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# Train schedule coordination at interchange station in an open railway market through agent negotiation

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## **Abstract**

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## **Keywords**

agent, interchange, schedule, coordination, market, station, railway, open, train, negotiation

## **Disciplines**

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# Train Schedule Coordination at an Interchange Station Through Agent Negotiation

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## Abstract

In open railway markets, coordinating train schedules at an interchange station requires negotiation between two independent train operating companies to resolve their operational conflicts. This paper models the stakeholders as software agents and proposes an agent negotiation model to study their interaction. Three negotiation strategies have been devised to represent the possible objectives of the stakeholders, and they determine the behavior in proposing offers to the proponent. Empirical simulation results confirm that the use of the proposed negotiation strategies lead to outcomes that are consistent with the objectives of the stakeholders.

*Key words:* agent negotiation; open railway markets; schedule coordination problem

## 1. Introduction

Modern railways have been embracing new opportunities and challenges ever since the introduction of open access. In open railway markets, the responsibilities of infrastructure provision and train operation are distributed to independent stakeholders. This has led to an infrastructure manager (IM) selling track capacity to a group of competing train operating companies (TOCs). By restructuring the conventional railway markets through disintegration (hence enabling competition), regulatory agencies anticipate improvement on the operational efficiency in their railway markets so that rail transportation is more responsive to market demands.

One approach to achieve the above objective is to promote seamless services. The availability of a direct transportation from source to destination is essential to compete with the door-to-door and

just-in-time services offered by road transportation. Consequently, removing barriers for seamless services becomes a key issue, especially in Australia and European countries where trains travel across jurisdictional boundaries. The attention on interoperability between railway markets can be realized from the National Competition Policy (of Australia) (BTRE, 2003) and the European Rail Directive 91/440/EEC (EC, 2006). However, barriers with respect to technical, corporate, jurisdictional and cultural interoperability have been identified as major impediments in promoting seamless services (Mulley and Nelson, 1997). While providing solutions for these barriers is a long-term process, the availability of coordinated train services between different TOCs facilitates the transportation across regions. In addition, even when seamless services are available, coordinated services can still compete with seamless services by providing an alternative choice for consumers.

As passengers are often discouraged by excessive waiting time during transit, schedule coordination mainly aims to reduce the passenger waiting time at interchange stations. This problem is not novel in railways, and it has been extensively modeled and examined in conventional railway markets. Minimization of waiting time is usually obtained by adjusting the commencement time of two services so that headways and traveling times are preserved to avoid degrading the quality of service of individual lines (Brucker et al., 1990; Burkard, 1986; Nachtigall, 1996; Nachtigall and Voget, 1996). In these studies, when coordinating schedules at a single station, the arrival times of a line at the station have been modeled by a set of vertices of a polygon within a unit circle (Brucker et al., 1990; Burkard, 1986). The problem is then to minimize the total arc lengths between the vertices on the circumference of the circle. On the other hand, when coordinating a set of trains at multiple interchange stations, the problem has been shown to be NP-hard (a problem yet to be solved deterministically in polynomial time), and it has been solved using a branch-and-bound algorithm (Nachtigall, 1996) or a genetic algorithm (Nachtigall and Voget, 1996).

Despite the effort of coordinating schedules in conventional markets, the introduction of open access has altered the nature of the problem. Firstly, railway lines are now operated by multiple

TOCs instead of a single authority. As a result, the alignment of schedules requires a mutual agreement from more than one party, whose operating constraints may be in conflict with those of the others. In addition, sensitive data, such as cost rates, are unlikely to be revealed, which means decisions are often made without complete information. Moreover, instead of generating a single solution (i.e. the optimal solution), the operators are now required to generate a set of offers for the negotiation process. Remodeling of the schedule coordination problem is thus essential to capture these new characteristics as a result of open access.

Multi-agent systems are particularly suitable for representing distributed problems as systems of software agents that are capable of social-like interactions such as negotiation (Jennings and Bussmann, 2003). Agent modeling has found many applications in transportation systems (Böcker et al., 2001; Teodorović, 2003; Tsang and Ho, 2004, 2006a, b, 2008; Zhang et al. 2004). In open railway markets, a multi-agent system was proposed to capture the distributed nature and negotiation behavior (Tsang and Ho, 2006a). Further, the negotiation between an IM and a single TOC for track access rights allocation was modeled and examined in details (Tsang and Ho, 2004, 2006a, 2008). A preliminary study (Tsang and Ho, 2006b) on schedule coordination was also performed. The study employed a simple negotiation protocol which effectively enabled TOC agents to propose, accept and reject offers. A negotiation strategy, called Strategy-PO (SPO), was then derived so that the resulting solution is guaranteed to be Pareto-optimal (i.e. a win-win situation) when an agreement is made. However, exhaustive searching, which lacks computational efficiency, was employed to generate offers during negotiation. In addition, not all TOCs are satisfied with reaching a Pareto-optimal agreement, but they may prefer to either exploit the negotiation partner for a more favorable offer (i.e. a win-lose situation) or reach an agreement within a few rounds of negotiations to reduce the administration cost (e.g. man-power, communication costs, etc.). As a result, it is beneficial to both improve the computation efficiency of SPO, and examine other potential negotiation strategies that can satisfy the particular needs of different TOCs.

The objective of this paper is to show the feasibility of modeling the behavior of TOCs in a schedule coordination negotiation. While developing a generic model for the coordination problem is beyond the scope of this paper, three negotiation strategies and an efficient algorithm for offer generation are proposed. Section 2 reviews the mathematical formulation of a schedule coordination problem involving two TOCs. Section 3 puts forward an agent negotiation model. Section 4 examines the rationality of the negotiation behaviors through a set of simulation studies. Section 5 presents a hypothetical case study to explore the benefits and limitations of applying the proposed setup for train planning in railway open markets. Section 6 delivers the conclusions.

## **2. Schedule Coordination Problem**

### *2.1. Assumptions*

The schedule coordination problem described here involves the alignment of two passenger train services  $L_i$  and  $L_j$ , operated by TOC- $i$  and TOC- $j$  respectively, at an interchange station  $X$  through negotiation. It assumes that the train operators only share common information on train traveling times. Sensitive data, such as cost rates, are only available to the operators themselves.

The model neglects the cost arising from the loss of punctuality of train services, which is usually recovered by an agreed penalty charge when forming a contract. In addition, quadratic function is used to model the relationship between expected passenger demand and waiting time at the interchange station. While it may be argued that other functions are feasible, and perhaps more accurate, quadratic functions are simple, and they have been employed to model passenger expectation on waiting and traveling times in railways (Murata and Goodman, 1998). In addition, regression analysis has been widely employed in transportation to obtain demand forecasts (Boyer, 1998). Thus, in practice, the required quadratic function may be generated from regression techniques using data collected in surveys on passengers' expectation.

## 2.2. Objective Function

The objective function for TOC- $i$  running  $L_i$  is defined in Equation (1).

$$\max R_i = a_i G_i(\zeta_i, \zeta_j) + a_j G_j(\zeta_j, \zeta_i) - F_i(\zeta_i) \quad \text{for } \zeta_i, \zeta_j \in \{0, 1, 2, \dots\} \quad (1)$$

$R_i$  is the revenue improvement for TOC- $i$  by coordinating its train service  $L_i$  with service  $L_j$ , operated by TOC- $j$ , at station  $X$ .  $G_i(\zeta_i, \zeta_j)$  denotes the estimated number of passengers transferring from  $L_i$  to  $L_j$  at  $X$ , when the commencement times of  $L_i$  and  $L_j$  are  $\zeta_i$  and  $\zeta_j$  respectively. Similarly,  $G_j(\zeta_j, \zeta_i)$  represents the expected passenger demand transferring from  $L_j$  to  $L_i$  at  $X$ .  $a_i$  is the average charge per passenger traveling with  $L_i$ , and  $F_i(\zeta_i)$  is the cost of idle time of rolling stock for  $L_i$  when the commencement time for  $L_i$  is  $\zeta_i$ .

