Energy Efficiency of Uplink Massive MIMO Systems With Successive Interference Cancellation

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Abstract—The energy efficiency (EE) of an uplink massive multiple input multiple output (MIMO) system depends strongly on the number of antennas at base stations (BSs) and the receiver architecture. The existing research has focused on linear receivers such as those based on zero forcing (ZF) or linear minimum mean squared error (LMMSE) detectors. In this letter, we investigate the EE when nonlinear successive-interference cancellation (SIC) receivers are employed at the BSs and provide an asymptotic analysis of the total transmit power with ZF-SIC. We show that to achieve the same spectrum efficiency, the SIC receivers require fewer antennas at BSs, but only moderate increase of the receiver complexity. As a result, the EE with the SIC receivers can be significantly higher than that with linear receivers, as demonstrated by the numerical results.

Index Terms—Energy efficiency, massive MIMO, successive interference cancellation.

I. INTRODUCTION

Energy efficiency (EE) has been a major concern in wireless communications in the past decade due to the increasing demands from customers and the rapid development of wireless networks [1]. Massive multiple input multiple output (MIMO) has been widely considered as one of the key techniques for the fifth generation (5G) wireless networks [2]. By employing a large number of antennas at the base stations (BSs), massive MIMO can significantly improve the rate and reliability of data transmission. The transmitted power for a given transmission rate is expected to decrease by using massive MIMO [3]. Thus, the overall EE may be improved if the increase of power consumption of radio frequency (RF) circuits is moderate.

However, the power consumption by the RF circuit and baseband processors can significantly influence the EE, especially for a massive MIMO system [4], [5]. Such power consumption can increase significantly due to the increased number of antennas and RF chains at the BSs. Meanwhile, the number of antennas needed to achieve a given quality of service (QoS) depends on the receiver architecture. To the best of our knowledge, the study to date has focused on linear receivers. It is found [6] that applying a more sophisticated linear receiver may reduce the number of BSs antennas needed, which results in lower power consumption of RF circuits and hence improve the overall EE. It is unclear whether non-linear receivers could further improve the EE.

In this paper, we study the uplink EE achievable when successive interference cancellation (SIC) based nonlinear receivers are applied, in contrast to the previous studies focusing only on linear receivers. Zero forcing (ZF)-SIC and minimum mean squared error SIC (MMSE-SIC) [7] are investigated and an asymptotic analysis of the total transmit power is provided for ZF-SIC receivers. We show that employing SIC receivers can noticeably reduce the number of needed BS antennas compared to linear receivers for given transmission rates. Meanwhile, the complexity of a SIC receiver can be kept comparable with the classical linear receivers. Consequently, the overall EE can be improved by employing the SIC receivers.

II. SYSTEM MODEL AND EE WITH A LINEAR RECEIVER

A. System Model

Consider an uplink single-cell MIMO system with $M$ antennas at the BS and $K$ single-antenna users (UEs). The system operates in a time-division duplex (TDD) mode with a bandwidth of $B$ Hz and the BS and UEs are perfectly synchronized. Assume that $U$ symbols are transmitted in total for the uplink and downlink within a time-frequency coherence block. Let the uplink ratio be $\zeta_{ul}$. Then $U\zeta_{ul}$ symbols are transmitted in the uplink. We assume that $\tau_{ul}K$ out of the $U\zeta_{ul}$ uplink transmission symbols are used for channel estimation, where $\tau_{ul}$ specifies the pilot length.

Following [8], a uniform gross rate $\bar{R}$ is assigned to each UE. All users are uniformly distributed in a circular cell with radius $d_{\text{max}}$. The path loss for user $k$ due to large scale fading is modeled as [8]

$$\beta_k = \frac{\bar{d}}{\|d_k\|_\kappa}, \quad d_{\text{min}} \leq d_k \leq d_{\text{max}}, 1 \leq k \leq K,$$

(1)

where $d_k$ is the distance between the $k$-th UE and the BS, $d_{\text{min}}$ is the minimum distance, $\kappa$ is the path loss exponent and $\bar{d}$ is used to regulate the channel attenuation at $d_{\text{min}}$ [8]. The signal model is given by

$$y = Hx + n,$$

(2)

where $y \in \mathbb{C}^M$ is the observed signal vector at the BS, $H \in \mathbb{C}^{M \times K}$ is the channel matrix, $x \in \mathbb{C}^K$ contains the transmitted symbols from the $K$ users, and $n \in \mathbb{C}^M$ is the additive white Gaussian noise (AWGN) with variance $\sigma^2$ (in Joule/symbol).

