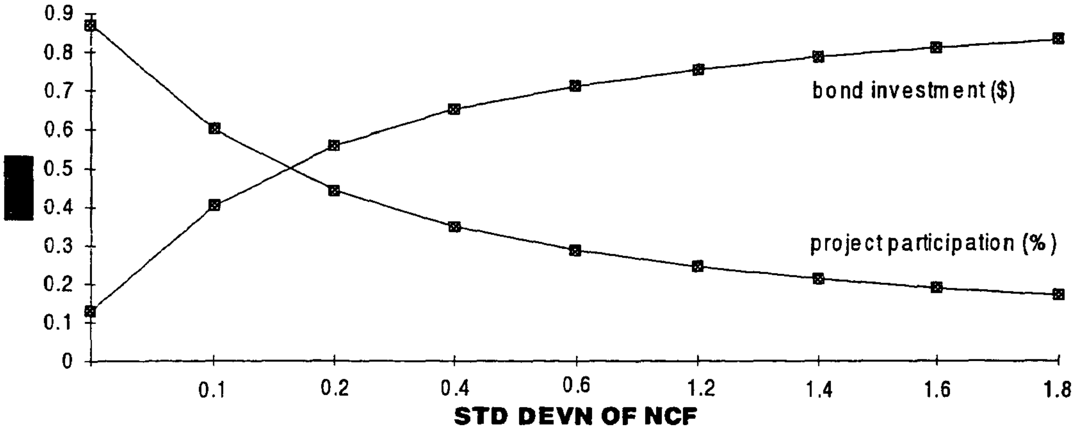
Volatility (s.d.)	0.10	0.15	0.20	0.25	0.30	0.40	0.50
Hedge							
Project Equity(%	0.99	0.95	0.72	0.79	0.72	0.60	0.51
Bonds Invest ($	0.01	0.05	0.28	0.21	0.28	0.40	0.49

* All tables based on multi-period case parameters: R=0.1178, SD_A=0.2, A=1, E=1, T=5

GRAPH 1 MINIMUM CASH LEVELS



GRAPH 2 HEDGE PROFILES FOR NCF STD DEVIATION LEVELS



Mathematical Appendix 1

As mentioned, the formation of a hedge security in a continuous time context requires restatement of equation (3) and the use Taylor series approximations for certain key expressions. Hence, restating equation (3), we have that:

$$\frac{[\Lambda(r^h-1)+\delta]W(\Lambda+\delta,t-h)}{2\delta}+\frac{[\Lambda(r^h-1)-\delta]W(\Lambda-\delta,t-h)}{2\delta}-r^h W(\Lambda,t)=0 \qquad (7)$$

where:

$W(A + \delta, t - h) = W_u$, $W(A - \delta, t - h) = W_d$ and $W(A, t) = W$.

If we expand $W(A + \delta, t - h)$ as a Taylor series about the point (A, t) we have (Apostol 1969, pp. 308-309):

$$W(A+\delta,t-h) \approx W(A,t) + \delta\frac{\partial W}{\partial \Lambda} + \frac{1}{2}\delta^2\frac{\partial^2 W}{\partial \Lambda^2} - h\frac{\partial W}{\partial t} + \text{-----} \qquad (8)$$

Similarly, for $W(A - \delta, t - h)$ we have:

$$W(A-\delta,t-h) \approx W(A,t) - \delta\frac{\partial W}{\partial \Lambda} + \frac{1}{2}\delta^2\frac{\partial^2 W}{\partial \Lambda^2} - h\frac{\partial W}{\partial t} + \text{-----}$$

(9)

Finally, expanding r^h as a Taylor series about the origin, we have (Apostol 1967, pp. 278-279):

$$r^h = 1 + h \log r + \text{------------}$$

(10)

Following Cox and Miller (1964, p. 206) we let $\delta = \sigma\sqrt{h}$, where σ^2 is the variance rate in the project's cash flows. Using this assumption in conjunction with equations (7) through (10), dividing by h and then letting h -----> 0, we have that the multi period version of the binomial model (3), satisfies the following partial differential equation:

$$\frac{1}{2}\sigma^2\frac{\partial^2 W}{\partial \Lambda^2} + i\Lambda\frac{\partial W}{\partial \Lambda} - \frac{\partial W}{\partial t} - i\,W(A, t) = 0 \tag{11}$$

(where i = log r is the continuously compounded annual risk free rate of interest.)

