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ABSTRACT

Hotelling's conceptual framework is expanded to incorporate the effects of a backstop technology on the planning horizon of the suppliers of an exhaustible resource and its price and quantity trajectories. It is shown that in the non-trivial case, the presence of a backstop technology shortens the planning horizon of the suppliers of the exhaustible resource in accordance with the resource suppliers' rate of time preference, backstop technology's rate of improvement and ratio of the initial resource spot price to the initial average production cost of the backstop substitute. As expected the presence of a backstop technology also lowers the spot prices of the exhaustible resource and accelerates its extraction and depletion. However, a decline of the initial average cost of producing the backstop substitute by a dollar leads to a decline of the exhaustible resource's initial spot price by less than a dollar.

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INTRODUCTION

The foundations of the economics of exhaustible resources have been layed by Hotelling (1931) in a paper entitled "The Economics of Exhaustible Resources". In this seminal paper Hotelling (*ibid*) has shown that in order to maximise the sum of the discounted profits the rate of change of the (net) marginal revenue of the exhaustible resource should be equal to its suppliers' discounting rate. This fundamental rule has been used and modified over the years to analyse a wide range of issues associated with the extraction of natural exhaustible resources. Hotelling's conceptual framework is expanded in this paper to incorporate the effects of backstop technology on the planning horizon of the suppliers of the exhaustible resource and its price and quantity trajectories.

A backstop technology is defined as a new technology producing a close substitute to an exhaustible resource by using relatively abundant production inputs and rendering the reserves of the exhaustible resource obsolete when the average cost of production of the close substitute falls below the spot price of the exhaustible resource. (Dasgupta and Heal, 1978) For instance, the technology of harnessing solar energy can be perceived as a backstop technology to oil, coal and natural gas. Hence, the development of a backstop technology shortens the planning horizon and, in turn, can accelerate the extraction and lower the spot prices of the exhaustible resource.

The paper is organised in four sections. The first section introduce Hotelling's fundamental rule and presents its implications for the equilibrium resource price and extraction trajectories when the resource is not threatened by a backstop technology. The second section presents the effect of the backstop technology on the planning horizon of the suppliers of the exhaustible resource. Incorporating the endogenously determined planning horizon, the third section presents the effects of the backstop technology on the exhaustible resource's price and extraction trajectories. Considering the effects of the backstop technology on both the planning horizon of the suppliers of the exhaustible resource and its initial price, the fourth section offers a framework for determining an efficient development path of the backstop technology.

FROM HOTELLING TO THE EQUILIBRIUM PRICE AND EXTRACTION TRAJECTORIES OF AN EXHAUSTIBLE RESOURCE UNTHREATEND BY A BACKSTOP TECHNOLOGY

In his aforementioned paper Hotelling (1931) have shown that, for a perfectly competitive industry where all the suppliers have the same discounting rate and planning horizon and maximise the sum of the discounted profits and when the variable costs of extraction are negligible, the rate of change of the spot price of the exhaustible resource (p) is equal to the discounting rate (r)

$$\frac{\dot{p}(t)}{p(t)} = r \tag{1}$$

or, equivalently,

$$p(t) = p_0 e^{rt}. \tag{2}$$

Since the present value of the spot price is time invariant

$$\arg\max \int_0^T e^{-rt} p(t) q_i(t) dt = p_0 R_{i0} \tag{3}$$

for any supplier i, where T is the end of the planning horizon, $q_i(t)$ the quantity extracted and supplied by i at time t, and R_{i0} the initial reserves in the i-th site. This in turn suggests that each supplier, and consequently the industry as a whole, exhausts the reserves by T.

By following Hotelling's no-arbitrage rule and assuming that the demand for the exhaustible resource is expected to be stable and isoelastic (i.e., $Q(t)^d = p(t)^{-a}$) there is no incentive for the suppliers to collude and form a cartel and the market clearing condition implies that

$$\int_0^T [p_0 e^{rt}]^{-a} dt = R_0 \tag{4}$$

where a denotes the demand elasticity and R_0 the initial level of reserves held by the industry. Hence, the initial spot price rises with the length of the planning period but declines with the industry's initial reserve of the resource and discounting rate as indicated by the following expression

$$p_0 = \left[\frac{1-e^{-arT}}{arR_0}\right]^{\frac{1}{a}} \tag{5}$$

Consequently, when the exhaustible resource is not threatened by a backstop technology its spot price path can be expressed as

$$p(t) = \left[\frac{1-e^{-arT}}{arR_0}\right]^{\frac{1}{a}} e^{rt} \tag{6}$$

and the trajectory of the quantity extracted and traded by the industry as a whole can be rendered as

$$Q(t) = \left[\frac{1 - e^{-arT}}{arR_0} \right]^{-1} e^{-art}. \tag{7}$$

WHEN DOES AN EXHAUSTIBLE RESOURCE BECOME OBSOLETE?