Let  $\hat{\zeta}_i$  be the release date of the rolling stock of  $L_i$ . If  $L_i$  commences at  $\hat{\zeta}_i$ , then the idle cost of  $L_i$  is zero. Also, let  $c_i$  be the unit cost of idle time for  $L_i$ . The idle cost is then modeled in Equation (2).

$$F_i(\zeta_i) = c_i(\zeta_i - \hat{\zeta}_i) \quad \text{for } \zeta_i \geq \hat{\zeta}_i \quad (2)$$

Let  $t_i$  be the time required for  $L_i$  to travel to  $X$  from the origin station, and  $d_i$  be the dwell time of  $L_i$  at  $X$ , then the arrival time  $A_i$  and departure time  $D_i$  of  $L_i$  at  $X$  are modeled in Equations (3) and (4) respectively.  $t_j$ ,  $d_j$ ,  $A_j$  and  $D_j$  can be similarly defined for  $L_j$ .

$$A_i = \zeta_i + t_i \quad (3)$$

$$D_i = \zeta_i + t_i + d_i \quad (4)$$

The passenger waiting time at the interchange station,  $w_{ij}$  and  $w_{ji}$ , for transferring to and from  $L_j$  and  $L_i$  at  $X$ , are expressed in Equations (5) and (6) respectively.  $\kappa_{ij}$  and  $\kappa_{ji}$  refer to the minimum time required to transfer to and from  $L_j$  and  $L_i$  at  $X$ .  $z_{ij}$  and  $z_{ji}$  are obtained

by substitution with Equations (3) and (4) as  $z_{ij} = t_j + d_j - t_i - \kappa_{ij}$  and  $z_{ji} = t_i + d_i - t_j - \kappa_{ji}$ .

$$w_{ij} = D_j - A_i - \kappa_{ij} = \zeta_j - \zeta_i + z_{ij} \quad (5)$$

$$w_{ji} = D_i - A_j - \kappa_{ji} = \zeta_i - \zeta_j + z_{ji} \quad (6)$$

$G_i(\zeta_i, \zeta_j)$  and  $G_j(\zeta_j, \zeta_i)$ , the expected passenger demands transferring between the two services, are modeled in Equations (7) and (8).  $G_{ij}^*$  and  $G_{ji}^*$  are the maximum expected demands and  $\hat{w}_{ij}$  and  $\hat{w}_{ji}$  are the waiting times when demands reach zero.

$$G_i(\zeta_i, \zeta_j) = G_{ij}^* \left[ 1 - \left( \frac{\zeta_j - \zeta_i + z_{ij}}{\hat{w}_{ij}} \right)^2 \right] \quad \text{for } 0 \leq \zeta_j - \zeta_i + z_{ij} \leq \hat{w}_{ij} \quad (7)$$

$$G_j(\zeta_j, \zeta_i) = G_{ji}^* \left[ 1 - \left( \frac{\zeta_i - \zeta_j + z_{ji}}{\hat{w}_{ji}} \right)^2 \right] \quad \text{for } 0 \leq \zeta_i - \zeta_j + z_{ji} \leq \hat{w}_{ji} \quad (8)$$

The objective of TOC- $i$  without considering the impacts to TOC- $j$  is thus to maximize the revenue improvement in Equation (1), subject to the constraints accompanied in Equations (1), (2), (7), and (8). Similarly, the objective function for TOC- $j$  can be generated by interchanging the indices of  $i$  and  $j$ . However, since the TOCs must agree on the decision variables  $\zeta_i$  and  $\zeta_j$  together, the individual optimal solutions may not be accepted by the other TOC, but are determined through a negotiation process. In other words, the schedule coordination problem considered here differs from conventional constrained optimization problems in that not only the optimal solution needs to be solved, but also a set of high quality solutions are required for negotiation purpose.

### 2.3. Negotiation Protocol

Negotiation is defined as the exchange of offers in a finite number of rounds. The TOC agent submitting the first offer is the initiator, while the agent submitting the second one is the responder. An offer  $O^k$  at round  $k$  is modeled in Equation (9), and it consists of the proposed commencement times  $\zeta_i^k$  and  $\zeta_j^k$  of the initiator  $i$  and responder  $j$  respectively. The revenue



improvement (utility value) for TOC- $i$  associated with  $O^k$  is represented by  $R_i^k$ .

$$O^k = \{\zeta_i^k, \zeta_j^k\} \quad (9)$$

The negotiation procedure is summarized in Figure 1. Both agents share a common action set  $Ac \in \{\text{PROPOSE}, \text{ACCEPT}, \text{FAILURE}\}$ . In the first round of negotiation ( $k = 1$ ), the initiator generates its optimal offer in Equation (1). If it exists, it is proposed to the responder. Otherwise, no action is taken. In all subsequent rounds, both agents evaluate the utility value and update  $O^{\hat{k}}$ , which is the offer received that has the highest utility value  $R_i^{\hat{k}}$  between the first round and the most recent round ( $\hat{k}$  corresponds to the round that has the highest utility value). In addition, the agent also computes the counteroffer  $O^*$  using a negotiation strategy. If no offer is found, the negotiation is terminated with action FAILURE. Where the offer exists, the agent proposes  $O^{k+1} = O^*$  if  $R_i^* > R_i^{\hat{k}}$ , and accepts  $O^{\hat{k}}$  otherwise.

## 2.4. Negotiation Strategies

### 2.4.1. Strategy-PO

Strategy-PO (SPO) was first proposed by Tsang and Ho (2006b), and its rationale is included here for the sake of completeness of discussions. In this strategy, the feasible offers are arranged in descending order of utility values, that is, for the initiator,  $R_1^1 \geq R_1^3 \geq \dots \geq R_1^{2m-1}$ , and for the responder,  $R_2^2 \geq R_2^4 \geq \dots \geq R_2^{2m}$ , where  $m$  denotes the ranking of the utility value.

SPO is intended to derive the Pareto-optimal solution. A solution is Pareto-optimal if there does not exist any alternative solution which improves the utility values of all negotiating parties (Ehtamo et al., 1996). In order to achieve Pareto-optimality, it requires both TOC agents to employ SPO, and the proof has been given by Tsang and Ho (2006b).

### 2.4.2. Strategy-MIN

Suppose an agent has just received an offer  $O^k$ . In Strategy-MIN (SMIN), the counteroffer  $O^*$  is derived from Equation (10), where  $O'$  and  $O''$  are offers with utility values  $R'$  and  $R''$  that

are found by Equations (11) and (12) respectively.  $R_i^{k-1}$  is the utility value associated with the previous offer  $O^{k-1} = \{\zeta_i^{k-1}, \zeta_j^{k-1}\}$ .  $R_{\partial\zeta_i}$  and  $R_{\partial\zeta_j}$  represent the utility values of the candidate offers  $O_{\partial\zeta_i} = \{\zeta_i, \zeta_j^{k-1}\}$  and  $O_{\partial\zeta_j} = \{\zeta_i^{k-1}, \zeta_j\}$  respectively.

$$O^* = \begin{cases} O' & \text{for } R' > R'' \\ O'' & \text{otherwise} \end{cases} \quad (10)$$

$$R' = \arg_{R_{\partial\zeta_i}} \{\min(R_i^{k-1} - R_{\partial\zeta_i})\} \quad (11)$$

$$R'' = \arg_{R_{\partial\zeta_j}} \{\min(R_i^{k-1} - R_{\partial\zeta_j})\} \quad (12)$$

SMIN attempts to reduce the concession made from the most recent offer proposed by the agent itself. Since the generated offers do not take the proponent's requirements into consideration, agents employing this strategy are expected to make fine steps of concession during negotiation.

#### 2.4.3. Strategy-MAX

Suppose an agent has just received the offer  $O^k = \{\zeta_i^k, \zeta_j^k\}$ . In Strategy-MAX (SMAX), the counteroffer  $O^*$  is also derived from Equation (10), but  $R'$  and  $R''$  are found by Equations (13) and (14) respectively.  $R_i^k$  is the utility value associated with the current offer  $O^k$ .  $R_{\partial\zeta_i}$  and  $R_{\partial\zeta_j}$  represent the utility values of the candidate offers  $O_{\partial\zeta_i} = \{\zeta_i, \zeta_j^k\}$  and  $O_{\partial\zeta_j} = \{\zeta_i^k, \zeta_j\}$  respectively.

$$R' = \arg_{R_{\partial\zeta_i}} \{\max(R_{\partial\zeta_i} - R_i^k)\} \quad (13)$$

$$R'' = \arg_{R_{\partial\zeta_j}} \{\max(R_{\partial\zeta_j} - R_i^k)\} \quad (14)$$

SMAX attempts to maximize the difference of utility value from the most recent offer received from the proponent agent. Since the generated offers are modified from the proponent's offers, which are likely to benefit to the proponent, agents employing this strategy are expected to make coarse steps of concession during the negotiation.

### 3. Algorithms for Generation of Offers

Exhaustive searching was proposed by Tsang and Ho (2006b) to generate offers to the proponent. The algorithm intuitively generates all possible offers in the solution space and arranges them in descending order of revenue improvement. This algorithm both imposes a high computational demand, and the majority of offers are in fact not proposed because the size of the solution space is usually much larger than the effective number of negotiation rounds.

In order to reduce the computation demand, a more efficient algorithm is employed in this study. The first stage of the algorithm generates the optimal offer by Lemke's Complementary Pivoting Algorithm (LCPA) (Bazaraa et al., 1993), and the second stage adopts a heuristic searching algorithm to generate a set of high quality solutions.

