Subcarrier allocation for OFDMA wireless channels using lagrangian relaxation methods

Gengfa Fang
Chinese Academy of Sciences

Yi Sun
Chinese Academy of Sciences

Jihua Zhou
Chinese Academy of Sciences

Jinglin Shi
Chinese Academy of Sciences

Zhongcheng Li
Chinese Academy of Sciences

See next page for additional authors
Subcarrier allocation for OFDMA wireless channels using lagrangian relaxation methods

Abstract
In this paper, we propose a practically efficient Subcarrier Allocation scheme based on Lagrangian relaxation to solve the problem of subcarrier allocation in OFDMA wireless channels. The problem of subcarrier allocation is formulated into an Integer Programming (IP) problem, which is relaxed by replacing complicating constraints with Lagrange multipliers using Lagrangian Relaxation. A subgradient method is used to optimize the Lagrangian dual function and a heuristic is designed to obtain the feasible solution. Lagrangian Relaxation Subcarrier Allocation (LRSA) is proven to be of polynomial complexity and it provides bounds on the value of channel efficiency. Numerical results show that compared with other algorithms proposed in the literature, LRSA can result in a significant improvement in channel efficiency, while at the same time guaranteeing minimum data rates of users.

Disciplines
Physical Sciences and Mathematics

Publication Details

Authors
Gengfa Fang, Yi Sun, Jihua Zhou, Jinglin Shi, Zhongcheng Li, and E. Dutkiewicz

This conference paper is available at Research Online: http://ro.uow.edu.au/infopapers/2995
Subcarri er Allocation for OFDMA Wire less Channels Using Lagrangian Relaxation Methods

Gengfa Fang, Yi Sun, Jihua Zhou, Jinglin Shi, Zhongcheng Li
Institute of Computing Technology
Chinese Academy of Sciences, Bejing, China
{gengfa.fang, sunyi, jhzhou, sjl, zcli}@ict.ac.cn

Eryk Dutkiewicz
Wireless Technologies Laboratory
University of Wollongong, Wollongong, Australia
eryk@uow.edu.au

Abstract—In this paper, we propose a practically efficient Subcarrier Allocation scheme based on Lagrangian relaxation to solve the problem of subcarrier allocation in OFDMA wireless channels. The problem of subcarrier allocation is formulated into an Integer Programming (IP) problem, which is relaxed by replacing complicating constraints with Lagrange multipliers using Lagrangian Relaxation. A subgradient method is used to optimize the Lagrangian dual function and a heuristic is designed to obtain the feasible solution. Lagrangian Relaxation Subcarrier Allocation (LRSA) is proven to be of polynomial complexity and it provides bounds on the value of channel efficiency. Numerical results show that compared with other algorithms proposed in the literature, LRSA can result in a significant improvement in channel efficiency, while at the same time guaranteeing minimum data rates of users.

I. INTRODUCTION

Unlike wireline channels, channels in broadband wireless access (BWA) networks are prone to frequency selective fading. Orthogonal Frequency Division Multiple Access (OFDMA) is a modulation and multiple access method used in IEEE 802.16 [1] and IEEE 802.11a and is considered for use in emerging 4th generation wireless networks because of its high immunity to inter-symbol interference and frequency selective fading. Unlike OFDM systems, where only a single user is allowed to use all of the subcarriers at any given time, OFDMA allows multiple users to share the subcarriers, which brings new challenges to the resource allocation problem in OFDMA wireless channels.

OFDMA is based on the Orthogonal Frequency Division Multiplexing (OFDM) technique where the total available bandwidth is divided into several narrow subcarriers. OFDMA may result in higher channel efficiency by exploiting the frequency diversity among subcarriers under the control of a properly designed subcarrier allocation algorithm. The main goal of subcarrier allocation in OFDMA wireless channels is, on one hand, to improve the average channel efficiency by exploiting both the time diversity in the time domain and frequency diversity in the frequency domain of the wireless channel. On the other hand, the subcarrier allocation algorithm should guarantee the prescribed QoS requirements of users, such as the minimum data rate considered in this paper.