Let $H_{mk}$ be the $(m, k)$-th entry of $H$, denoting the channel gain between the $k$-th UE and the $m$-th BS antenna. Similar to [8], we assume a channel model with $H_{mk} = \sqrt{\beta_k} H_{mk}$,
where $\beta_k$ represents the path loss defined in (1) and $H_{mk}$ characterizes independent Rayleigh fading between the $k$-th UE and the $m$-th BS antenna. We can write

$$H = \bar{H}\text{diag}(\sqrt{\beta_1}, \sqrt{\beta_2}, \ldots, \sqrt{\beta_K}),$$  

where $\bar{H}$ is the Rayleigh fading component of $H$.

### B. Power Consumption Model

We adopt the system-level power consumption model of [8] to take RF chains into account. The total power consumption arises from the transmission and the circuits, i.e., $P_{total} \triangleq P_{TX} + P_{circuits}$.

The power consumed by the power amplifiers at the UEs in uplink transmission is denoted by $P_{TX}$. Let $\eta$ be the power amplifier efficiency at each UE, the uplink transmission power

$$P_{TX} = \frac{B\zeta_{inal}}{\eta}\mathbb{E}\{1^T_Kp\},$$

where $B$ is the bandwidth and $1_K$ is a $K$ dimensional unit vector. $p$ in Joule/symbol is the vector of transmitted power from the $K$ UEs, which can be calculated by (7), (12) and (15), respectively, for the linear and SIC receivers.

$P_{circuits}$ consists of the circuit power consumptions at BS and UEs. Specifically, according to [8], $P_{circuits} \triangleq MP_{BS} + KP_{UE} + P_{SYN} + P_{CE} + P_{CD} + P_{BT} + P_{FX} + P_{SD}$. $P_{BS}$ and $P_{UE}$ denote the power consumption per RF circuit at the BS and UE, respectively. $P_{CD}$ and $P_{BT}$ are consumed by channel coding/decoding and the backhaul including control signalling in uplink transmission, respectively. $P_{SYN}$ is required by local oscillator at the BS and $P_{FX}$ is consumed by site cooling etc. $P_{SD}$ depends on the complexity of signal detection and will be detailed. The uplink channel estimation power consumption, $P_{CE}$, is modified from [8] by ignoring the downlink part.

In this letter, we investigate the influence of the BS receiver on $P_{TX}$, $P_{circuits}$ and EE. Before detailing the results with nonlinear receivers in Section III, we first review the analysis for linear receivers. We assume that perfect uplink channel state information (CSI) is known at BSs. The receiver weight matrix $G$ of the ZF and LMMSE receivers are given by

$$G = \begin{cases} H(H^HH)^{-1}, & \text{ZF} \\ (HPH^H + \sigma^2I_M)^{-1}HP, & \text{LMMSE} \end{cases}$$

where $P = \text{diag}(p_1, p_2, \ldots, p_K)$ is the diagonal matrix with the power of transmitted signals from the $K$ UEs on its diagonal. The resulting filter output, i.e., the estimated transmitted symbol, is: $\hat{x} = GHx$. The achievable uplink transmission rate of the $k$-th user is described as [8]

$$R_k = \zeta_{ual} \left(1 - \frac{\tau_{ual} K}{U_{ual}}\right) \tilde{R}_k,$$

where the term $1 - \frac{\tau_{ual} K}{U_{ual}}$ characterizes the decrease of effective transmission rate due to the overheaded of training, and $\tilde{R}_k = \hat{R}, k = 1, 2, \ldots, K$, are the same gross rates of the $K$ UEs. Let $g_k$ and $h_k$ be the $k$-th columns of $G$ and $H$, respectively. The optimal power allocation for achieving $\hat{R}_k = \hat{R}, k = 1, 2, \ldots, K$, satisfies [8]

$$p = \sigma^2D^{-1}1_K,$$

where

$$D_{k,l} = \begin{cases} \frac{|g_k^Hh_l|^2}{(2^{R/B} - 1)||g_k||^2}, & k = l, \\ -\frac{|g_k^Hh_l|^2}{||g_k||^2}, & k \neq l. \end{cases}$$

