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**LESSONS FROM THE FINANCIAL THEORY OF HORSE
RACING**

by

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Lessons From the Financial Theory of Horesracing

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Introduction. Gambling, Investing, and Information

Throughout history, man's understanding of Risk appears to have been led by those seeking to accumulate wealth through games of chance, and, much later, through investment¹. Generally, there was little development in the understanding of Risk or Chance until the 18th century, when mathematicians such as Bernoulli, Pascal, Laplace, and others began to investigate and characterise even the most elementary properties of coins and dice.

One of the first notions introduced at that time, which we shall make frequent use of here, was that of *Expected Value*, or *Expectation*. Simply stated,

Expected Value is the average of possible outcomes, weighted by the relative likelihoods of those outcomes

For example, the *expected* number of 'Heads' occurring in one toss of a fair coin is .5, or $(\frac{1}{2})0 + (\frac{1}{2})1$. As Bernoulli first proved², independent repetition of a chance experiment many times, would yield with certainty the same average result. In proving this, he made use, as we shall here, of a quantity known as the *variance*, or the expected squared-distance of an outcome from its expected outcome. In the case of a single toss of a fair coin, this is $(\frac{1}{2})(0 - \frac{1}{2})^2 + (\frac{1}{2})(1 - \frac{1}{2})^2 = \frac{1}{4}$. Note for example that an experiment that returned the result of .5 with certainty on each trial would have the same expectation as the number of 'Heads' in a single toss of a fair coin, but would have a *variance* of zero, indicating **no** variability at all.

As Bernoulli did (though not necessarily in the same terms), we shall here make

1. For an interesting and entertaining review of the historical development of man's understanding of Risk, see Bernstein's *Against the Gods*

2. *Ars Conjectandi* (1713)

the distinction between Gambling and Investing, with regard to return: If there exists some function of an individual's wealth, increasing at a rate which is staying the same or decreasing³ which is expected to improve with some (voluntary) financial transaction or game, that transaction is regarded as a *rational*; whereas if there exists no such function, the transaction is regarded as a *irrational*⁴. In the sequel, we shall refer to all rational financial transactions as *Investments*, and irrational ones as *Gambles*.

In the early 1950's, Harry Markowitz (relying on results of Laplace) noticed that for a repetition of many independent chance outcomes⁵, the character of total chance outcome (which represents our evolving wealth) will be bell-curve distributed, having an expectation which is the sum of all the expectations of the smaller chance outcomes, and a variance which is likewise the sum of the variances of the smaller chance outcomes. Since the bell-curve variable (sometimes called the 'Normal') depends only on its expectation and variance, it is in many cases sufficient to restrict one's attention in financial transactions to merely the Expectation-Variance character of those financial transactions. Generally, in investment, one wishes to maximise return and minimise risk. Since return translates most directly to expectation and risk to variance, the problem of Investment in the presence of Uncertainty may be reduced to acting so as to maximise expectation and minimise variance.

While seemingly trivial (given the work of the early 18th century), this observation has had enormous impact, and may even be said to dominate thinking and practice in Finance from the time of Markowitz's original paper to the present.

To fix ideas, we shall argue that (like virtually all Casino games) a bet at the Roulette table is irrational (i.e., a *Gamble*). For a bet on 'black' assume a win, of

3. Such a function is sometimes called a *normative utility function*, in that it does not necessarily represent an individual's preferences, but might be usefully applied as if it did.

4. In fact, Bernoulli appears to have believed, like the author of the present paper, that the only truly rational normative Utility function is logarithmic.

5. Under a wide variety of assumptions

amount equal to the amount bet, occurs with probability $\frac{18}{38}$, and a loss with probability $\frac{20}{38}$. The expectation for a dollar bet, then, is

$$\left(\frac{18}{38}\right)(+1) + \left(\frac{20}{38}\right)(-1) = -\frac{2}{38} \doteq -.05,$$

a loss of 5 cents on the dollar. Clearly, the bet also increases the variability of one's total wealth, making it *doubly* irrational (i.e., decreasing expected wealth *and* increasing risk or variability). Conversely, the individual who makes such a bet not only increases the expected wealth of the Casino, but decreases its average risk (through diversification).