The problem of subcarrier allocation in OFDMA systems has been recently studied quite extensively. In [2], subcarriers are allocated to users based on the current channel condition and current buffer state as well as the measured ratio of the arrival rate to throughput for each user. In [3], a delay utility function based subcarrier allocation scheme is presented for real time services, where the main goal of the proposed scheme is to maximize the aggregate utility. However the above schemes do not take any QoS requirements into account, thus they provide no guarantee of the minimum service quality for users. In reference [4], Wong et. al. present a new method to solve the subcarrier allocation problem in OFDMA system by proportionally distributing the subcarriers among users based on their QoS requirements. Wong’s algorithm is conservative in the sense of channel efficiency improvement although it can guarantee a proportional fairness among users. Other works on this topic i.e. [5]∼[7], try to combine the subcarrier allocation with the power allocation problem, and offer no better solution to the subcarrier allocation problem.

In this paper, we focus on subcarrier allocation in wireless OFDMA channel where users have QoS requirements in terms of the minimum data rates. The subcarrier allocation process is formulated into an Integer Programming (IP) problem with constraints. We solve the problem based on the Lagrangian relaxation theory and propose an efficient new scheme called Lagrangian Relaxation Subcarrier Allocation: LRSA. The properties and performance of LRSA are verified through simulations.

The rest of the paper is organized as follows. In Section II, the system model of subcarrier allocation in OFDMA wireless channels is introduced and the subcarrier allocation problem is formulated. In Section III, we analyze the subcarrier allocation problem and propose our solution using Lagrangian relaxation methods. Simulation results are given in Section IV, which is followed by our conclusions in Section V.

II. SYSTEM MODEL

In this paper, we consider the downlink OFDMA cell system, where there is only one base station (BS) and multiple users. The wireless channel between the BS and users consists of K subcarriers, which are shared by M users under the control of the subcarrier allocation algorithm in BS. The users measure the current Channel State Information (CSI) based on the received signals and then send SCI back to BS in a predefined feedback channel. During each subcarrier allocation cycle, the scheduler in BS allocates the subcarriers to users.
based on the SCI and QoS requirements of users. Based on the result of subcarrier allocation, bits of users’ data are then modulated into the subcarriers allocated to them. All the subcarriers belonging to different users are then combined into a single symbol and broadcasted to users. Next, we list some notations used in this paper and then formulate the subcarrier allocation problem.

\( i \) Index of users in the cell. \( i \in \{1,2,...,M\} \).

\( j \) Index of subcarriers for the downlink channel. \( j \in \{1,2,...,K\} \).

\( c_{i,j} \) The achievable data rate of user \( i \) at subcarrier \( j \) in the current time slot for a given BER and transmission power.

\( x_{i,j} \) Indication of subcarrier \( j \) allocation to user \( i \). \( x_{i,j} \) equals to 1 if subcarrier \( j \) is allocated to user \( i \), otherwise, \( x_{i,j} \) is equal to 0.

\( R_{i}^{min} \) Minimum data rate in bits per second that user \( i \) should receive in order to guarantee its service quality. This is the only QoS parameter considered in this paper.

\( r_{i}^{-} \) The data rate that user \( i \) has received before the current subcarrier allocation cycle.

\( r_{i}^{+} \) The data rate of user \( i \) after the current subcarrier allocation cycle.

\( r_{i}^{min} \) The minimum data rate that user \( i \) should get in the current subcarrier allocation cycle in order to meet its QoS requirement.

The total bandwidth of the wireless channel is \( B \) Hz and if we use \( \Delta f \) Hz to stand for the bandwidth of each subcarrier, we have \( \Delta f = B/K \). The achievable data rate of user \( i \) at subcarrier \( j \), i.e., \( c_{i,j} \), can be calculated as follows:

\[
    c_{i,j} = \Delta f \log_{2}(1 + \beta \rho_{i,j})
\]

where \( \rho_{i,j} \) is the signal to noise rate (SNR) of user \( i \) on subcarrier \( j \) in current time slot and \( \beta = -1.5 \ln(5+BER) \) [2][3]. In this paper, we assume that power is equally distributed over all the subcarriers so that the maximum feasible data transmission rate is only decided by the current channel quality in SNR and BER. We also assume that the scheduler in the base station knows exactly the channel state information of the subcarriers at the beginning of each subcarrier allocation cycle, i.e., \( c_{i,j} \), is available at the beginning of each scheduling cycle.