The power consumption of signal detection, including the power $P_{WM}$ for computing the weight matrix $G$, is evaluated as

$$P_{SD} = B \zeta_{ual} \left(1 - \frac{K\tau_{ual}^2}{\zeta_{ual}^2U}\right) \frac{2KM}{L_{BS}} + P_{WM},$$

where $L_{BS}$ denotes the computational complexity of [8]. For the ZF receiver [8], $P_{W_MZF} = \frac{B}{\sigma^2U} H_k^3(K^3 + 3MK^2 + MK)_{BS}$, whereas for an LMMSE receiver, $P_{W_MLMMSE} \approx QP_{W_MZF}$, and $Q$ is the number of iterations in the power allocation.

### III. EE With SIC Receivers

We now analyze the EE when the SIC receivers are employed at the BS. The power consumptions of the transmitters and signal detection are now different, as analyzed below.

#### A. Transmitter Power Consumption

When SIC is used to decode the $K$ UEs’ signals, the signals from already decoded UEs are assumed ideally canceled. We assume that at the $k$-th decoding stage, user 1 to $k-1$’s signals are already decoded and canceled. With ZF-SIC, the detector for user $k$ is given by

$$g_k = H_k(H_k^HH_k)^{-1} \bar{g}_k,$$

where $H_k$ denotes a matrix formed by the last $K - k + 1$ columns of $H$ and $\bar{g}_k$ means taking the first column of a matrix. It can be verified that $g_k^HH_k = 1, \forall k$, while $g_k^Hh_l = 0, \forall l > k$. Under the assumption of ideal interference cancellation, the $k$-th UE’s achievable rate is

$$\tilde{R}_k = B \log \left(1 + \frac{p_k}{\sigma^2||g_k||^2}\right).$$

Assuming $\tilde{R}_k = R, \forall k$, the corresponding transmit power should be allocated as

$$p_k = (2^{R/B} - 1)\sigma^2||g_k||^2.$$  

With the MMSE-SIC receiver, the signals from the remaining UEs are treated as noise. Assuming ideal interference cancellation, the $k$th UE’s detector is given by

$$g_k = p_k\left(\sum_{j=k+1}^K p_jh_jh_j^H + \sigma^2I_M\right)^{-1}h_k.$$  

The achievable rate is

$$\tilde{R}_k = B \log \left(1 + p_k h_k^H \left(\sum_{j=k+1}^K p_jh_jh_j^H + \sigma^2I_M\right)^{-1}h_k\right),$$

and the transmit power should be allocated as

$$p_k = \frac{2^{R/B} - 1}{\left(\sum_{j=k+1}^K p_jh_jh_j^H + \sigma^2I_M\right)^{-1}}.$$
In the above we have assumed a natural ordering for detection, i.e., UEs 1, 2, \ldots, K are decoded successively. The performance may be improved significantly by optimizing the detection ordering. Among the various ordering strategies [7], the norm-based ordering represents a sub-optimal but low-complexity solution. With the norm-based ordering, UEs with higher channel gains are decoded first, i.e., the columns of $H$ are sorted such that

$$||h_1||^2 \geq ||h_2||^2 \geq \cdots \geq ||h_K||^2. \quad (16)$$

In order to sort the columns, the Frobenius norms of the $K$ columns of $H$ need to be computed.