#### 3.1. Lemke's Complementary Pivoting Algorithm

The objective function in Equation (1) contains  $G_i(\zeta_i, \zeta_j)$  and  $G_i(\zeta_j, \zeta_i)$  which represents the expected demands transferring between the train services. According to Equations (7) and (8), these demands have domains defined by  $0 \leq \zeta_j - \zeta_i + z_{ij} \leq \hat{w}_{ij}$  and  $0 \leq \zeta_i - \zeta_j + z_{ji} \leq \hat{w}_{ji}$  respectively. When both passenger waiting times are out of these ranges, the demands cease and the utility value  $R_i$  becomes zero. When either  $G_i(\zeta_i, \zeta_j)$  or  $G_i(\zeta_j, \zeta_i)$  is invalid (i.e.  $\zeta_j - \zeta_i + z_{ij} < 0 \leq -1$  or  $\zeta_i - \zeta_j + z_{ji} < 0 \leq -1$ ), it corresponds to the two situations of unidirectional transfer. When both terms are valid, the transfer is bidirectional. In other words, the optimal revenue improvement  $R^*$  can be computed by Equation (15), where  $R_{i \rightarrow j}$  is the optimal value for the problem  $\mathbf{P}_{i \rightarrow j}$  (unidirectional transfer from  $L_i$  to  $L_j$ ),  $R_{j \rightarrow i}$  is the optimal value for the problem  $\mathbf{P}_{j \rightarrow i}$  (unidirectional transfer from  $L_j$  to  $L_i$ ), and  $R_{i \leftrightarrow j}$  is the optimal value for problem  $\mathbf{P}_{i \leftrightarrow j}$  (bidirectional transfer between  $L_i$  and  $L_j$ ). These three sub-problems are defined in Table 1.

$$R^* = \max(R_{i \rightarrow j}, R_{j \rightarrow i}, R_{i \leftrightarrow j}) \quad (15)$$

When the integer constraints on  $\zeta_i$  and  $\zeta_j$  associated with Equation (1) are neglected, these sub-problems can be expressed in the standard form for quadratic programming in Equation (16), where  $\mathbf{x} = [\zeta_i \ \zeta_j]^T$  is the vector containing the decision variables. A summary of  $\mathbf{c}$ ,  $\mathbf{H}$ ,  $\mathbf{A}$  and  $\mathbf{b}$  is given in Table 2.

$$\min \{f(\mathbf{x}) = \mathbf{c}^T \mathbf{x} + \frac{1}{2} \mathbf{x}^T \mathbf{H} \mathbf{x} : \mathbf{A} \mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}\} \quad (16)$$

Although not all quadratic programming problems can be solved analytically, it has been shown that if  $\mathbf{H}$  is positive semi-definite, the problem can be reduced to a linear programming problem supplemented by a complementary constraint, which is solved efficiently by Lemke's Complimentary Pivoting Algorithm (LCPA) (Bazaraa et al., 1993). For the special case of a  $2 \times 2$  matrix  $\mathbf{H}$ ,  $\mathbf{H}$  is positive semi-definite if and only if  $h_{11} \geq 0$ ,  $h_{22} \geq 0$ ,  $h_{11}h_{22} - h_{12}h_{21} \geq 0$  (where  $h_{ij}$  is the element of  $\mathbf{H}$  at row  $i$  and column  $j$ ). As the  $\mathbf{H}$ -matrices for the sub-problems can be shown to satisfy this condition, LCPA is used to generate the optimal solution for the relaxed (non-integer) problems in Equation (1). Nevertheless, for the purpose of negotiation, it is still necessary to generate a sequence of offers. To obtain such a set of potential offers, a heuristic searching algorithm is proposed.

### 3.2. Algorithm for Strategy-PO

Instead of searching for the entire search space, this heuristic algorithm extracts only the portion of solutions satisfying Equation (17). In other words, the revenue improvement of the generated offers  $R$  is no less than  $(\alpha \times 100)\%$  of the optimal solution  $R^*$ .

$$R \geq \alpha R^*, \text{ for } \alpha \in [0, 1] \quad (17)$$

The search is organized as a tree diagram as shown in Figure 2. The nodes at levels 1 and 2 correspond to the optimal solutions evaluated by LCPA. However, it should be noted that these solutions may be infeasible because the integer constraints on  $\zeta_i$  and  $\zeta_j$  are neglected. To obtain a set of solutions satisfying Equation (17), the integer constraints are considered in the nodes at

levels 3 and 4.

### 3.2.1. Evaluation at level 3 of search tree

At this level,  $\zeta_i$  is assigned (in ascending order) with an integer value to node indexed by  $u = \{1, 2, \dots\}$ . The objective is to determine the optimal solution among its leaf nodes. Since  $\zeta_i$  is a constant at this level, the sub-problems in Table 1 are reduced to single-variable optimization problems of  $\zeta_j$  coupled with linear constraints. These are easily solved by standard constrained optimization techniques by finding the derivative of the cost function and comparing the utility values at the local maximum and the boundary cases.

To avoid evaluating all instances of  $\zeta_i$ , nodes are pruned by using the heuristics depicted in Figure 3. Each box represents the revenue improvement of a decision variable pair  $\zeta_i$  and  $\zeta_j$ . In Figure 3a, the effects on revenue improvement associated with demands  $G_i(\zeta_i, \zeta_j)$  and  $G_i(\zeta_j, \zeta_i)$  are shown. According to Equations (7) and (8), a unit increase in both  $\zeta_i$  and  $\zeta_j$  results in no change in passenger demands. This forms the constant contours represented by the dotted lines. Also, as the revenue improvement grows when passenger demand is increased, a rise in  $G_i(\zeta_i, \zeta_j)$  diagonally downwards (refer to Equation (7)) will contribute to an increase in revenue improvement. Similarly, a rise in  $G_i(\zeta_j, \zeta_i)$  diagonally upwards (refer to Equation (8)) will contribute to an increase in revenue improvement. Similar sketch can be obtained in Figure 3b for idle cost  $F_i(\zeta_i)$ , when considering Equations (1) and (2).

The resulting effects of the three factors may either increase (+) or decrease (–) the revenue improvement. However, for the problems  $\mathbf{P}_{i \rightarrow j}$  and  $\mathbf{P}_{i \leftrightarrow j}$ , the center box with  $\zeta_i = \zeta_i^u$  in Figure 3c (where  $\zeta_i^u$  is the commencement time corresponding to one of the node  $u$  at level 3) is considered. With the trends shown in Figures 3a and 3b, the utility value at the upper right diagonal box is always lower because the demands are constant but the idle cost is increasing. Although the

change in the adjacent and lower diagonal boxes are uncertain, if these boxes are infeasible values (i.e. beyond the boundary constraints), all columns beyond  $\zeta_i^u$  will not contain any solution satisfying the search criteria in Equation (17). In other words, all nodes  $\zeta_i > \zeta_i^u$  may be pruned if the condition shown in Figure 3c is detected.

Similarly, for problem  $\mathbf{P}_{j \rightarrow i}$  (Figure 3d), the entire column at  $\zeta_i^{u+1}$  has the revenue improvement reduced, so that the columns beyond  $\zeta_i^u$  can be pruned without the need of reaching the boundary constraint.

### 3.2.2. Evaluation at level 4 of search tree

At this level,  $\zeta_i$  is inherited from the parent node at level 3, and  $\zeta_j$  is assigned (in ascending order) with an integer value to node indexed by  $v = \{1, 2, \dots\}$ . Since both values are constants at this level, the utility of a node is directly computed by Equation (1).

To avoid evaluating all instances of  $\zeta_j$ , nodes are pruned when  $\zeta_j > \zeta_j^v$  (where  $\zeta_j^v$  is the commencement time corresponding to one of the node  $v$  at level 4) if the revenue improvement  $R^v$  at node  $v$  has already violated Equation (17). The value of  $\zeta_j^v$  is determined by comparing against the boundary constraints of  $\mathbf{P}_{i \rightarrow j}$ ,  $\mathbf{P}_{j \rightarrow i}$  and  $\mathbf{P}_{i \leftrightarrow j}$  in Table 1, which can be determined by Equations (18), (19) and (20) respectively.

$$\zeta_j^v = \hat{w}_{ij} + \zeta_i - z_{ij} \quad (18)$$

$$\zeta_j^v = \min\{(\zeta_i + z_{ji}), (\zeta_i - z_{ji} - 1)\} \quad (19)$$

$$\zeta_j^v = \min\{(\zeta_i + z_{ji}), (\hat{w}_{ij} - \zeta_i - z_{ij})\} \quad (20)$$

### 3.3. Algorithms for Strategy-MIN and Strategy-MAX

In these strategies, the initial offer proposed in round 1 can be generated by LCPA discussed above. However, to ensure that the resulting offer is feasible (i.e.  $\zeta_i$  and  $\zeta_j$  satisfying the integer constraints), the optimal offer is obtained by comparing the direct neighboring solutions of

$O_1 = \{Dn(\zeta_i), Dn(\zeta_j)\}$  ,  $O_2 = \{Dn(\zeta_i)+1, Dn(\zeta_j)\}$  ,  $O_3 = \{Dn(\zeta_i), Dn(\zeta_j)+1\}$  , and  $O_4 = \{Dn(\zeta_i)+1, Dn(\zeta_j)+1\}$  , where  $Dn(\bullet)$  rounds down the value to the nearest integer. In other words,  $O^1 = O_i$ ,  $i = \arg\{\max\{R_i\} | i = \{1, 2, 3, 4\}\}$ .

