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Limit analysis of ground anchor forces

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Abstract
An anchored sheet wall is commonly used in retaining deep excavations. Anchor force is one of the critical design parameters in practice. Based on the kinematical admissible failure mechanism, a limit analysis approach to determine the anchor force is presented. The explicit formula for the anchor force is given, which makes it easy to calculate using a simple calculation program such as MS Office Excel. Anchor force is mainly influenced by seven parameters: the internal friction angle; cohesion of the soil; wall friction angle; surcharge on the ground surface; dip angle of the anchor; penetration depth of the wall; and depth from the anchor force action point on the wall to the ground surface. The relevant quantitative calculation can be performed by the proposed method. In addition, the design anchor force under a specified design safety factor and the anchor forces of multiple rows of anchors are also illustrated in this paper. To verify that the method is reasonable, the predicted and measured anchor forces are compared in two classical soil-nailed wall experiments. The result shows that the presented method is applicable.

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analyses of a soil slope and a uniformly reinforced soil slope were conducted completely using a kinematical approach (Michalowski, 1995, 1997). The stability analysis of a nailed soil slope was also conducted using limit analysis by Hong (2001). Similarly, the stability of an anchored slope was analysed with limit analysis by Li et al. (2012), and the design of soil-nailed structures using kinematical limit analysis was conducted by Juran et al. (1990). However, no kinematical method has been reported for application to an anchored sheet wall, which is different from a slope or a reinforced slope. Therefore, to develop a more suitable method that is easy to follow for the practical design of anchor force on an anchored sheet wall, a new approach based on the limit analysis upper bound theorem is presented in this paper. The proposed method is applicable to both cohesive and cohesionless soil and can be used to determine the anchor force of one row of anchors or multiple rows of anchors. The presented approach is also applied to two classical soil-nailed walls to predict the tensile anchor forces of nails, and the predicted and experimental results are compared.

2. Analysis model and derivation of anchor force

An anchored sheet wall is used to support a deep excavation as shown in Figure 1. The anchor force \( F \) must be large enough to keep the soil stable. The resistance force \( F_r \) (on the penetration part of the sheet wall) is a single resultant of Rankine’s passive pressure. It is assumed that the velocity discontinuity line passes through the tip of the anchored wall; the wall and the soil mass on its right side (see Figure 1) slide together as one body synchronously. The maximum value of the anchor force is resolved in the framework of the kinematically admissible failure mechanism (Chen, 1975) for the unified body. It is further stipulated that the critical slip surface passes through the tip of the wall and of a logarithmic spiral (based on resolving Euler’s equations in the variational approach (Pula et al., 2005)) described by

\[ r(\theta) = r_0 \times e^{(\theta-\theta_0)\tan \phi} \]

The work rate of external forces encompasses \( W_G \), \( W_q \), \( W_F \) and \( W_p \) concerning gravity, surcharge on the ground surface, anchor force and resistance force, respectively. For the kinematically admissible rigid sliding body, the work rate of the external forces is equal to the internal energy dissipation rate, \( E_c \).

\[ W_G + W_q + W_F + W_p = E_c \]

Next, according to the upper bound theorem of limit analysis, superposition principle and the log-spiral slip line formulated in Equation 1, the work rate of \( W_G \), \( W_q \), \( W_F \) and \( W_p \) is deduced as

\[ W_G = \pi r_0^2 (f_1 - f_2 - f_3) \omega \]

\[ W_q = q \frac{(r_0 \cos \theta_0)^3 - (r_h \cos \theta_h)^3}{2} \]

\[ W_F = -F[\cos \beta (r_0 \sin \theta_0 + z) - \sin \beta (r_h \cos \theta_h)] \]

\[ W_p = -C_F \omega \]

where coefficients \( f_1 \), \( f_2 \) and \( f_3 \) are provided in the Appendix, and \( C_F \) is given by

\[ C_F = 2 \epsilon \sqrt{k_p z_p} \left[ (r_0 \sin \theta_0 + h - \frac{z_p}{2}) \cos \delta + r_h \cos \theta_h \sin \delta \right] + \frac{1}{2} k_p z_p \left[ (r_0 \sin \theta_0 + h - \frac{z_p}{3}) \cos \delta + r_h \cos \theta_h \sin \delta \right] \]

where \( k_p = \tan \left( \frac{2(45^\circ + \phi/2)}{3} \right) \).
Using an integration method (Chen, 1975), the internal energy dissipation rate can also be derived as

\[ E_i = \frac{c r^2}{2 \tan \phi} \left\{ c^2 (\theta_0 - \theta_h) \tan \phi - 1 \right\} \theta \]