Since such bets are only offered by profit-seeking entities, it is safe to assume that any of those on offer without limit would be irrational ventures.

By contrast, horserace betting, while generally grouped with Casino gambling, can be argued to be potentially rational, given the correct information and application thereof. While betting at random on the NSW totalizator yields a negative expectation of approximately 20%, suggesting that horserace betting is a 'Gamble' for the uninformed punter, it may in fact be possible for a clever form student to use information to tip the odds in his/her favour. Such an individual, if he or she existed, would then be regarded as an *investor*, rather than a *gambler* (at least, according to our definition).

Since most money in the financial markets may be expected to grow, on average, this money is generally regarded as 'invested' money. Still, most experts would agree that astute application of information to the financial markets may be used to increase return and decrease risk over what would have been achieved by investing at random.

Unfortunately, with regard to international investment opportunities, the nature of the interrelationships between the many relevant observable variables across international boundaries and over time is so complex, that a pure study of the relationship between information and investment is intractable. By contrast, horseraces appear to be well-modeled by independence, across time and space, while retaining the same essential elements of the relationship between Information and Investment as are contained in the more complicated problem of investment in the global financial environment. Since the most fundamental strides in science appear to come from the simplest

of examples, by focusing on the horserace bet as potential investment, we are in good company⁶.

In the sequel, we shall introduce the various notions and motivate principles using idealised horseracing examples.

Information

We shall begin with a mathematical definition of *Information*.

$$\text{Information(Event)} = -\text{Log}(\text{Assigned Probability(Event)})^7$$

If someone reports the result of a race, the degree of ‘how interesting’ the result was is in keeping with the amount of Informational Content in their message. For instance, if the horse that won was regarded beforehand as an ‘absolute certainty’ (translating, at least theoretically, to a probability near 1), the Information is near

$$-\text{Log}_2(1) = 0,$$

indicating no informational content in the message.

6. Not surprisingly, horserace betting figures prominently in the primary current reference on the subject of Information Theory: *the Elements of Information Theory*, by Prof. Thomas Cover, who (incidentally) owns racehorses.

7. The *logarithm* (base 2, say) of a number a is that number b such that $2^b = a$ [it may be useful to think of logarithm function as the ‘opposite’ of raising to a power].

Logarithms base 2

Number	1	2	4	8	16	32
Log ₂ (Number)	0	1	2	3	4	5

Logarithms base 10

Number	1	10	100	1,000	10,000	100,000
Log ₁₀ (Number)	0	1	2	3	4	5

On the other hand, if the horse which won was even money (probability 50% of winning), the Information is

$$-\text{Log}_2\left(\frac{1}{2}\right) = \text{Log}_2(2) = 1$$

Continuing, if the horse which won was 3-to-1 (probability 25%), the Information is

$$-\text{Log}_2\left(\frac{1}{4}\right) = \text{Log}_2(4) = 2,$$

in some sense 'twice' the Information as in the previous case. To see the logic in this, suppose the results of two (independent) races in which even-money horses had won were reported. Then the Information contained in this report is either

$$-\text{Log}_2\left(\frac{1}{2}\right) + \{-\text{Log}_2\left(\frac{1}{2}\right)\} = 1 + 1 = 2$$

or

$$-\text{Log}_2\left(\frac{1}{2} \cdot \frac{1}{2}\right) = -\text{Log}_2\left(\frac{1}{4}\right) = 2$$

In fact, the Logarithm function is the only function for which the above two methods of computation would agree (hence their use in the mathematical definition of Information).

Optimal Investment in Favourable Games and Investments

Like interest rates, the *exponential rate of growth* is the primary measure of performance in investment. Alternatively, it may be said that in order to optimise your growth of wealth in the presence of reinvestment, one optimises the *Logarithm* of one's wealth.