As suggested by Dasgupta and Heal (1979), the development of a backstop technology is expected to shorten the planning horizon of the suppliers of an exhaustible resource to $\bar{T}$, which is given by the intersection of the downward sloping average cost curve of producing the backstop substitute and the exhaustible resource's spot price trajectory displayed in Figure 1. Assuming that the prices of the production factors are time-invariant, the improvement of the backstop technology is reflected by a gradual and continuous decline of the average cost. For tractability it is assumed that the rate of decline of the average cost of the backstop technology is constant.

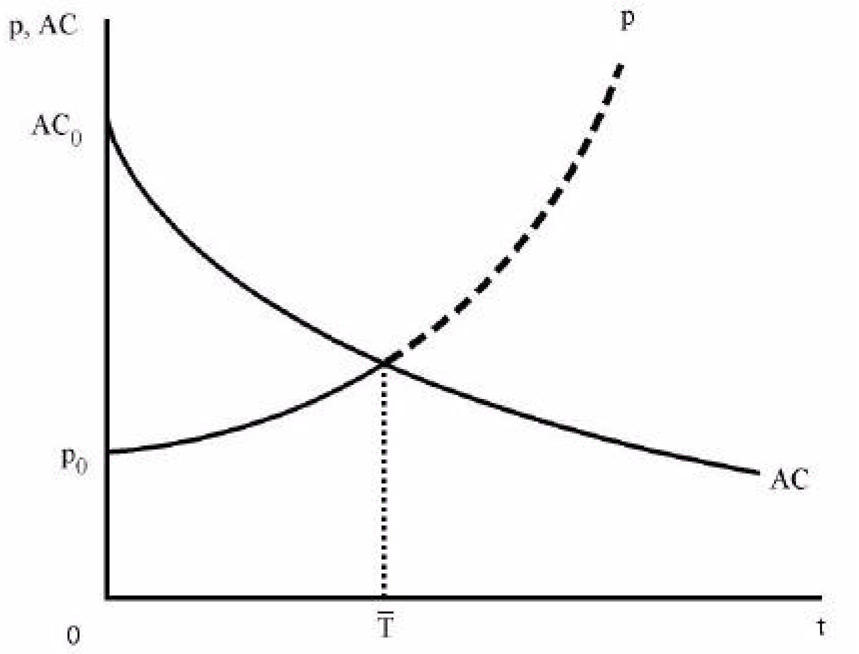

Figure 1. The spot price and the backstop technology average cost trajectories

In this setting the investment in research and development is manifested in a constant rate of decline, d, of the average costs of producing the backstop technology product from an initial level AC_0

$$AC(t) = AC_0 e^{-dt}. \tag{8}$$

The exhaustible resource becomes obsolete at a point in time $\bar{T}$ for which its spot price is equal to the average cost of producing the backstop substitute

$$p(\bar{T}) = AC(\bar{T}) \tag{9}$$

and in recalling equation (2) and equation (8),

$$\bar{T} = \frac{1}{r+d} \ln\left(\frac{AC_0}{p_0}\right). \tag{10}$$

In terms of Figure 1, a higher discounting rate is reflected by a steeper natural resource's price trajectory and a higher initial price of the natural resource is represented by a shift outward of the natural resource trajectory. Everything else remains the same, they imply a shorter period for extracting the exhaustible natural resource. Likewise, a higher rate of decline in the average production cost of the backstopping product is depicted by a steeper average cost trajectory and a lower initial average cost is portrayed by downward shift in the average cost trajectory. Everything else remains the same, they also imply a shorter period for extracting the natural exhaustible resource.

WHAT IS THE EFFECT OF A BACKSTOP TECHNOLOGY ON THE EXHAUSTIBLE RESOURCE'S PRICE AND EXTRACTION TRAJECTORIES?