For the subsequent offers, according to the definitions of SMIN and SMAX, the counteroffer  $O^*$  is obtained by comparing  $O'$  and  $O''$ . Since  $O''$  corresponds to minimizing/maximizing the utility value by holding the commencement time  $\zeta_i$  constant for the sub-problems in Table 1, it can be found by standard optimization techniques as discussed at level 3 of the search tree. Similarly, as  $O'$  corresponds to minimizing/maximizing the utility value by holding  $\zeta_j$  constant, it can also be obtained by standard optimization techniques.

#### 4. Simulation Setup and Results

The simulation set-up described below examines the performance of the strategies in terms of their quality of solutions and the duration of negotiations. Five cases have been constructed according to Table 3, and all combinations of strategy pairs are simulated in each case. If  $(S_1, S_2)$  denotes the strategies employed by TOC-1 and TOC-2, where  $S_1, S_2 \in \{SPO, SMAX, SMIN\}$ , a total of nine combinations are available.

The simulated cases represent scenarios from a spectrum of extreme conditions. In case 1, the traveling time of train services are set up so that, without coordination, bidirectional transfer is impossible. Case 2 is deliberately set up so that only unidirectional transfer can be achieved, even when the train services are coordinated. In case 3, the release date of TOC-1 is set to a large value to resemble the scenario when the two TOCs begin the negotiation with substantial operational differences. Cases 4 and 5 examine the consequences when the idle cost of rolling stock is high and low respectively.

#### 4.1. *Quality of Solutions*

Figure 4 displays the frequency distribution of the quality of solutions. A ‘win-win’ solution refers to an agreement that is Pareto-optimal. When the utility value of one TOC is improved at the expense of the other one, the solution is considered as ‘win-lose’. On the other hand, when the utility values of both TOCs are lower than the Pareto-optimal solution, these solutions are denoted by ‘lose-lose’. Finally, ‘none’ refers to cases that are terminated without reaching any agreement.

##### 4.1.1. *Strategy-PO*

The solutions obtained by (SPO, SPO) are used as the reference for the other pairs since these solutions are by definition Pareto-optimal. While Figure 4 suggests that other strategy pairs may also obtain the Pareto-optimal solution occasionally, it is important to note that the concession curves (i.e. the sequence of proposed offers) of (SPO, SPO) are always monotonically decreasing. An example is illustrated in Figure 5 where the introduction of SMIN causes the utility values of both TOCs to ripple downwards. In other words, SPO always guarantees Pareto-optimality.

##### 4.1.2. *Strategy-MIN*

Figure 4 also shows that the use of SMIN by at least one TOC usually results in either win-win or win-lose solution. In SMIN, since the generation of potential offers is restricted by holding one of the commencement time constant, the agent is only able to search within a limited set of offers. This contrasts to SPO which is capable of selecting the next best offer from the entire solution space. Therefore, SMIN has the risk of proposing (or revealing) a less favorable solution during the negotiation. In the example depicted in Figure 5, although the offer  $\{7, 5\}$  contributes a lower utility value to TOC-2, the utility value of its proponent is higher. As a result, TOC-1 prefers the offer over the Pareto-optimal one  $\{8, 5\}$ .

Nevertheless, the frequency of reaching a sub-Pareto-optimal (i.e. win-lose/lose-loss) offer is not exceedingly high. In addition, even if the negotiation ends with a sub-Pareto-optimal offer, the quality of solution is usually close to the Pareto-optimal one. In this aspect, SMIN seems to be



capable of approximating the operation for SPO in most scenarios, but it introduces a small opening of exploiting (and being exploited by) the negotiating partner.

#### 4.1.3. *Strategy-MAX*

According to Figure 4, more than half of the negotiations involving SMAX are sub-Pareto-optimal. Moreover, two negotiations have no solution. Thus, the results suggest that SMAX is less favorable than SPO and SMIN in terms of the quality of solution attained.

Despite the similarities between SMAX and SMIN, there are now significantly fewer negotiations leading to the Pareto-optimal solution when employing SMAX. Since SMAX uses the proponent's offer ( $O^k$ ) instead of the more favorable one ( $O^{k-1}$ ) when generating the counteroffers, it is less likely to reach the Pareto-optimal agreement.

When using the strategy pair (SMAX, SMAX), both agents may suffer from a reduction in utility value because they are both manipulating the proponent's offer to generate their counteroffers. In other words, neither agent is consistently benefiting from the operation. Without any logical modification of the counteroffers, the final agreement may eventually be unfavorable to both parties.

#### 4.2. *Duration of Negotiation*

Figure 6 shows the frequency distribution of the negotiation duration. 'Equal' refers to the same number of rounds as the solution obtained by (SPO, SPO), while 'faster' and 'slower' correspond to requiring fewer and more number of negotiation rounds respectively. Figure 7 shows the distribution of the average negotiation round computed as percentage of the result obtained from (SPO, SPO).

##### 4.2.1. *Strategy-PO*

As shown in Figure 6, employing SMIN or SMAX usually improves the negotiation speed. In fact, in the five simulated cases, (SPO, SPO) often requires a substantial number of rounds (up to 804) before the negotiations are settled. If exhaustive search was used in the simulation instead of

LCPA with heuristic search, the simulation time would further be increased because an extensive number of evaluations is needed (Table 4).

#### 4.2.2. *Strategy-MIN*

According to Figure 6, about half of the negotiations employing SMIN complete the transaction with fewer rounds. Since there is no need to propose the offers in monotonically decreasing order of utility values, SMIN is likely to skip some of the intermediate solutions while still being able to reach the Pareto-optimal or a sub-Pareto-optimal agreement. However, in the other half of the cases, SMIN requires the same number of rounds as the reference negotiation. As a result, SMIN may also be regarded as a good approximation to SPO in terms of negotiation duration.

#### 4.2.3. *Strategy-MAX*

In Figure 6, almost all negotiations involving SMAX produce faster negotiation. The main reason is that the proponent's commencement time ( $\zeta_j$ ) is usually unchanged as the counteroffer is modified from the proponent's offer. In other words, the proponent is more likely to accept the counteroffer. Hence, the number of negotiation rounds is lowered.

The average number of negotiation rounds required by SMAX is usually lower than SMIN. In Figure 7, when SMAX is employed as at least one of the strategies, the average number of negotiation round required is only 20-80% of the result employing (SPO, SPO). On the other hand, when SMIN is employed by one agent, the average round of negotiation required is about 80-90%.

#### 4.3. *Remarks*

The simulation results find that SPO guarantees the Pareto-optimal solution, but it often requires an extensive number of negotiation rounds during negotiation. Although the offer generation process has already been improved using LCPA with heuristic searching algorithm, the large number of negotiation rounds in practice is often infeasible since it will induce a large administration cost (e.g. man-power and communication costs). The use of SMIN generally reduces the number of negotiation rounds by introducing a small opening to exploit a win-lose or lose-win

solution. SMAX further reduces the negotiation rounds, but it has the highest risks of reaching a lose-lose agreement.

## **5. Case Study**

This section demonstrates how the developed simulation software and findings may be employed as a tool for planning and evaluation in practice. However, it should be clearly stated that the simulation data used below is hypothetically created and not collected from any official organizations. Thus, the results only demonstrate the applicability of the simulation models but not necessarily reflect any current situation. Through the following description, it is intended that the potential benefits and limitations of the proposed agent negotiation model will be appreciated.

### *5.1. Background*

In the UK, Network Rail is the infrastructure manager, and a number of passenger train operating companies seek access to this network. Network Rail is responsible for managing 17 major interchange stations. An example is the Liverpool Lime Street station. Intercity services are provided by TransPennine Express and Virgin Trains, while regional services are offered by Central Trains and Northern Rail. A schematic diagram for the lines serviced by these operators is shown in Figure 8. The intercity service providers compete in the northern England including cities at Lancaster, Preston, Liverpool, Manchester, Sheffield, Leeds, York and Newcastle. On the other hand, the regional services encounter only limited competition on the Liverpool-Manchester-Sheffield corridor.

As a consequence, the two regional service operators may consider coordinating their schedules to attract an additional demand for the cross-regional services. This would create a yardstick competition with the seamless intercity services. For example, the journey from Preston to Birmingham via Virgin Trains takes about 1 hour 40 minutes while the trips from Preston to Liverpool via Northern Rail and Liverpool to Birmingham via Central Trains are approximately 1 hour and 1 hour 45 minutes respectively. In other words, the minimum journey time for the

coordinated service is 2 hour 45 minutes. If the combined train fares for the regional services are lower than the intercity one, and the passenger waiting time is kept reasonably short, it is possible that some passengers will use the coordinated service instead of the seamless one.

## 5.2. Setup

The schedule coordination problem at the Liverpool Lime Street station involving Northern Rail and Central Trains is examined using the negotiation model presented in this study. It is assumed that the simulation is conducted from the perspective of Northern Rail whose train planners attempt to determine the operating conditions for its service from Preston to Liverpool if schedule coordination with Central Trains is possible.