Based on the definitions above, we define the subcarrier allocation problem in each allocation cycle as follows: There are \( K \) subcarriers to be allocated to \( M \) users. Each subcarrier \( j \) must be allocated to one and only one user. Each user \( i \) can occupy multiple or no subcarriers during each allocation cycle. A data rate of \( c_{i,j} \) in bits per second will be achieved when subcarrier \( j \) is allocated to user \( i \). During each allocation cycle, a data rate of \( r_{i}^{min} \) in bits per second must be assigned to user \( i \) as a result of subcarrier allocation. We have \( r_{i}^{min} \geq 0 \). The main goal of subcarrier allocation in each cycle is to maximize the overall data rate (or channel efficiency) as a result of subcarriers allocations subject the constraints above. The subcarrier allocation problem can be then formulated as follows:

\[
    f(x) = \max \sum_{i=1}^{M} \sum_{j=1}^{K} c_{i,j} x_{i,j} \tag{2a}
\]

\[(IP) \quad s.t. \quad \sum_{i=1}^{M} x_{i,j} = 1; \quad j = 1,2,...,K \tag{2b}\]

\[
    \sum_{j=1}^{K} c_{i,j} x_{i,j} \geq r_{i}^{min}; \quad i = 1,2,...,M \tag{2c}
\]

\[
    x_{i,j} \in \{0,1\}; \quad i = 1,2,...,M; \quad j = 1,2,...,K. \tag{2d}
\]

Objective function \( f(x) \) is the total data rate as a result of the current subcarrier allocation cycle. Constraints (2b) ensure that each subcarrier can only be allocated to one user. Constraints (2c) are the minimum data rate constraints of users in the current scheduling cycle.

III. LRSA SUBCARRIER ALLOCATION

In this section, we describe in detail how to solve the above subcarrier allocation problem using Lagrangian relaxation. A service tracking and minimum data rate requirement estimation method is also proposed. Following that we present our LRSA algorithm.

A. Lagrangian Relaxation based Subcarrier Allocation

The goal of subcarrier allocation is to maximize the total channel efficiency while at the same time guaranteeing minimum data rates of users as defined in the IP problem in the previous section. Clearly the IP problem is a 0-1 integer optimization problem. Unfortunately, as proven in many references [8], the IP problem is NP-hard. However, by inspection, we find that if we remove the constraints of (2c) in the IP problem, then we get a simplified IP’ problem defined as follows:

\[
    \text{(IP')} \quad f(x) = \max \sum_{i=1}^{M} \sum_{j=1}^{K} c_{i,j} x_{i,j} \quad \text{s.t. constraints} \ (2b) \text{ and } (2d). \tag{3b}
\]

The solution of the IP’ problem above becomes feasible in polynomial time, where the optimal solution is to choose a user \( i^{*} \) with maximum \( c_{i,j} \) for each of the subcarriers in current subcarrier \( j \), i.e.,

\[
    i^{*} = \arg \max_{i} c_{i,j}; \quad j = 1,2,...,K.
\]

Constraints of (2c) are called complicating constraints, which make the IP problem polynomial time unfeasible. Based on the above, it is clear that if we can find a method which can remove constraints of (2c) in a specific way and at the same time keep the linear property of the IP problem then the IP problem becomes polynomial time feasible. This can be achieved using Lagrange relaxation. Next, we briefly explain the main idea in Lagrangian relaxation methodology and construct the Lagrangian dual problem.
Lagrangian relaxation is a mathematical programming technique for solving constrained optimization problems. The main idea of Lagrange relaxation is to create a relaxed problem by replacing complicating constraints with Lagrange multipliers. The relaxed problem then can be decomposed into subproblems, which are much easier to solve compared to the original problem and solutions can be efficiently obtained by dynamic programming. The multipliers are then adjusted iteratively based on the degree of constraint violation while at the same time the subproblems are resolved based on the new multipliers. In mathematical terms, a dual function is maximized or minimized when the multipliers are updated. The value of the dual function is the lower or upper bound of the original problem.

However there are several difficulties when applying this method for solving discrete variable problems such as the subcarrier allocation in the OFDMA channel as defined above. Firstly, the dual function is not differentiable everywhere, so specialized methods for optimizing this nondifferentiable dual function must be employed, such as the subgradient method used in this paper. Secondly, even if the optimum of the dual function is obtained, the corresponding result at that point may not be feasible, so adjustments may be needed to make sure that all the constraints are met.