Further, we have the <u>initial</u> condition:

$$W(A,0) = \begin{cases} 0 & \text{if } A \geq E \\ E - A & \text{if } A < E \end{cases} \tag{12}$$

The requirement is now to find a unique solution to equation (11) which satisfies the above initial condition (12). To convert equation (11) to solvable form, we make the following substitutions:

$$W(A, t) = e^{-it}F(\xi,\eta)$$

where:

$$\xi = \frac{\sqrt{2}[E - \Lambda e^{it}]}{\sigma} \tag{13}$$

and:

$$\eta = \frac{e^{2it} - 1}{2i}$$

thus reducing equation (11) to the diffusion equation of mathematical physics [Crank (1975, p 11)]:

$$\frac{\partial^2 F}{\partial \xi^2} = \frac{\partial F}{\partial \eta} \tag{14}$$

Further, since:

$$\xi(A,0) = \frac{\sqrt{2}[E - \Lambda]}{\sigma}$$

the initial condition becomes:

$$F(\xi,0) = \begin{cases} \frac{\sigma}{\sqrt{2}}\xi & \text{if } \xi \geq 0 \\ 0 & \text{if } \xi < 0 \end{cases} \tag{15}$$

The solution to the partial differential equation (14) is (Weinberger 1965, pp. 320, 328):

$$F(\xi,0) = \frac{1}{\sqrt{4\pi\eta}} \int_0^\infty F(y) \exp[-\frac{(\xi - y)^2}{4\eta}] dy$$

If we make the substitution:

$$y = \xi + z\sqrt{2\eta}$$

and use the initial condition (15), we obtain the following unique solution to the problem (11), (12):

$$W(A, t) = [Ee^{-it} - A]N(d) + \sigma\sqrt{\frac{1 - e^{-2it}}{4\pi i}} e^{-\frac{1}{2}d^2} \tag{16}$$

where:

$$d = \frac{Ae^{it} - E}{\sigma\sqrt{\frac{e^{2it} - 1}{2i}}} \tag{17}$$

and:

$$N(d) = \frac{1}{\sqrt{2\pi}} \int_d^\infty e^{-\frac{1}{2}y^2} dy \tag{18}$$

is the value of the complementary accumulated standard normal distribution.

The sum result of the above analysis is that in continuous time, the hedge security is formed by investing

$$e^{-it}[EN(d) + \sigma\sqrt{\frac{e^{2it} - 1}{4\pi i}} e^{-\frac{1}{2}d^2}] \tag{19}$$

in government bonds and selling short the proportionate investment N(d), in the underlying project.

Endnotes

1. Even in the continuous case, the probability density specifications may be made to refer to the minimum level concept rather than to the probability density function of the whole cash flows.

2. See Bird and Tippett [1986] for a simple introduction to portfolio insurance, as it affects equity portfolios. Benninga and Blume (1985) contains a more advanced and detailed treatment.

3. See Cox, Ross and Rubenstein [1979] for a very clear exposition of the assumptions underlying the Black-Scholes model.

4. See Cox and Miller [1964, pp. 205-208] for a good introduction to this topic.

5. The method of proof used in this section (as distinct from the proof itself), was first suggested by Cox, Ross and Rubenstein [1979, p. 254] and amplified on in Cox and Rubenstein [1985, pp. 208-209].

6. The disadvantage of mathematical complexity for practical application is more apparent than real. The model, once developed, can be programmed into a spread sheet in less than half an hour.

References

Apostol, T., 1967, *Calculus*, Volume 1 (Xerox College Publishing Waltham, Massachusetts).