The scheduling problem is illustrated in Figure 9. TOC-1 and TOC-2 represent Northern Rail and Central Trains respectively. Northern Rail operates a service from Preston to Liverpool which requires a journey time of 60 minutes and a dwell time of 15 minutes at Liverpool station. On the other hand, the service provided by Central Trains from Liverpool to Birmingham consists of a journey time of 105 minutes and a dwell time of 10 minutes. The minimum transfer time between the two services is 5 minutes. Since Liverpool Lime Street is the terminal station for the Northern Rail's service, the case shown in Figure 9a represents a unidirectional passenger transfer from Northern Rail to Central Trains. In addition, according to the past timetabling experience, the commencement time of the service operated by Central Trains is likely to be 70 minutes later than the commencement time of the Northern Rail's service. According to Equation (5), with  $\zeta_j - \zeta_i = 70$ ,  $t_j = 0$  (Central Trains' service departs directly from the interchange station),  $d_j = 10$ ,  $t_i = 60$ ,  $\kappa_{ij} = 5$ , the default passenger waiting time  $w_{ij}$  is computed to be 15 minutes.

Suppose the current average train fares for the Northern Rail and Central Trains services are £8.00 and £17.00 respectively. These train fares are expected to give rise to a maximum demand of 50 passengers when the waiting time is zero, and the demand will cease when the waiting time exceeds 30 minutes. Moreover, the current estimation of idle costs for the rolling stock of Northern

Rail and Central Trains are £20/min and £25/min respectively. The base case for the situation described is denoted as Case A in Table 5. Simulation of this case yields the probable outcome derived by negotiation. Case B refers to the situation when Northern Rail attempts to increase the passenger demand by reducing the average train fare by £2.00. Finally, Case C demonstrates an example of bidirectional transfer if the same set of rolling stock is used for the backward journey as shown in Figure 9b.

Having devised the situations intended for investigation, the train planners of Northern Rail can generate results using agent negotiation. While it is possible to simulate the scenarios using all combinations of strategy pairs, the assumption is that the proponent (i.e. Central Trains) will use SPO to represent the fact that it has no intention to make any concession to Northern Rail. On the other hand, SPO and SMAX are chosen for Northern Rail to obtain the expected best and worst outcomes respectively. The simulation results are summarized in Table 6.

### 5.3. Results and Findings

#### 5.3.1. Case A

The solution obtained in this case using the strategy pair (SPO, SPO) is  $\{2, 70\}$ , meaning that Northern Rail is willing to postpone its service by 2 minutes, while Central Trains keep its commencement time unchanged. The Pareto-optimal solution has reduced the waiting time by 2 minutes (from 15 to 13 minutes). With the balance between the income generated from an increased passenger demand of 40.6 and the 2-minute idle cost of rolling stock, the overall revenue gained by the stakeholder is found to be £289.89. On the other hand, the solution obtained from the strategy pair (SMAX, SPO) is  $\{7, 70\}$ . As SMAX aims to reduce the negotiation time by sacrificing Pareto-optimality, the commencement time for Northern Rail is further delayed to 7 minutes which leads to a higher idle cost. Although the passenger demand has been increased further to 46.4 (i.e. about 6 more passengers) due to a shorter waiting time of 8 minutes, the overall revenue gained is lowered to £231.56. Nevertheless, since both simulated negotiations lead to a considerable gain in

revenue, conducting a negotiation with Central Trains in practice seems to be beneficial.

### 5.3.2. *Case B*

The reduction of train fare has increased the revenue of Northern Rail to £292.80 using (SPO, SPO) and £250.13 using (SMAX, SPO). Although this may encourage the stakeholder to lower the train fare, the expected gain is not substantial (only £3 - £20). Thus, the stakeholder may retain the basic train fare to avoid the additional administration cost of modifying the charging scheme.

### 5.3.3. *Case C*

The possibility of bidirectional transfer has provided a reasonable increase in revenue for Northern Rail. Having an additional demand of almost 50 passengers in the backward journey, Northern Rail is willing to postpone the commencement time by about 20 minutes instead of only 2 minutes in Case A.

### 5.3.4. *Remarks*

Based on the simulation results, the recommendation to Northern Rail is to explore the possibility of schedule coordination with Central Trains. Preferably, the rolling stock should also be used for the backward journey. However, the stakeholder should pay serious attention to the possible errors in their estimation or prediction (e.g. passenger demand). It is also recommended that Northern Rail should negotiate in a cautious manner if adequate time is available for negotiation.

## **6. Conclusions**

We have presented an agent negotiation model for the schedule coordination problem in open railway markets. The model consists of a simple protocol and the Train Operating Company (TOC) agents are able to incorporate Strategy-PO (SPO), Strategy-MIN (SMIN) or Strategy-MAX (SMAX). The offer generation problem is resolved using Lemke's Complementary Pivoting Algorithm (LCPA) with a heuristic searching algorithm, which is more efficient than exhaustive search. Through the negotiation process, the TOC agents are able to decide whether coordinating

the schedules between the train services is favorable.

Simulations have been conducted to evaluate the Pareto-optimality and negotiation length. The findings confirm that SPO guarantees a Pareto-optimal (win-win) agreement but requires the highest negotiation demand. The performance of SMIN is similar to that of SPO but it introduces a small opening to exploit the win-lose or lose-win solution. SMIN is thus suitable for ambitious TOCs aiming to obtain a solution that is more favorable than the Pareto-optimal solution by exploiting the benefits of the negotiating partner. For TOCs that are keen on obtaining a deal quickly, SMAX is a fast means to complete a negotiation with a higher risk of reaching a lose-win or lose-lose agreement.

Although the proposed model considers coordinating passenger train services, it may also be applied to coordinating freight train services. Instead of using passenger demands in Equation (1), freight demands (e.g. measured in tons) can be used. Nevertheless, since the relationship between demands and waiting time for freight consignments may be significantly different from that of passenger services, and extra costs are usually associated with handling the transfer of consignments, the objective function may become more complicated. In fact, the objective function, even for passenger train coordination, can be more complex than the model presented in this paper when TOCs wish to consider the dependency cost (e.g. train delays of other TOCs) or to model passenger demand more accurately using non-quadratic functions. In such case, a different algorithm is required to generate the sequence of offers, even though the agent modeling and the definitions on negotiation strategies remain applicable. Further research on devising more complex and generic objective function (hence algorithm) will greatly improve the usability of the model.

In addition, the model has considered schedule coordination involving neither more than 2 TOCs nor a set of regular services on different headways. Furthermore, the need for coordinating trains with multiple trains at multiple platforms, and the granting of track access rights by the Infrastructure Manager (IM) have not been considered in this study. It is therefore not our intention to apply the model to resolve any practical problems currently experienced by the railway open

markets, but to demonstrate how the objectives and behavior of the railway stakeholders can be captured by agent modeling. Further development based on the proposed model is believed to be a valuable tool to assist the planning of policy makers before the actual negotiation is conducted.

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**Table 1 Optimization Problems**

Problem	Objective	Constraints
$\mathbf{P}_{i \rightarrow j}$	$\max R_u^{i \rightarrow j} = a_i G_{ij}^* \left[ 1 - \left( \frac{\zeta_j - \zeta_i + z_{ij}}{\hat{w}_{ij}} \right)^2 \right] - c_i (\zeta_i - \hat{\zeta}_i)$	$\begin{aligned} \zeta_i &\geq \hat{\zeta}_i \\ \zeta_j - \zeta_i + z_{ij} &\geq 0 \\ \zeta_j - \zeta_i + z_{ij} &\leq \hat{w}_{ij} \\ \zeta_i - \zeta_j + z_{ji} &\leq -1 \\ \zeta_j &\geq 0 \end{aligned}$
$\mathbf{P}_{j \rightarrow i}$	$\max R_u^{j \rightarrow i} = a_i G_{ji}^* \left[ 1 - \left( \frac{\zeta_i - \zeta_j + z_{ji}}{\hat{w}_{ji}} \right)^2 \right] - c_i (\zeta_i - \hat{\zeta}_i)$	$\begin{aligned} \zeta_i &\geq \hat{\zeta}_i \\ \zeta_i - \zeta_j + z_{ji} &\geq 0 \\ \zeta_i - \zeta_j + z_{ji} &\leq \hat{w}_{ji} \\ \zeta_j - \zeta_i + z_{ij} &\leq -1 \\ \zeta_j &\geq 0 \end{aligned}$
$\mathbf{P}_{i \leftrightarrow j}$	$\begin{aligned} \max R_u^{i \leftrightarrow j} &= a_i G_{ij}^* \left[ 1 - \left( \frac{\zeta_j - \zeta_i + z_{ij}}{\hat{w}_{ij}} \right)^2 \right] \\ &+ a_i G_{ji}^* \left[ 1 - \left( \frac{\zeta_i - \zeta_j + z_{ji}}{\hat{w}_{ji}} \right)^2 \right] - c_i (\zeta_i - \hat{\zeta}_i) \end{aligned}$	$\begin{aligned} \zeta_i &\geq \hat{\zeta}_i \\ \zeta_j - \zeta_i + z_{ij} &\geq 0 \\ \zeta_j - \zeta_i + z_{ij} &\leq \hat{w}_{ij} \\ \zeta_i - \zeta_j + z_{ji} &\geq 0 \\ \zeta_i - \zeta_j + z_{ji} &\leq \hat{w}_{ji} \\ \zeta_j &\geq 0 \end{aligned}$