According to the definition of Lagrange relaxation, we relax the complicating constraint (2c) in IP using Lagrange multiplier \( \lambda_i (i = 1, 2, \ldots, M) \) and then get the following Lagrange Relaxation problem LR:

\[
\begin{align*}
    f_{LR}(\lambda) &= \max \sum_{i=1}^{M} \sum_{j=1}^{K} c_{i,j}(1 + \lambda_i)x_{i,j} - \sum_{i=1}^{M} \lambda_i \hat{r}_{i}^{\text{min}} \\
    \text{s.t. constraints (2b) and (2d)}. 
\end{align*}
\]

So we have:

\[
\forall \lambda \geq 0, \Rightarrow f_{LR}(\lambda) \geq f_{IP}(x). 
\]

Expression (5) indicates that LR is an upper bound on the optimal value of the original IP problem and \( \forall \lambda > 0 \) can be used to produce the upper bound. In order to get the optimal lowest upper bound, our goal is to make the upper bound as close to the optimal value of the original IP problem as possible. We define problem LD as the optimal upper bound of IP and we have:

\[
\begin{align*}
    (LD) \quad f_{LD} &= \min_{\lambda \geq 0} f_{LR}(\lambda) \\
    &= \min_{\lambda \geq 0} \max_{x} \left( \sum_{i=1}^{M} \sum_{j=1}^{K} c_{i,j}(1 + \lambda_i)x_{i,j} - \sum_{i=1}^{M} \lambda_i \hat{r}_{i}^{\text{min}} \right). 
\end{align*}
\]

Problem LD is also called the Lagrange dual problem of IP, which is concave and piecewise linear. Several steps will be presented in order to get the near optimal solution. In the next subsections, we provide the solutions to the subproblems, to the Lagrange dual problem LD and to obtaining a feasible solution.

1) Solving Subproblems : Given Lagrange multiplier \( \lambda \), the Lagrangian dual problem LD leads to the decomposed subproblem for each subcarrier \( j \) as follows:

\[
\begin{align*}
    (SP) \quad \max \sum_{i=1}^{M} c_{i,j}(1 + \lambda_i)x_{i,j} \\
    \text{s.t. constraints (2b) and (2d)}. 
\end{align*}
\]

where the Lagrange multipliers can be interpreted as the gain that user \( i \) uses subcarrier \( j \). Each subcarrier \( j \) tends to be used by user \( i \), who has the best channel quality, i.e., large \( c_{i,j} \), while user \( i \) will also win a gain in terms of \( \lambda_i \) for using subcarrier \( j \). Through inspection, we can solve the subproblem as follows:

\[
i^*_j = \arg \max_i c_{i,j}(1 + \lambda_i); \quad j = (1, 2, \ldots, K).
\]

and let

\[
x_{i,j} = \begin{cases} 
    1; & \text{if } i = i^*_j \\
    0; & \text{otherwise}
\end{cases}
\]

The complexity of solving all the \( K \) subproblems above is \( O(K) \) in contrast to the NP-hard complexity of the original problem, which indicates that the above algorithm is very efficient.

2) Subgradient Based Solution of Dual Problem: Since the subproblems involves discrete variables, the objective function \( f_{LD} \) defined in Lagrange dual problem LD is concave, piecewise linear and may not be differentiable at certain point in the \( \lambda \) space. The subgradient method is commonly used to solve this kind of optimization problem. A subgradient method finds the optimal solution of \( f_{LD}(\lambda) \) by iterative method in which starting with some \( \lambda^0 \), a sequence of \( \lambda^n \), which eventually converges to the optimal solution, is constructed according to:

\[
\lambda^{n+1} = \lambda^n + Q^n S^n. 
\]

where \( n \) is the iteration index, \( Q^n \geq 0 \) is a suitable step length and \( S^n \) is the subgradient vector of \( f_{LD}(\lambda) \) at \( \lambda^n \). We have:

\[
S^n = c x^n - \hat{r}^{\text{min}}. 
\]

where \( x^n \) is a solution of problem SP defined above given \( \lambda^n \), \( \hat{r}^{\text{min}} = (\hat{r}_1^{\text{min}}, \hat{r}_2^{\text{min}}, \ldots, \hat{r}_M^{\text{min}}) \) and \( c \) is the achievable data rate matrix. We have:

\[
\sum_{n=1}^{\infty} Q^n = \infty, \quad \text{and} \quad Q^n \to 0, \quad n \to \infty.
\]