Apostol, T., 1969, *Calculus*, Volume 2 (Xerox College Publishing Waltham, Massachusetts).

Benninga, S. and M. Blume., 1985, On the optimality of portfolio insurance, Journal of Finance, XL, 5. 1341-1352.

Bird, R. and M. Tippett., 1986, Portfolio insurance: What it is and how is it implemented? JASSA, September, 3-5.

Black, F. and M. Scholes., 1973, The pricing of options and corporate liabilities", Journal of Political Economy, 81, 637-654.

Brennan, M. and E.Schwarz, 1985, Evaluating natural resource investments, Journal of Business, 58, 2, 135-157.

Brennan, M. and E.Schwarz, 1985b, A new approach to evaluating natural resource investments, Midland Corporate Finance Journal, 3/1, 37-47.

Cox, J., S. Ross and M, Rubenstein., 1979, Option pricing: A simplified approach", Journal of Financial Economics, 7, 3, 229-263.

Cox, J. and M. Rubenstein., 1985, Options markets, (Prentice-hall, Englewood Cliffs, New Jersey).

Cox, D. and H. Miller., 1964, The Theory of stochastic processes, (Chapman and Hall, London).

Crank, J., 1975, The Mathematics of diffusion, (Clarendon Press, Oxford).

Dixit, A. and R. Pindyck, 1994, Investment Under Uncertainty, (Princeton University Press, New Jersey).

Kulatilkaka, N. and A. Marcus, 1992 Project valuation under uncertainty: When does DCF fail? Journal of Applied Corporate Finance, 5, 3, 92-100.

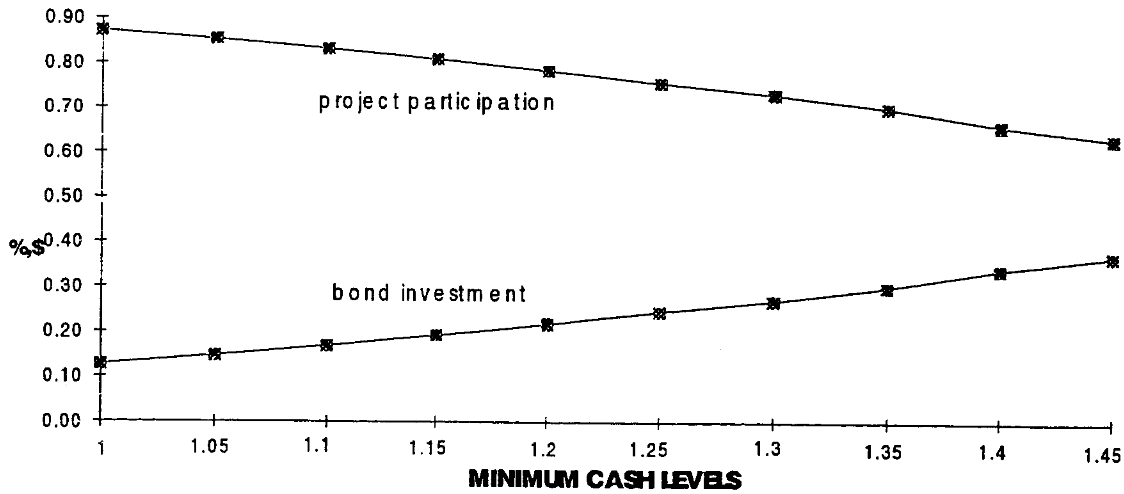
Sick, G., 1989, Capital budgeting with real options, (Monograph 3, Salomon Brothers Center, NYU)

Trigeorgis, L., 1988, A Conceptual options framework for capital budgeting, Advances in futures and options research, 3, 145-167.

Weinberger, H., 1965, Partial differential equations, Lexington, (Xerox College Publishing, Massachusetts).

Winsen, J., 1994 Project NPV as a portfolio of derivative securities: A discrete time analysis, Paper presented to 7th Australasian Finance and Banking Conference, University of New South Wales, December.

FIGURE 1 MINIMUM CASH LEVELS



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