Substituting Equations 3–8 into Equation 2, the force \( F \) is deduced as

\[ F = \frac{\gamma h}{(f_1 - f_2 - f_3)} + q \left[ \frac{(r_0 \cos \theta_0)^2}{2} - \frac{(r_0 \cos \theta_h)^2}{2} \right] - \frac{C_p + \frac{1}{2 \tan \phi} \left\{ c^2 (\theta_0 - \theta_h) \tan \phi - 1 \right\}}{\sum_{i=1}^{n} \cos \beta_i (r_0 \sin \theta_0 + z_i) - \sin \beta_i (r_0 \cos \theta_0)} \]

The maximum \( F \) is obtained using the first derivatives of \( F \) with the angles \( \theta_0 \) and \( \theta_h \) (see Figure 1)

\[ \begin{align*}
\frac{\partial F}{\partial \theta_0} &= 0 \\
\frac{\partial F}{\partial \theta_h} &= 0 
\end{align*} \]

Given multiple rows of anchors and identical anchor force of each row, the \( F \) can be deduced as

\[ F = \frac{\gamma h}{(f_1 - f_2 - f_3)} + q \left[ \frac{(r_0 \cos \theta_0)^2}{2} - \frac{(r_0 \cos \theta_h)^2}{2} \right] - \frac{C_p + \frac{1}{2 \tan \phi} \left\{ c^2 (\theta_0 - \theta_h) \tan \phi - 1 \right\}}{\sum_{i=1}^{n} \cos \beta_i (r_0 \sin \theta_0 + z_i) - \sin \beta_i (r_0 \cos \theta_0)} \]

Alternatively, provided that the ratio of anchor force \( \zeta_i \) for the \( i \)th row over the first row (nearest to the ground surface) is specified in a practical design, the following can be obtained

\[ F_i = \frac{\gamma h}{(f_1 - f_2 - f_3)} + q \left[ \frac{(r_0 \cos \theta_0)^2}{2} - \frac{(r_0 \cos \theta_h)^2}{2} \right] - \frac{C_p + \frac{1}{2 \tan \phi} \left\{ c^2 (\theta_0 - \theta_h) \tan \phi - 1 \right\}}{\sum_{i=1}^{n} \cos \beta_i (r_0 \sin \theta_0 + z_i) - \sin \beta_i (r_0 \cos \theta_0) \zeta_i} \]

Equations 9, 11 or 12 are readily resolved using Solver Macro (of MS Office Excel 2010) by way of a spreadsheet. The anchor force \( F \) is usually determined in several iterations. The authors also obtained the anchor force by developing a computer program in Microsoft Visual C++ 2008.

### 3. Two engineering examples

An anchored sheet wall was proposed to retain deep excavation in cohesionless soil with a unit weight \( \gamma \) of 19 kN/m\(^3\) (Pula et al., 2005). The wall system is characterised by \( \beta = 0^\circ, \delta = 0^\circ, h = 12 \text{ m}, z = 2.5 \text{ m}, z_p = 3 \text{ m} \) and \( q = 0 \text{ kPa} \). With respect to angles of internal friction \( \phi \) (= 27\(^\circ\) to 32\(^\circ\)) of the soil, the values of anchor force \( F \) (kN/m) are determined (see Table 1) using the newly proposed limit analysis method. The associated slip lines are shown in Figure 2. Table 1 indicates that the values of the anchor force and some geometric features of the slip line obtained by the proposed method are in good agreement with the variational calculus, despite the differences at \( \phi = 27^\circ, 34^\circ \) and \( 35^\circ \). At \( \phi = 30^\circ \) to \( 33^\circ \), the anchor forces obtained by the proposed method are also similar to those obtained from the limit equilibrium method (without considering moment equilibrium) based on Rankine’s pressure theory. The impact of the shape of the slip line (log-spiral in the current method, see Figure 2) is limited on the force \( F \) as against the latter based on a linear slip line. This is especially true at a large internal friction angle (e.g. \( \phi > 32^\circ \)) as the corresponding log-spiral slip line approaches closely to a straight line and exhibits the sliding mode of a coarse soil slope. Overall, the proposed method is acceptable in most cases.