In order to get the optimal solution, we may need an infinite number of iterations, so to obtain a reasonable suboptimal solution as quickly as possible, we define \( Q^n \) as follows:

\[
Q^n = Q^0 \rho^n, \quad 0 < \rho < 1.
\]
when the change of $\lambda$ is small enough, i.e., $\lambda < \varepsilon$ where $\varepsilon$ is a predefined system parameter.

3) Obtaining Feasible Solution: Since the minimum data rate constraints have been relaxed, the solution of the dual problem above may be infeasible, i.e., constrains (2c) may not be met. We present a heuristic to convert the infeasible subcarrier allocation result to a feasible one. The main idea of our heuristic is to do the adjustment for each subcarrier, such that it leads to the least decrease in channel efficiency. We call the heuristic Least Lose First (LLF), which works as follows:

**Step 1:** Let $SC$= the set of all subcarriers.
**Step 2:** Pick a subcarrier $j$ randomly from $SC$, Let $i^*$ be the MS that subcarrier $j$ is allocate to, i.e., $x_{i^*,j} = 1$ and $U$ be the remaining.
**Step 3:** If there is MS $i$, where $i \in U$ and $r_i^+ < i^+_i$, we select a MS $i'$ according to: $i' = \arg \min_U (\lambda_i)$.
**Step 4:** Assign subcarrier $j$ to MS $i'$ and do:
   
   
   
   
   
   where $\mu$ is a parameter defined in next subsection.
**Step 5:** Set $SC=SC\backslash j$. If $SC = \phi$, stop, else go to Step 2.

The selection of MS in step 2 comes from the idea that users with small $\lambda_i$ may have relatively better channel quality, so that the exchange may result in the least decrease in channel efficiency.

B. Service Tracking and Overall Algorithm

The above algorithm relies on the availability of $\hat{r}_i^{\min}$ for users at the beginning of each subcarrier allocation cycle. $\hat{r}_i^{\min}$ is calculated based on the history information of subcarrier allocation and the QoS requirements of users in terms of $R_i^{\min}$. Next we provide a method to calculate $\hat{r}_i^{\min}$ based on the commonly used sliding window mechanism. We use $T$ to stand for the size of the sliding window in terms of time slots and we have:

$$r_i^+ = \frac{T - 1}{T} r_i^- + \frac{1}{T} \hat{r}_i. \tag{11}$$

where $\hat{r}_i$ is the data rate received in current subcarrier allocation cycle. In order to guarantee the minimum data rate requirements of users, we have:

$$r_i^+ \geq R_i^{\min}. \tag{12}$$

Let $\mu = \frac{1}{T}$, then we have:

$$(1 - \mu) r_i^- + \mu \hat{r}_i \geq \frac{R_i^{\min}}{\mu} \Rightarrow \hat{r}_i \geq \frac{R_i^{\min} - (1 - \mu) r_i^-}{\mu}. \tag{12}$$

$\hat{r}_i^{\min}$ equals to the minimum value of $\hat{r}_i$, while at the same time $\hat{r}_i^{\min}$ must be larger than zero, so we have:

$$\hat{r}_i^{\min} = \begin{cases} 0 & \text{if } R_i^{\min} - (1 - \mu) r_i^- < 0 \\ R_i^{\min} - (1 - \mu) r_i^- & \text{otherwise} \end{cases} \tag{13}$$

Based on the Lagrangian relaxation subcarrier allocation framework above, we present the overall algorithm, i.e., LRSA as follows:

1. Choose a starting point $\lambda^0$, $Q^0$, and let $n=0$;
2. Calculate $\hat{r}_i^{\min}$ according to (13) for all users, and solve the subproblem $SP(\lambda^n)$ by:

$$f_{LR}(\lambda^n) = \max \left\{ \sum_{i=1}^{M} \xi_{i,j}(1 + \lambda^n) x_{i,j} - \sum_{i=1}^{M} \lambda_i^n \hat{r}_i^{\min} \right\}.$$ 

Let $x^n$ be the solution of this subproblem.