B. Asymptotic Transmitted Power Consumption With ZF-SIC

In this subsection, we derive a tractable approximate closed-form expression for the transmitted power consumption when the ZF-SIC receiver is applied, which allows fast evaluation and optimization of the EE for a large $M$. We assume $M > K$ for a ZF-SIC system, which is slightly different from the ZF requirement, i.e. $M \geq K$. Recall that the transmit power of user-$k$ is allocated as (12). It can be verified from (10) that

$$||g_k||^2 = \left( \left( H_k^H H_k \right)^{-1} \right)_{1,1} = \frac{1}{\beta_k \left( H_k^H H_k \right)_{1,1}}, \quad (17)$$

where $[\cdot]_{1,1}$ means the first element of a matrix and $H_k$ denotes the small-scale fading part of $H_k$, i.e., $H_k = \tilde{H}_k \text{diag}(\sqrt{\beta_k}, \sqrt{\beta_k} + 1, \ldots, \sqrt{\beta_K})$. Under the assumption that $\tilde{H}_k$ consists of independent, identically distributed complex Gaussian entries with zero mean and unit variance, $H_k^H \tilde{H}_k$ is a central complex Wishart matrix and we have the following approximation [9]

$$\left( \left( H_k^H \tilde{H}_k \right)^{-1} \right)_{1,1} \approx \frac{1}{M - (K - (k - 1)).} \quad (18)$$

The accuracy of (18) is high when $M$ is large enough, as indicated by our simulation results. With the above approximation, the transmit power of user-$k$ when the ZF-SIC receiver is applied can be approximated by

$$p_k \approx \frac{(2^{R/B} - 1)\sigma^2}{\beta_k (M - (K - 1) + k)}, \quad (19)$$

from which we can approximately compute the total transmitted power consumed by the $K$ UEs as

$$p_{\text{total}, \text{zf}} \approx (2^{R/B} - 1)\sigma^2 \sum_{k=1}^{K} \frac{1}{\beta_k (M - (K - 1) + k)}. \quad (20)$$

From [8] we can verify that when the linear ZF receiver is applied, the total transmit power is approximated as

$$p_{\text{total}, \text{zf}} \approx (2^{R/B} - 1)\sigma^2 \sum_{k=1}^{K} \frac{1}{\beta_k (M - K)}. \quad (21)$$

Comparing (20) and (21), it can be seen that SIC can reduce the overall transmitted power consumption when $M$ is fixed. On the other hand, when the total transmitted power is fixed, i.e., $p_{\text{total}, \text{zf}} = p_{\text{total}, \text{zf}}$, applying SIC can reduce the number $M$ of receiver antennas.

We can also investigate the influence of ordering on the overall transmit power. When $M$ is large enough, according to the central limit theorem [9], $||h_k||^2 = \beta_k ||h_k||^2 \approx \beta_k M$.

With the norm-based ordering in (16), $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_K$, is true with a high probability. Note that the set of values of $\{\beta_k\}$ are fixed given the realization of the channel gains. With the norm-based ordering, a larger value of $\beta_k$ is matched to a smaller value of $M - K - 1 + k$ in the summation of (20). From the rearrangement inequality, it can be verified that norm-based ordering can lead to reduced total transmit power in (20) as compared with the natural detection order.

C. Power Consumption of Signal Detection

Compared to linear receivers, SIC receivers achieve higher transmission rate at the cost of signal processing complexity due to ordering and filter computation. Since the filter and detection order need to be computed in each coherence block, the effect of the increased complexity on EE is moderate under the proper coherence block length assumption.

We start with the detection order of SIC. The squared norms $||h_k||^2$ are sorted for the detection order at a complexity of

$$C_{\text{order}} = K^2 + 2MK. \quad (22)$$

Below we analyze the complexity for computing the ZF-SIC and MMSE-SIC filters $g_k$ separately. For the ZF-SIC, a major cost for finding $g_k$ is to compute $\Phi_k \triangleq (H_k^H H_k)^{-1}$ in (10). This can be implemented recursively by starting from $k = K$ and exploiting the Sherman-Morrison formula as

$$\Phi_k \triangleq \frac{a_k b_k^H}{\Psi_{k+1} b_k} \left( \Phi_{k+1} \right)^{-1} - \frac{b_k^H \Psi_{k+1} b_k}{\Psi_{k+1} b_k + c_k} \Phi_{k+1}, \quad \text{where} \quad a_k = h_k^H h_k, b_k = H_k^H h_k, c_k = a_k - b_k^H \Psi_{k+1} b_k.$$  