**Table 2 Matrices for Optimization Problems**

Problem	$\mathbf{c}$	$\mathbf{H}$	$\mathbf{A}$	$\mathbf{b}$
$\mathbf{P}_{i \rightarrow j}$	$\frac{2a_i G_{ij}^* z_{ij}}{\hat{w}_{ij}^2} \begin{bmatrix} c_i \hat{w}_{ij}^2 / 2a_i G_{ij}^* z_{ij} - 1 \\ 1 \end{bmatrix}$	$\frac{2a_i G_{ij}^*}{\hat{w}_{ij}^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 1 & -1 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}$	$\begin{bmatrix} -\hat{\zeta}_i \\ z_{ij} \\ \hat{w}_{ij} - z_{ij} \\ -1 - z_{ji} \end{bmatrix}$
$\mathbf{P}_{j \rightarrow i}$	$\frac{2a_i G_{ji}^* z_{ji}}{\hat{w}_{ji}^2} \begin{bmatrix} c_i \hat{w}_{ji}^2 / 2a_i G_{ji}^* z_{ji} + 1 \\ -1 \end{bmatrix}$	$\frac{2a_i G_{ji}^*}{\hat{w}_{ji}^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ -1 & 1 \\ 1 & -1 \\ -1 & 1 \end{bmatrix}$	$\begin{bmatrix} -\hat{\zeta}_i \\ z_{ji} \\ \hat{w}_{ji} - z_{ji} \\ -1 - z_{ij} \end{bmatrix}$
$\mathbf{P}_{i \leftrightarrow j}$	$2a_i \begin{bmatrix} \frac{G_{ji}^* z_{ji}}{\hat{w}_{ji}^2} - \frac{G_{ij}^* z_{ij}}{\hat{w}_{ij}^2} + \frac{c_i}{2a_i} \\ \frac{G_{ij}^* z_{ij}}{\hat{w}_{ij}^2} - \frac{G_{ji}^* z_{ji}}{\hat{w}_{ji}^2} \end{bmatrix}$	$2a_i \begin{bmatrix} h & -h \\ -h & h \end{bmatrix}$ where $h = \frac{G_{ij}^*}{\hat{w}_{ij}^2} + \frac{G_{ji}^*}{\hat{w}_{ji}^2}$	$\begin{bmatrix} -1 & 0 \\ 1 & -1 \\ -1 & 1 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}$	$\begin{bmatrix} -\hat{\zeta}_i \\ z_{ij} \\ \hat{w}_{ij} - z_{ij} \\ z_{ji} \\ \hat{w}_{ji} - z_{ji} \end{bmatrix}$

**Table 3 Simulation Setup**

Case	Idle Cost Rates (£/min)		Commencement Time (min)		Average Train Fare (£/person)		Travel Time from Origin (min)		Station Dwell Time (min)		Max. Demand (persons)		Min. Transfer Time (min)		Passenger Waiting Time (min)	
	$c_1$	$c_2$	$\hat{\zeta}_1$	$\hat{\zeta}_2$	$a_1$	$a_2$	$t_1$	$t_2$	$d_1$	$d_2$	$G_{12}^*$	$G_{21}^*$	$\kappa_{12}$	$\kappa_{21}$	$\hat{w}_{12}$	$\hat{w}_{21}$
1	50	60	7	5	15	22	20	30	5	7	100	80	2	2	20	20
2	50	60	7	5	15	22	20	30	1	1	100	80	2	2	20	20
3	50	60	60	5	15	22	20	30	5	7	100	80	8	8	20	20
4	250	60	7	5	15	22	20	30	5	7	100	80	2	2	20	20
5	1	60	7	5	15	22	20	30	5	7	100	80	2	2	20	20

**Table 4 Comparison of Computation Requirement between Exhaustive Search and LCPA with Heuristic Search**

Dimension of $\zeta_i$ and $\zeta_j$	Number of Nodes Evaluated	
	Exhaustive Search	LCPA with Heuristic Search ( $\alpha = 0.8$ )
60	3600	48
120	14,400	105
240	57,600	682
480	230,400	2255

**Table 5 Simulation Setup for Schedule Coordination at Liverpool Lime Street Station**

Case	Description	Commencement time (min)		Average Train Fare (£/person)		Max. Demand (persons)		Idle Cost Rates (£/min)	
		$\hat{\zeta}_1$	$\hat{\zeta}_2$	$a_1$	$a_2$	$G_{12}^*$	$G_{21}^*$	$c_1$	$c_2$
A	Unidirectional transfer <i>Default schedules lead to waiting time of 15 minutes</i>	0	70	8	17	50	0	20	25
B	Unidirectional transfer <i>Reduced train fare to increase passenger demand</i>	0	70	6	17	70	0	20	25
C	Bidirectional transfer <i>Same rolling stock is used for the backward journey</i>	20	0	8	17	50	50	20	25

**Table 6 Simulation Results for Schedule Coordination at Liverpool Lime Street Station**

	Strategy Pair	Case A	Case B	Case C
Solution	(SPO, SPO)	{2, 70}	{3, 70}	{39, 0}
$\{\zeta_1, \zeta_2\}/\text{min}$	(SMAX, SPO)	{7, 70}	{7, 70}	{40, 0}
Revenue gained by TOC-1	(SPO, SPO)	289.89	292.80	359.11
$R_1$ (£)	(SMAX, SPO)	231.56	250.13	344.44
Revenue gained by TOC-2	(SPO, SPO)	789.56	999.60	1570.61
$R_2$ (£)	(SMAX, SPO)	690.39	1105.38	1581.94
Number of negotiation rounds	(SPO, SPO)	147	167	19
	(SMAX, SPO)	47	57	14
Waiting time $L_1 \rightarrow L_2$	(SPO, SPO)	13	12	11
(min)	(SMAX, SPO)	8	8	10
Waiting time $L_2 \rightarrow L_1$	(SPO, SPO)	-	-	4
(min)	(SMAX, SPO)	-	-	5
Demand for $L_1 \rightarrow L_2$	(SPO, SPO)	40.6	58.8	42.3
(persons)	(SMAX, SPO)	46.4	65.0	44.4
Demand for $L_2 \rightarrow L_1$	(SPO, SPO)	-	-	49.1
(persons)	(SMAX, SPO)	-	-	48.6

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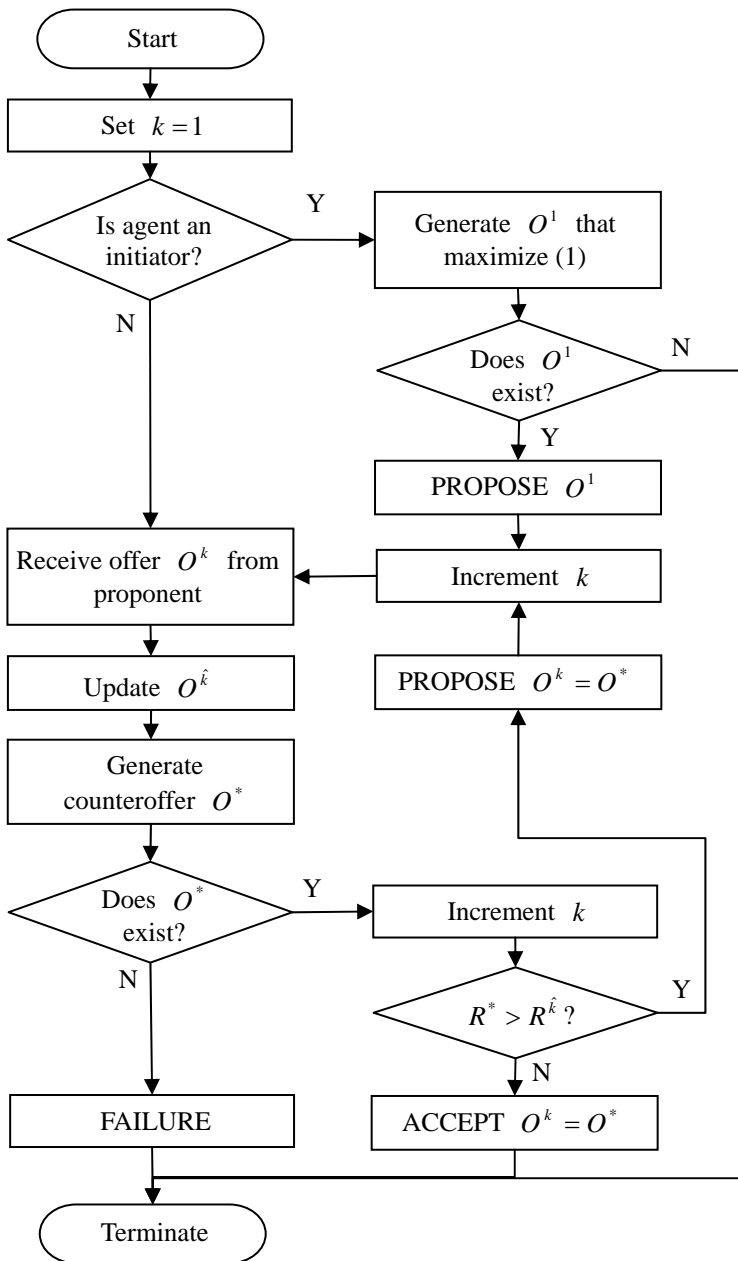