Next, an anchored sheet wall is adopted to retain deep excavation engineering in cohesive soil (Pula et al., 2009). The clay has a unit weight, \( \gamma \), of 20·11 kN/m\(^3\), cohesion, \( c \), of 30 kPa and internal friction angle, \( \phi \), of 17\(^\circ\). The wall system has the features of \( \beta = 0^\circ, h = 15 \text{ m}, z = 3 \text{ m} \) and \( z_p = 3 \text{ m} \). Given the conditions of no surcharge, \( q = 0 \), on the upper and lower ground surface, and smooth wall (with a friction angle \( \delta = 0^\circ \)), the current limit analysis method offers an anchor force, \( F \) of 177·041 kN/m. This force agrees well with \( F = 164·3 \text{ kN/m} \) and 174 kN/m estimated using the classical limit equilibrium method and variational calculus, respectively (see Table 2). Figure 3 shows similar curved slip lines between the proposed method and variational calculus, but both are different from the straight line determined using the classical limit equilibrium method.

### 4. Parameter study and discussion

Based on Equation 9, 11 or 12, the effects of related parameters on the anchor force are further analysed for the above-mentioned example problems.

Figures 4, 5, 6 and 7 provide the dimensionless \( F_i(0·5\gamma h^2) \) for typical internal friction angles of the soil, \( \phi \), plotted against...
wall friction angle, $\delta$, dip angle of anchor, $\beta$, normalised cohesion, $c/\gamma h$, or surcharge, $q/\gamma h$, and $z/h$ or $z_p/h$, respectively. Figure 4 indicates that $F/(0.5\gamma h^2)$ reaches the maximum value for a frictionless wall ($\delta = 0$), and the associated anchor force is on the conservative side. Figure 5 shows that the influence of the dip angle of the anchor on $F/(0.5\gamma h^2)$ depends to a great extent on the internal friction angle, $\phi$, of the soil. At $\phi > 29^\circ$, the $F/(0.5\gamma h^2)$ shows slight increase with the dip angle;
Figure 5. Relationship between $F(0.5 \gamma h^2)$ and dip angle of anchor $\beta$ ($h = 12$ m)

Figure 6. Relationships between $F(0.5 \gamma h^2)$ and dimensionless factors $c/(\gamma h)$ and $q/(\gamma h)$ ($h = 12$ m): (a) $F(0.5 \gamma h^2)$ plotted against $c/(\gamma h)$; (b) $F(0.5 \gamma h^2)$ plotted against $q/(\gamma h)$

otherwise at $\phi < 29^\circ$, the increase is remarkable. An increase in $c/(\gamma h)$ reduces the $F(0.5 \gamma h^2)$ (see Figure 6(a)); and increasing $q/(\gamma h)$ causes a linear increase in $F(0.5 \gamma h^2)$ (see Figure 6(b)). The ratios $z/h$ and $z_p/h$ greatly influence $F(0.5 \gamma h^2)$ (see Figure 7). The $F(0.5 \gamma h^2)$ decreases with increasing $z/h$, but the trend is not distinct at $\phi > 29^\circ$. In contrast, the $F(0.5 \gamma h^2)$ decreases with increasing $z_p/h$ for the internal friction angles investigated.

The effects of essential parameters $\beta$, $q$, $c$, $z$, $z_p$, and $F_e$ on anchor force are illustrated in the following subsections.