The effects of the backstop technology on the price and extraction trajectories of the natural resource is explored for the non-trivial case where $\bar{T} < T$. In this case, the whole reserve of the exhaustible resource should be extracted and sold between 0 and $\bar{T}$ in order to maximise the sum of the discounted profits for the individual suppliers. Recalling the market clearing condition and that the demand for the exhaustible resource is assumed to be stable and isoelastic the equality between the cumulative extracted natural resource and its initial reserve can be rendered as

$$\int_0^{\frac{1}{r+d}\ln\left(\frac{AC_0}{p_0}\right)} p(t)^{-a}\,dt = R_0 . \qquad (11)$$

By substituting the solution to Hotelling no-arbitrage rule, equation (2), for $p(t)$ into equation (11) and solving the integral it is obtained that in the presence of the backstop technology the efficient initial price of the exhaustible resource should satisfy the following equality:

$$p_0^{-a}\left[1-\left(\frac{p_0}{AC_0}\right)^{\frac{ar}{r+d}}\right] = arR_0 . \qquad (12)$$

A close-form solution for equation (12) can be obtained for the case where the backstop technology does not improve over time, i.e., $d = 0$:

$$p_0 = \left[\frac{1}{arR_0 + AC_0{}^{-a}}\right]^{\frac{1}{a}}. \tag{13}$$

As can be seen, this simple case provides a straightforward modification of the implications of the Hotelling rule for the initial spot price of the exhaustible natural resource which were encapsulated by equation (5) in the introduction. A comparison between equation (13) and equation (5) reveals that in the presence of a binding backstop technology the planning horizon is endogenously determined rather than a predetermined T and that the initial spot price of the exhaustible resource is moderated by the initial average cost associated with the production of its backstopping substitute. Furthermore, by substituting equation (13) into equation (2) and recalling that the demand for the natural resource is assumed to be isoelstic, the price and traded quantity trajectories of the natural resource for the simple case of a time-invariant backstop technology can be displayed as

$$p(t) = \left[\frac{1}{arR_0 + AC_0{}^{-a}}\right]^{\frac{1}{a}} e^{rt} \tag{14}$$

and

$$Q(t) = [arR_0 + AC_0{}^{-a}]e^{-art}. \tag{15}$$

Reconsidering the case of a constant rate of improvement of the backstop technology it can be argued that a decline of the initial average cost of producing the backstop substitute by a dollar leads to a decline of the exhaustible resource's initial price by *less* than a dollar. This claim can be proved by totally differentiating equation (12):

$$\frac{dp_0}{dAC_0} = \frac{p_0 \,/\, AC_0}{1 + a\Delta \,/\, \Omega} \tag{16}$$

where Δ denotes the term in the brackets on the left hand side of equation 12 and therefore positive, and

$$\Omega = \left(\frac{ar}{r+d}\right)\left(\frac{p_0}{AC_0}\right)^{\frac{ar}{r+d}} > 0\,. \tag{17}$$

Recalling that $p_0 < AC_0,\ 0 < \dfrac{dp_0}{dAC_0} < 1.$

Equation (11) indicates that as can be expected intuitively the greater d the lower the exhaustible resource's initial price. The underlying rationale is that the higher the rate of the decline of the backstop substitute's average cost the shorter the effective planning horizon for the suppliers of the natural resource. Hence, these suppliers accelerate the extraction of the resource by lowering its initial price so as to sell the whole reserve in a shorter planning horizon and maximise their sum of discounted profits. In formal terms, this relationship between the exhaustible resource's initial price and the rate of decline of the average cost of production of its backstopping substitute is obtained by totally differentiating equation (12):

$$\frac{dp_0}{dd} = \frac{p_0 \dfrac{r}{(r+d)^2} \ln\left(\dfrac{p_0}{AC_0}\right)}{\Delta + \Omega \,/\, a} < 0 \tag{18}$$

since $\ln\left(\frac{p_0}{AC_0}\right) < 0$.