Figure 1 Negotiation Procedure for TOC-TOC Transaction

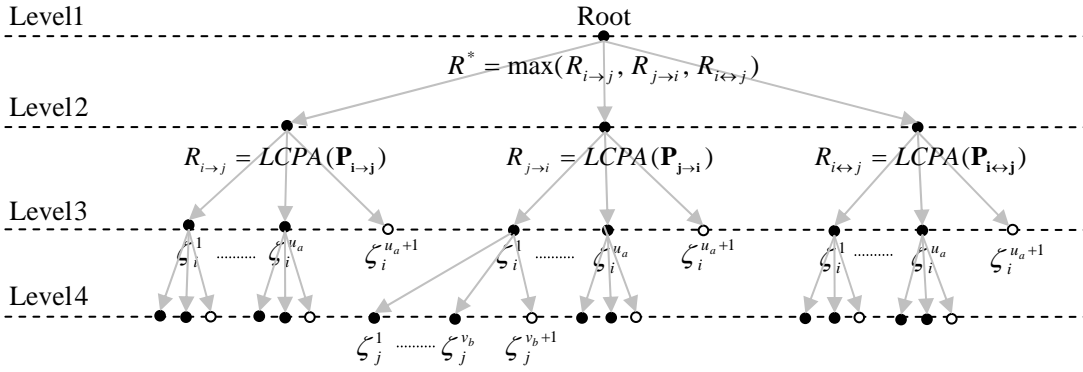


Figure 2 Structure of Pruning Tree

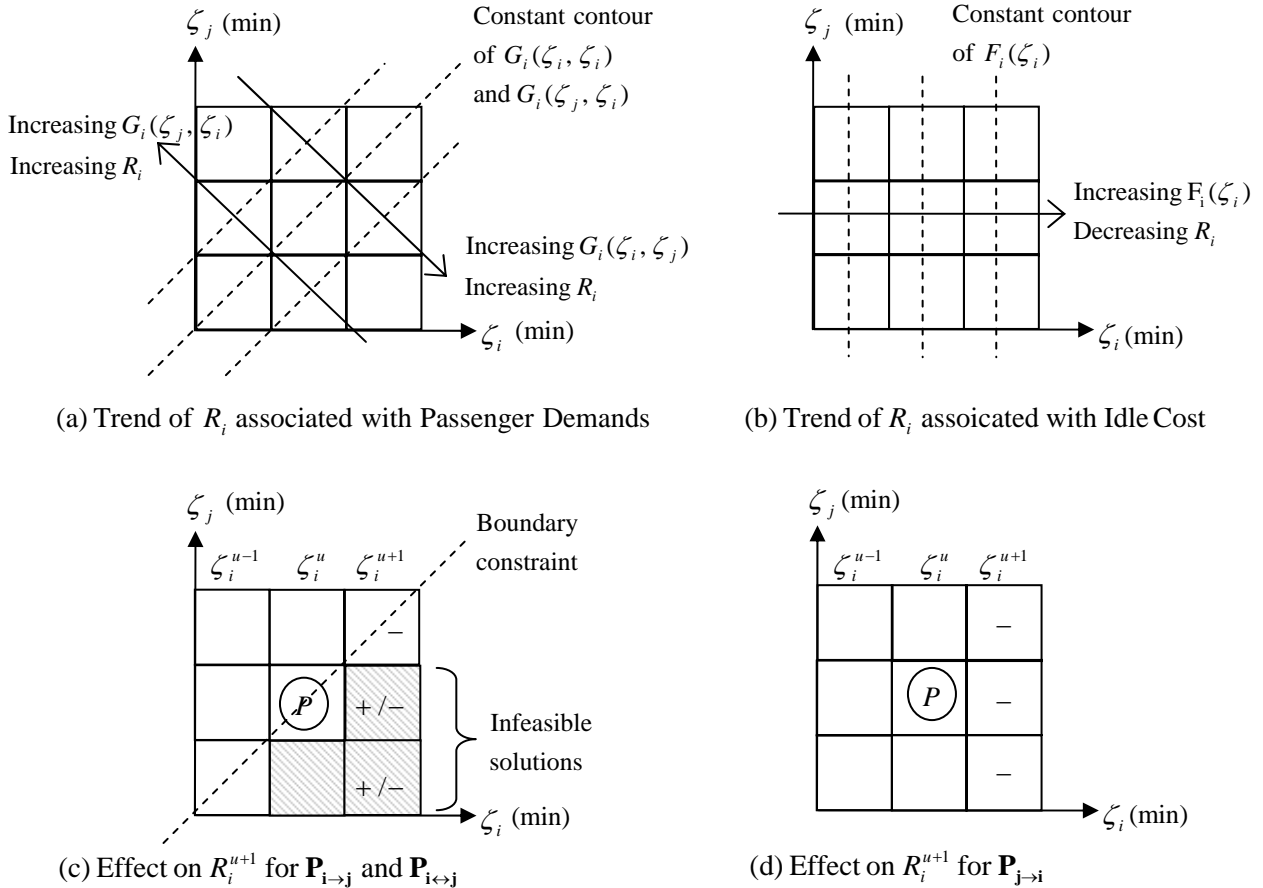


Figure 3 Pruning Conditions at Level 3

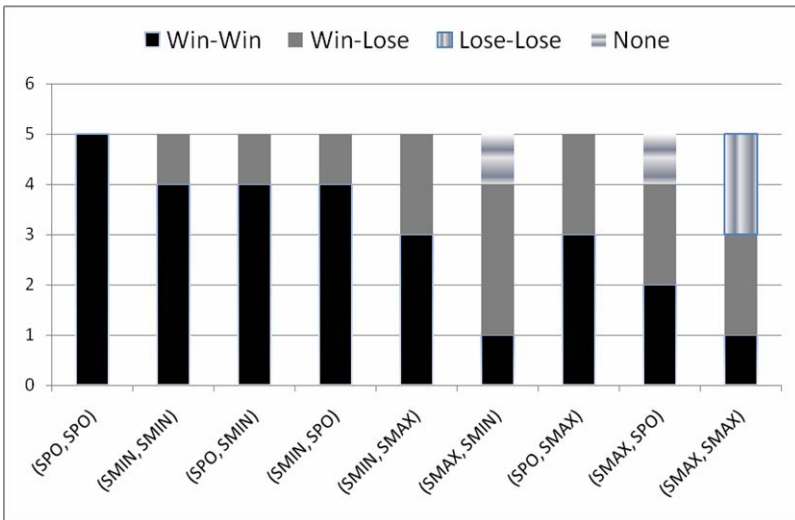


Figure 4 Frequency Distribution of Quality of Solutions

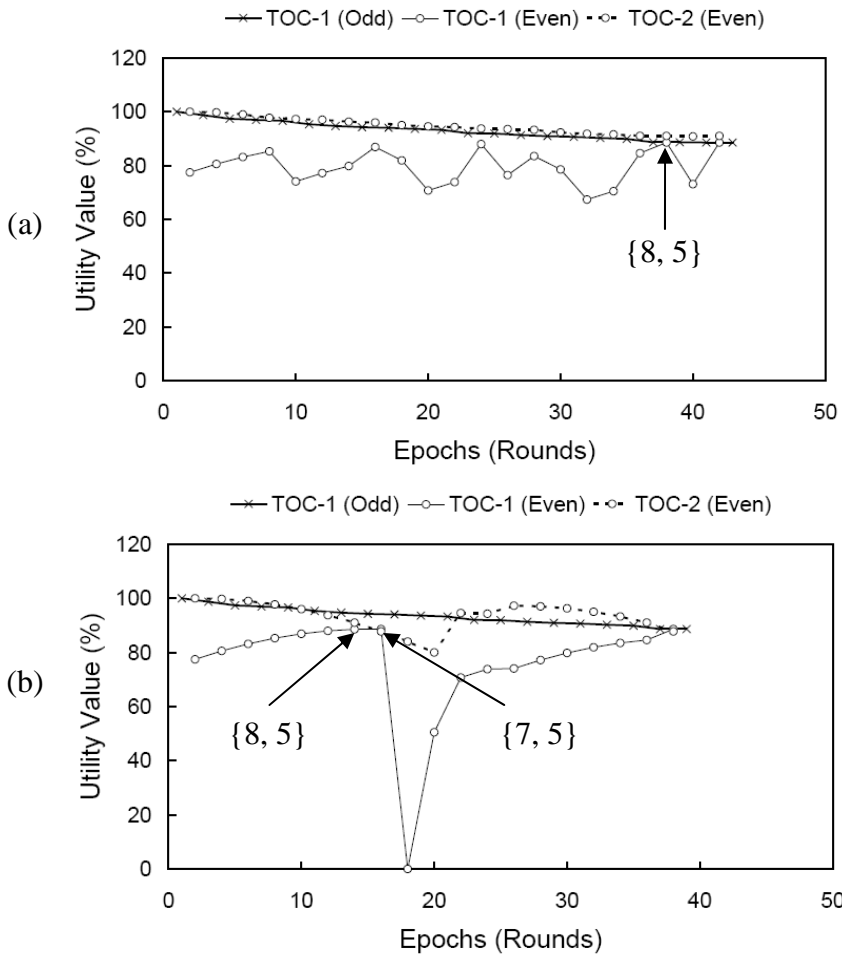


Figure 5 Concession Curves in Case 2 (a) (SPO, SPO) (b) (SPO, SMIN)