4.1 Dip angle $\beta$ of anchor

Under various dip angles of the anchor, the anchor force obtained by the proposed method is shown in Figure 8, together with those obtained by variational calculus (Pula et al., 2005). There is a small difference between the two methods at $\phi = 35^\circ$. The overall comparison is satisfactory; in
particular, the presented method is much more readily calculated than the variational calculus.

4.2 Surcharge $q$ on the ground surface

As shown in Figure 9(a), the anchor force increases ‘linearly’ with increasing surcharge.

4.3 Cohesion $c$ of soil

According to Equations 9, 11 and 12, it is clear that a higher cohesion of the soil reduces the anchor force. This reduction is generally non-linear, as is evident from Figures 9(b1) and 9(b2). Figure 9 also shows that the forces obtained by the proposed method are larger than those calculated using the limit equilibrium method (ignoring the moment equilibrium on the wall) based on Rankine’s pressure theory. The latter estimation is not safe for practical engineering. Figure 9(b3) demonstrates that the log-spiral slip line under the condition of $c = 5$ kPa also gradually approaches a straight line with an increasing internal friction angle of the soil.
4.4 Depth of anchor force action point on the wall
Figure 10(a) shows that the anchor force gradually decreases with increasing depth of the anchor force action point on the wall (below the ground surface). The decreasing rate is high at \( \phi < 29° \), otherwise the rate is small.

4.5 Penetration depth \( z_p \) of the wall
Figure 10(b) indicates that the anchor force decreases approximately linearly (at a similar rate) with increasing penetration depth of the wall when the internal friction angle is higher than 30°.

4.6 Design safety factor \( F_s \)
The anchor force as discussed above is for the critical state of soil. In practice, determining design anchor force in a working condition of soil (with a factor of safety, \( F_s \)) is of much greater significance. According to the shear strength reduction method (Zienkiewicz et al., 1975), the design safety factor, \( F_s \), is defined as

\[
F_s = \frac{c}{c_t} = \frac{\tan \phi}{\tan \phi_t}
\]

Equation 13 is recast into

\[
\phi_t = \arctan \left( \frac{\tan \phi}{F_s} \right)
\]

The design anchor force for a safety factor, \( F_s \), is determined by simply substituting \( c_t \) and \( \phi_t \) for \( c \) and \( \phi \) in Equations 9, 11 or 12. Figure 11 (for instance for one row of anchors) shows the design anchor force grows non-linearly as the design safety factor increases for both cohesionless soil and cohesive soil.
4.7 Multiple rows of anchors
In retaining deep excavation, multiple rows of anchors are commonly adopted. Given identical anchor force and identical dip angle of each row of anchors (see Equation 11), it is safe to say that anchor force depends mainly on the sum, $z_s$, of the anchor depths (e.g. $z_1$ and $z_2$) of all rows and is not directly influenced by the specific depth of each row. The anchor force decreases as the sum $z_s$ increases (see Figure 12). The rate of the decrease is negligible at $\phi > 30^\circ$, and the anchor force is nearly independent of the anchor depth. Alternatively, the ratio of anchor force of each row over the first row (nearest to the ground surface) is assumed as 1:1:1, 1:0·9:0·8 and 1:1·1:0·9.

Figure 13. Relationship between sum of multiple rows of anchor forces and dip angle of anchor in the case of a specified ratio for anchor force of each row over the first row: (a) $\phi = 27^\circ$; (b) $\phi = 29^\circ$; (c) $\phi = 31^\circ$; (d) $\phi = 33^\circ$. (e) Sketch map of multiple rows of anchors. I, II, III represents case number.
The resulting sum of anchor forces of each row is obtained using Equation 12 and is shown in Figure 13. This result indicates that the ratio has negligible effect on the sum except for the conditions of $\phi = 27^\circ$ and $\beta \geq 15^\circ$ (see Figures 13(a)–13(d)), and the anchor force of each row can be regarded as identical in practical design under these conditions.