3. Calculate subgradient vector $S^n$ at $\lambda^n$ by

$$S^n = c x^n - \hat{r}_i^{\min}. \tag{14}$$

4. If $S^n \geq 0$, $\lambda^n$ is the optimal solution then stop, otherwise go to Step 5.
5. If stop condition $\lambda < \varepsilon$ is met, then stop, otherwise go to Step 6.
6. Let $\lambda^{n+1} = \max \left\{ \lambda^n + Q^n S^n, 0 \right\}$, $Q^n = Q^0 \rho^n$, $n=n+1$, go to Step 2.
7. Obtain the feasible solution following the LLF algorithm presented earlier above.

IV. SIMULATION RESULTS

In this section, we study performance of LRSA by simulations. Our simulation is based on the implementation of IEEE 802.16. We consider a single cell with one Base Station and a varying number of Mobile Stations. We assume that there are 32 subcarriers (subchannels) in the wireless channel between the Base Station and the Mobile Stations. The frequency selective fading wireless channels are emulated by a nine-state Markov chain [9]. Different states represent different channel qualities and data rates by which users can transmit their data. In this paper, we are concerned with data services, that have prescribed minimum data rates requirements as defined in the IEEE 802.16 standard.

A. Channel Efficiency

Figure 1 shows channel efficiency as function of the number of users. We compare the channel efficiency of LRSA with that of Best Channel First (BCF), Wong’s algorithm [4] and SAMDRA [10]. BCF chooses the best channel quality user on each subcarrier, so it is the optimal solution in the sense of channel efficiency. As shown in Figure 1, channel efficiency increases when the number of users increases. This is because frequency diversity increases when more users enter into the network, so that each subcarrier can be allocated to relatively better channel quality users.

Compared to the other algorithms, Wong’s algorithm has the lowest channel efficiency. This is because Wong’s algorithm restricts the maximum number of sucarriers that can be allocated to a user during each allocation cycle in order to guarantee the weighted proportional fairness among the users.

We also can see that channel efficiency of SAMDRA and LRSA is very high and is almost equal to that of the BCF.
optimal solution when the number of admitted users is less than 25. On the other hand, channel efficiency improvement decreases when the number of user is more than 25, for at this moment, the system becomes overloaded and the subcarrier allocation algorithm must guarantee bad channel quality users’ the minimum data rates at the expense of channel efficiency. LRSA has a higher channel efficiency compared to SAMDRA especially when the number of users is larger than 30. Figure 1 also shows the effect of system parameter $\varepsilon$ in LRSA, which reflects the trade-off between channel efficiency improvement and computing complexity.

B. Minimum Data Rate Guarantee

Figure 2 shows the average data rates received by users where users have different channel qualities. There are 30 users in total in the system. Channel quality decreases as the user index increases from user 1 to user 6. The 24 other users have middle level channel quality as that of user 3. We can see that LRSA can allocate more subcarriers to better channel quality users than the other algorithms, while guaranteeing their minimum data rates. This is the reason why LRSA has high channel efficiency as shown in Figure 1.

In Figure 3, we assign user 1, user 2 and user 3 with different channel qualities, i.e., user 1 with high channel quality, user 2 with middle and user 3 with low. The three users have their minimum data rate requirement set to 30kbps (480 packet per second) 20kbps (320 packet per second) and 10kbps (160 packet per second), respectively. As indicated in Figure 2, when the number of users increases, the date rates of user 1 user 2 and 3 decrease to the minimum data rate prescribed. When the system payload is increasing, data rates of bad channel quality users decrease to their minimum data rates earlier. This is because LRSA is more likely to allocate the subcarriers to users with better channel qualities in order to improve the overall channel efficiency.

V. CONCLUSION

In this paper, we have considered the subcarrier allocation problem in OFDMA wireless channels. A practically efficient Lagrangian Relaxation based Subcarrier Allocation (LRSA) scheme is proposed. LRSA is proven to be of polynomial complexity and provides bounds on the value of the objective function, i.e., channel efficiency. Numerical results show that, in comparison with other algorithms, LRSA can result in significant improvement in channel efficiency while at the same time guaranteeing the minimum data rate of users.

REFERENCES