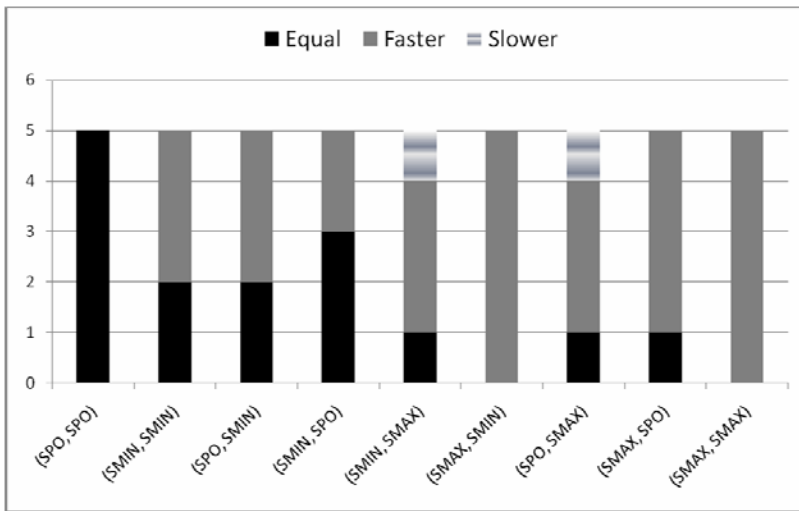


Figure 6 Frequency Distribution of Negotiation Duration

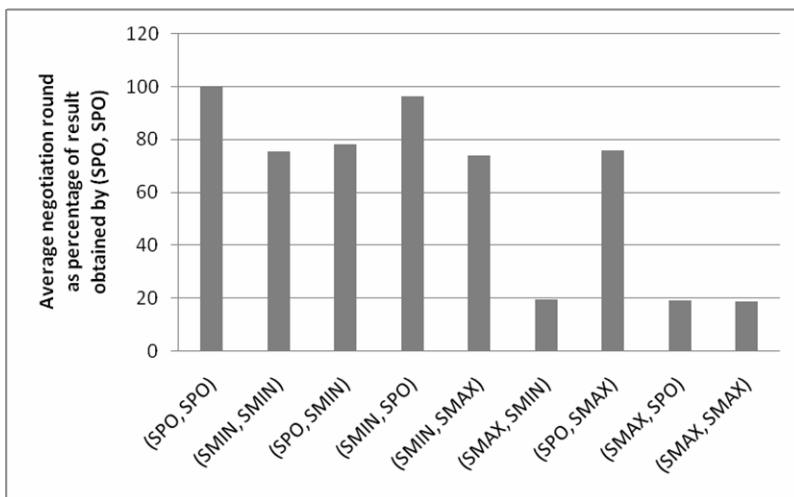


Figure 7 Distribution of Average Negotiation Round



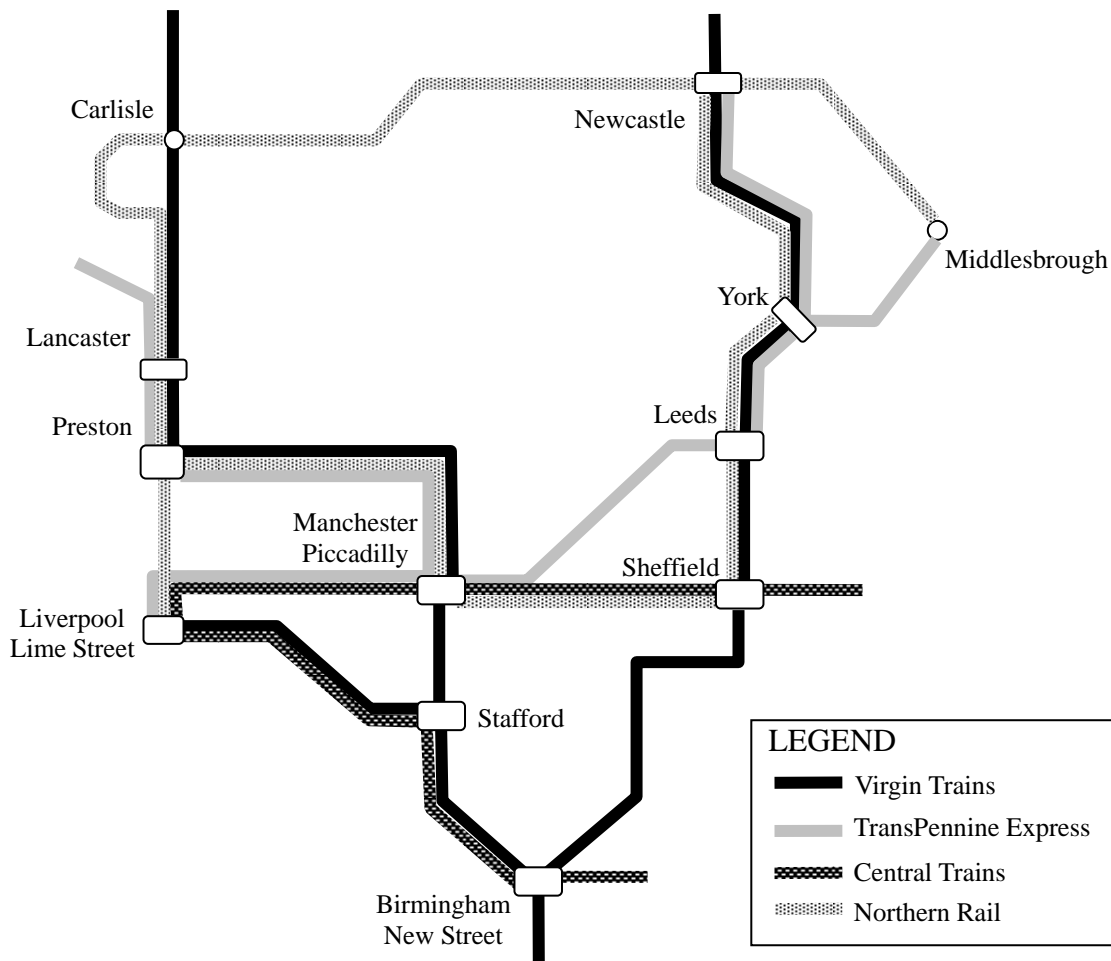


Figure 8 Schematic Diagram for Major Railway Lines of Four TOCs in the UK

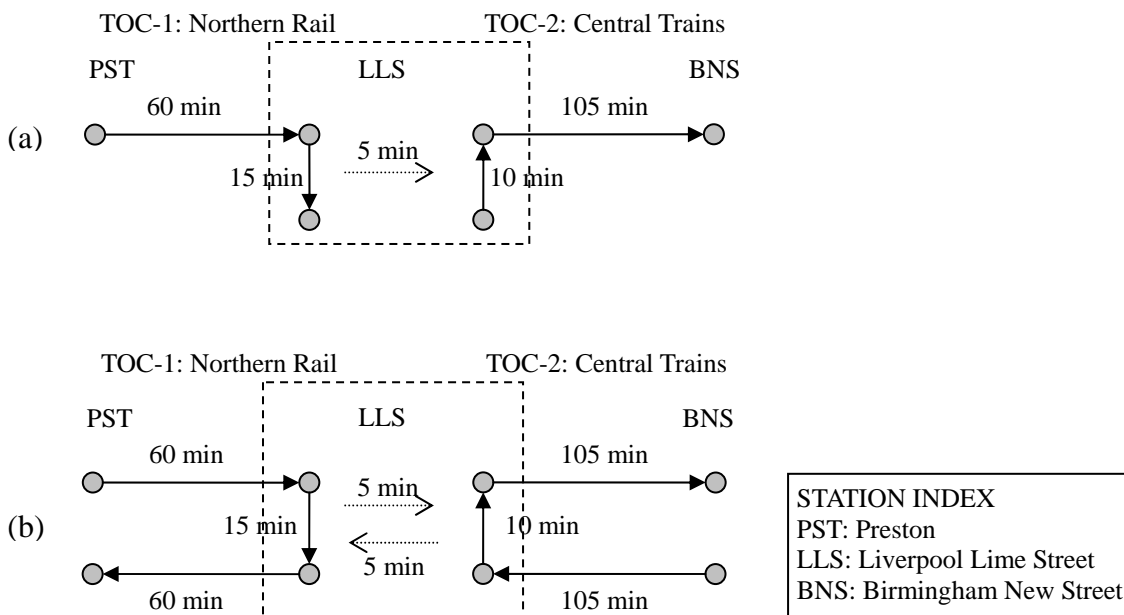


Figure 9 Transfer at Liverpool Lime Street Station