Figure 14. Comparison of anchor forces between the predicted and measured results: (a1) CEBTP wall ($h = 7$ m); (a2) CEBTP wall ($h = 5$ m), (a3) CEBTP wall ($h = 3$ m); (b) Davis wall ($h = 9.2$ m); (c) diagrammatic sketch of the CEBTP wall; (d) diagrammatic sketch of the Davis wall.
5. Comparison between theoretical predictions and experimental results

The proposed method is here compared with the tests on two classical soil-nailed walls: a Centre d’Expertise du Bâtiment et des Travaux Publics (CEBTP) wall in sand soil (Plumelle, 1986) and a Davis wall in cohesive soil (Shen et al., 1981). The relative ratios of anchor force of each row over the first row are prescribed in advance as unity and as identical to measured data (termed ‘Calibrated’), respectively, in order to calculate the anchor force of each row. First, taking an identical anchor force among different rows, the predicted (tensile) anchor force of a soil nail is approximately equal to the measured maxima in the CEBTP wall experiment at a wall height of 7 m or 5 m (see Figure 14). The predictions, however, overestimated the measured maximum by 12% and 18% for the 3 m-high CEBTP wall and the Davis wall, respectively. Next, the ratios of anchor force of each row over the first row are taken as the experimental ratios for the CEBTP wall, the tensile anchor forces in the soil nails were re-estimated, and are plotted in Figures 14(a1), 14(a2) and 14(a3) as ‘Calibrated’ force. As expected, the ‘calibrated’ tensile anchor forces for all rows follow a similar trend to the measured data. They are slightly higher than the measured values owing to the proposed method being underpinned by the upper bound theorem of limit analysis.

6. Conclusions

For an anchored sheet wall, the limit analysis method based on a kinematically admissible displacement mechanism is suitable for analysing the anchor force, and an explicit formula for the force can be obtained. The proposed method is easily applied. The following conclusions can be drawn.

(a) The internal friction angle and cohesion of the soil and the depth of excavation have major effects on the anchor force. The surcharge on the ground surface and the penetration depth of the wall also significantly affect the anchor force. Relatively speaking, the depth from the anchor force action point to the ground surface has a minor impact on the anchor force. The wall friction angle affects slightly the anchor force and its calculation result is conservative if the angle is assumed to be zero degrees. For multiple rows of anchors, the anchor force is linked much more closely with the sum of the depth of each row of anchors than with the specific depth of each row.

(b) On the one hand, the anchor force gradually grows with the increasing dip angle of the anchor. Thus, the dip angle of the anchor should perhaps be as small as is practically possible. On the other hand, the influence depends greatly on the internal friction angle of the soil and is distinctive when the friction angle is smaller than 30°.

(c) Based on the design safety factor defined by shear strength reduction, the anchor force increases non-linearly with the safety factor for both cohesive and cohesionless soil.

(d) Whether the soil is cohesionless or not, the log-spiral slip line approaches a straight line with increasing internal friction angle of the soil. In addition, the anchor force obtained by the limit equilibrium method (without considering moment equilibrium) based on Rankine’s pressure theory is not conservative for practical engineering, when compared with the proposed method.

Appendix

In Figure 1, there is the following geometric relationship

15. \[ \frac{h}{r_0} = e^{(\theta_i - \theta_0)} \tan \phi \times \sin \theta_0 - \sin \theta_0 \]

According to the concept of the gravity work rate, the coefficients of the gravity work rate can be expressed as follows

16. \[ s = r_0 \cos \theta_0 - r_0 \cos \theta_0 \]

17. \[ f_1 = \frac{(3 \tan \phi \cos \theta_0 + \sin \theta_i) e^{(\theta_i - \theta_0) \tan \phi} - 3 \tan \phi \cos \theta_0 - \sin \theta_0}{3(1 + 9 \tan^2 \phi)} \]

18. \[ f_2 = \frac{1}{3} \frac{h e^{(\theta_i - \theta_0) \tan \phi}}{r_0} \cos^2 \theta_0 \]

19. \[ f_3 = \frac{s}{6r_0} \left( \cos \theta_0 + e^{(\theta_i - \theta_0) \tan \phi} \times \cos \theta_0 \right) \times \sin \theta_0 \]

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REFERENCES


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