It can be seen that the major computations include the calculation of $b_k$ and $\Psi_{k+1} b_k$ for each $k$ and the overall complexity for finding the ZF-SIC filter is approximately

$$C_{\text{pw}}^{\text{ZF,SIC}} = \frac{5}{3} K^3 + 4K^2 M - \frac{5}{2} K^2 - 7KM. \quad (24)$$

Given the detection order, the Sherman-Morrison formula is also useful for power allocation with MMSE -OSIC receivers. Let $\Phi_k \triangleq \sum_{j=k+1}^{K} p_j h_j h_j^H + \sigma^2 I_M$. Then $\Phi_k = \Phi_{k+1} + p_{k+1} h_k h_k^H$. By applying Sherman-Morrison formula,

$$\Phi_k^{-1} = \Phi_{k+1}^{-1} - \frac{\Phi_{k+1}^{-1} p_k h_k h_k^H \Phi_{k+1}^{-1}}{1 + p_k h_k h_k^H \Phi_{k+1}^{-1} h_k}, \quad (25)$$

with $\Phi_{K+1}^{-1} = 1/\sigma^2 I_M$, this can be exploited for computing (15). The complexity of the power allocation and filter calculation for MMSE-SIC is approximately

$$C_{\text{pw}}^{\text{MMSE,SIC}} = \frac{5}{3} K^3 + \frac{9}{2} K^2 M + 4 K M^2 - \frac{3}{2} K^2 - 4 M^2 + \frac{5}{2} K M. \quad (26)$$

The detection of the $K$ users’ signal for each information-carrying symbol using SIC costs [10]

$$C_{\text{symbol}} = 4 MK - K^2 - 2M \quad (27)$$

flops for each channel use. Then the total power consumption
Circuit: ZF

The number of antennas at the BS when the SE is less than 6 bits/s/Hz. This is because the MMSE-OSIC SIC improves the EE of the linear receivers and ordering with SE greater than 2.5 bits/s/Hz, all SIC-based non-linear receivers have higher EE than the widely studied linear receivers. Specifically, non-linear receivers. It shows that the approximate analysis values of SE for the system with the linear and the SIC based receivers in an uplink single cell scenario.

Fig. 2. Circuit power consumption and transmission power consumption with natural detection order the value of C allocation for each coherence block. For the SIC receiver with power consumed by sorting, filter computation and power consumption by using fewer antennas, to improve the overall EE performance.

IV. Numerical Results

We now demonstrate the uplink EE of a single-cell case with the ZF, LMMSE, ZF-SIC, and MMSE-SIC receivers. We assume K = 90 UEs (we observed similar trends for other values of K) and perfect CSI at the BS. The number M of BS antennas is varied from 1 to 220. The transmission rate \( \bar{R} \), i.e., the spectral efficiency (SE) per user, is increased. For each value of \( \bar{R} \), the EE values for \( M = 1, 2, \ldots, 220 \) are evaluated, among which the highest is chosen as the best EE performance for a given transmission rate. The value of the parameters introduced in Section II are chosen from [8].

Fig. 2. Circuit power consumption and transmission power consumption with linear and SIC receivers in an uplink single cell scenario.

Fig. 1. EE with linear and SIC receivers in an uplink single cell scenario.

number of antennas at the BS must be greater than or equal to the number of UEs for ZF-OSIC.

Fig. 2 shows the circuit (\( P_{circuits} \)) and transmission power consumption (\( P_{TX} \)) for all receivers and Fig. 3 shows the corresponding number of antennas. The MMSE-OSIC receiver leads to the lowest \( P_{circuits} \), as it requires fewer antennas compared with the other receivers shown in Fig.3. In addition, Fig. 2 implies that the RF circuit power consumption becomes a major part in the overall power consumption for a massive MIMO system due to the large number of antennas used at BSs. Based on this, it is reasonable to consider low-complexity non-linear receivers, which can reduce the RF circuit power consumption by using fewer antennas, to improve the overall EE performance.

V. Conclusion

We have investigated the influence of the SIC-based non-linear receivers on the EE for an uplink massive MIMO system. The results indicate that the low-complexity SIC receivers can improve the EE compared to the linear receivers, as the SIC receivers require fewer antennas than linear receivers and thus consume less RF circuit power.

REFERENCES