If money is invested at (fixed) 5%, then the logarithm of wealth will be proportional to $.05 t$, where t denotes the length of time invested. Naturally, the rate, or factor of t which is the largest should be chosen. While this is true for fixed rate, a result

(Bernoulli-Kelly-Breiman) is that it holds for random rates as well. This result is stated as follows

In order to achieve an arbitrary fixed financial goal as soon as possible by investing repeatedly (assuming similar independent, favourable opportunities), choosing investment so as to maximise the expected logarithmic increase in wealth at each stage is optimal

This fact is sometimes referred to as the 'Optimality of the Kelly Criterion'⁸

Equivalently, it may be shown that by maximising the so-called 'Kelly Criterion' you will eventually become wealthier, with probability one, than anyone else with similar opportunities using any other system for investing, *regardless of initial wealth.*

Example How much to bet on an even money horse which has probability .6 of winning its race?

Ans: 20% of the betting pool, which will be assumed to be 1 unit.

Try 10%: $E(\text{Log}_2(W)) = .6\text{Log}_2(.9 + 2(.1)) + .4\text{Log}_2(.9) \doteq .022$

Try 20%: $E(\text{Log}_2(W)) = .6\text{Log}_2(.8 + 2(.2)) + .4\text{Log}_2(.8) \doteq .029$

Try 30%: $E(\text{Log}_2(W)) = .6\text{Log}_2(.7 + 2(.3)) + .4\text{Log}_2(.7) \doteq .021$

suggesting that 20% of one's wealth optimises the tradeoff between expectation and risk [though note expected increase is higher for the 30% investment, at 6%, as compared to 4% for the optimal one].

Example (cont'd) Suppose another horse, offered at 2-to 1, has a 30% chance of winning the race (profit margin: $.30(2) - .7(1) = -.1$). Should one bet on it also?

Ans: Yes. Consider 30% investment in the even money horse and 10% investment in

8. Though the notion was evident in the writings of Bernoulli in the early 1700's, and first proved rigorously by Breiman in the 1960's, it was Kelly who resurrected the idea in the early 1950's, and whose name is generally attached to it.

the 2-to 1 (unprofitable) horse

$$E(\text{Log}_2(W)) = .6\text{Log}_2(.6 + 2(.3)) + .3\text{Log}_2(.6 + 3(.1)) + .1\text{Log}_2(.6) \doteq .039$$

showing that it can sometimes be advantageous to invest in the unprofitable! [Note that in the first scheme, with probability .4 one loses 20% of one's wealth, whereas in the second, with probability .9 at least 90% of one's wealth is retained.]

With the above example, we find the beginnings of Portfolio Theory.

(Paradoxical) Example If you are offered the opportunity of either doubling or halving your entire wealth, each with equal probability, as many time as you wish, is this a good offer?

Ans: No. While the expected increase in your wealth would be by a factor of $(\frac{1}{2})2 + (\frac{1}{2})(\frac{1}{2}) = 1.25$ (representing a 25% increase), the expected logarithmic increase of your wealth would be

$$(\frac{1}{2})\log_2(2) + (\frac{1}{2})\log_2(\frac{1}{2}) = 0,$$

indicating a zero growth rate.

Suppose, on the other hand, you were allowed to avail yourself of the opportunity, but with *half* of your wealth, rather than all of it. The expected increase in your wealth in this case would be by a factor of $(\frac{1}{2})(1.5) + \frac{1}{2}(.75) = 1.125$, or by 12.5%. The expected logarithmic increase in your wealth in this case is

$$(\frac{1}{2})\log_2(1.5) + (\frac{1}{2})\log_2(.75) = .085,$$

decidedly favourable. After 25 plays, the lower probability quartile of your wealth following this betting scheme is a 37% increase, as compared to a 90% decrease for the previous (full-investment) case. At 100 plays, the corresponding lower quartiles are a 250% increase versus a 99% decrease. Clearly the fractional betting scheme represents better risk management in this case.

Doubling Rate

In the previous section, we presented the recipe for financial optimisation, and applied it to some betting examples. We did not discuss the meaning of the resulting index. For the first example of that section, we achieved an expected logarithm of .029, and for the second an expected logarithm of .039. How do these numbers compare? This number is sometimes called the *doubling rate* of the investment. This is because if one bet in n races with expected logarithm .029, the wealth would be expected to increase by a factor of approximately $2^{.029n}$. For this factor to be 2 (indicating a doubling of one's wealth), the product $.029n$ would have to be 1, showing that $n = 1/.029 \doteq 35$ repetitions would be required. For the second (hedged) case, the n required would be approximately $1/.039$, or approximately 25 repetitions, a tangible improvement.