AN EFFICIENT RATE OF TECHNOLOGICAL IMPROVEMENT

The notion of efficiency considered in this section is consistent with the assertion that the owners of the backstop technology are profit maximisers. The planning horizon of the owners of the backstop technology comprises two periods. The first period is the incubation period which is terminated when the average cost of producing the substitute becomes equal to the spot price of the exhaustible resource as indicated by equation (10). During this period the backstop technology is developed but there are no sales of the substitute. The instantaneous costs of developing the backstop technology are assumed to increase with the rate of moderation of the cost of production d in accordance to some convex function $c(d)$ for which $c' > 0$ and $c'' > 0$. During the second period the exhaustible resource is obsolete and the owners of the backstop technology enjoy an absolute monopolistic power and expect the price of their product to decline with the quantity supplied as displayed by the inverse demand function $P(y)$ with $P' < 0$. They also expect, however, that at some date t their monopolistic power and profit would vanish as a large number of imitators enters the industry. Correspondingly, the fixed rate of technological improvement that maximises the sum of the discounted profits accruing to the owners of the backstop technology can be found by maximising

$$\Pi = -\int_{0}^{\frac{1}{r+d}\ln\frac{AC_0}{p_0}} e^{-rt}c(d)dt$$

$$+\int_{\frac{1}{r+d}\ln\frac{AC_0}{p_0}}^{t} e^{-rt}[P(y(t)) - AC_0 e^{-d\left(\frac{1}{r+d}\ln\frac{AC_0}{p_0}\right)}]y(t)dt \tag{19}$$

with respect to d and y, where r is the rate of time preference of the owners of the backstop technology and where the exhaustible resource's initial price p_0 is endogenous and depends on d as displayed by equation (18).

Using Leibniz's formula the first-order conditions for maximum can be represented as:

$$\frac{\partial\Pi}{\partial y} = \int_{\frac{1}{r+d}\ln\frac{AC_0}{p_0}}^{t} e^{-rt}\left\{\left[P(y(t)) - AC_0\left(\frac{AC_0}{p_0}\right)^{\frac{-d}{r+d}}\right] + P'(y(t))y(t)\right\}dt = 0 \tag{20}$$

$$\frac{\P\Pi}{\P d} =$$

$$\left(\frac{AC_0}{p_0}\right)^{\frac{-r}{r+d}}\left\{c(d)\left(\frac{1}{r+d}\right)+\left[P(y(t))-AC_0\left(\left(\frac{AC_0}{p_0}\right)^{\frac{-d}{r+d}}\right)\right]y(t)\right\}\Theta$$

$$-\int_0^{\frac{1}{r+d}\ln\frac{AC_0}{p_0}} e^{-rt}c'(d)dt$$

$$-\int_{\frac{1}{r+d}\ln\frac{AC_0}{p_0}}^{t} e^{-rt}dAC_0\left(\frac{AC_0}{p_0}\right)^{\frac{-d}{r+d}}\Theta y(t)dt = 0 \qquad (21)$$

where,

$$\Theta \equiv \frac{1}{r+d}\ln\frac{AC_0}{p_0}+\frac{1}{p_0}\frac{\P p_0}{\P d} \qquad (22)$$

and $\frac{\P p_0}{\P d}$ is as given by equation (18).

If Π is concave in d and y there exists an internal solution and the profit maximizing values of d and y are those satisfying the necessary conditions. Their relationship with the model's parameters can be implicitly expressed as

$$d^* = f_1(r, r, AC_0, t) \tag{23}$$

and

$$y^* = f_2(r, r, AC_0, t) \tag{24}$$

but due to the complexity of the problem the properties of f_1 and f_2 cannot be derived analytically or simulated.

CONCLUSIONS

Hotelling's conceptual framework was expanded to incorporate the effects of a backstop technology on the planning horizon of the suppliers of the exhaustible resource and its price and quantity trajectories. Hotelling's fundamental rule and its implications for the equilibrium resource price and extraction trajectories when the resource was not threatened by a backstop technology were presented as a benchmark. It was shown that in the presence of a backstop technology the planning horizon of the suppliers of the exhaustible resource might be shortened and inversely related to the resource suppliers' rate of time preference, backstop technology's rate of improvement and ratio of the initial resource spot price to the initial average cost of production of the backstop substitute. By incorporating this endogenously determined planning horizon, the effects of the backstop technology on the exhaustible resource's price and extraction trajectories were analysed. As intuitively expected, the presence of a backstop technology lowers the spot prices of the exhaustible resource and accelerates its extraction and depletion. However, a decline of the initial average cost of producing the backstop substitute by a dollar leads to a decline of the exhaustible resource's initial spot price by less than a dollar. Finally, by incorporating the effects of the backstop technology on both the planning horizon of the resource suppliers and the resource initial price, a framework for determining an efficient development path of the backstop technology was offered.

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