Incidentally, in finance, it is more common to use *natural* logarithms generally, and if one does so, the 'doubling rate' translates to a 'growth rate' which may be compared directly with fixed rates (e.g., interest rates).

Financial Value of Information

We have not yet related Information directly to profitability. To this end, we motivate the relationship with a betting example.

An Example. Four horse race.

In order to report the results of a four-horse race, two bits⁹ are necessary and sufficient:

Horse	Code
1.	10
2.	11
3.	00
4.	01

Suppose, however, that it were known that horses 1, 2, 3, and 4 had respective

9. 'bit' is an abbreviation of 'binary digit', which is equivalent to the answer of one 'Yes/No' question

probabilities $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{8}$ of winning the race. Then it would be possible to arrange a code that would require fewer bits to report, on average, than the two required by the above two-bit one:

Horse	Code	Prob.
1.	1	$\frac{1}{2}$
2.	01	$\frac{1}{4}$
3.	001	$\frac{1}{8}$
4.	000	$\frac{1}{8}$

On average, the number of bits required to report the results of the race is

$$1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 3 \cdot \left(\frac{1}{8} + \frac{1}{8}\right) = \frac{7}{4},$$

an amount of $\frac{1}{4}$ less than 2.

Next, suppose that odds of 3-to-1 (i.e., gross payoff of \$4.00 per unit bet) are offered for each of the four horses. It must next be decided which horses should be bet, and the amounts.

To this end, we shall, for simplicity, begin by making the (common sense) assumption that no money should be bet on the least profitable (in sense of expected return per unit of investment) horses, in this case horses 3 and 4. The expected log-return assuming initial wealth of 1 unit and betting x_1 units on horse 1 and x_2 units on horse 2, is given by

$$\frac{1}{2} \log_2 \{1 + 4x_1 - (x_1 + x_2)\} + \frac{1}{4} \log_2 \{1 + 4x_2 - (x_1 + x_2)\} + \frac{1}{4} \log_2 \{1 - (x_1 + x_2)\}$$

This function may be plotted as a function of betting amounts x_1 and x_2 , and may in fact be optimised, either analytically or numerically, with the result that the maximum occurs when $x_1 = \frac{3}{8}$ and $x_2 = \frac{1}{8}$. At this point, the expected log-return is equal to

$$\begin{aligned} & \frac{1}{2} \log_2 \left(4 \cdot \frac{3}{8} + \frac{1}{2}\right) + \frac{1}{4} \log_2 \left(4 \cdot \frac{1}{8} + \frac{1}{2}\right) + \frac{1}{4} \log_2 \left(4 \cdot 0 + \frac{1}{2}\right) \\ &= \frac{1}{2} \log_2(2) + \frac{1}{4} \log_2(1) + \frac{1}{4} \log_2\left(\frac{1}{2}\right) = \frac{1}{4} \end{aligned}$$

It will be recalled that this value is precisely the amount of reduction in bit length achieved above using superior knowledge of the probabilities.

This is a demonstration of a more general result:

Achievable Financial Rate of Return = Information Improvement
--

Thus, in this case, it appears that knowledge which enables a reduction in description length of $\frac{1}{4}$ bit also enables investment which achieves a doubling rate of $\frac{1}{4}$ (the latter meaning that it requires 4 plays of this game, on average, to double one's wealth).

Applications and Extensions

The relation between profitability and coding (description length) may be used to fit probability models.

A probability model which utilises the minimum description length criterion is much easier to fit than one which might be based on optimal investment, since determination of optimal portfolios for all various candidate probability distributions would be required for the latter but not the former.

It may be argued that a logistic regression or Artificial Neural Network model based on Maximum Likelihood would achieve this result, provided the model class were rich enough. This is, in fact, been applied successfully to Australian and U.S. horseracing form data, and is being extended to the financial markets.

It should be mentioned that the basic identity between Information and Doubling Rate has been extended to continuous random variables, such as bell-curve distributed variables. Thus Financial Market returns may be analysed in this way as well, but these results are mathematically beyond the present discussion.

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