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ENDOGENEITY, KNOWLEDGE AND DYNAMICS OF LONG RUN CAPITALIST ECONOMIC GROWTH

E.J. Wilson and D.P. Chaudhri *

ABSTRACT

The revival of interest in economic growth and technological leadership issues has resulted in the re-examination of the theoretical foundations of the economics of growth. The neoclassical concerns with steady state paths and neo-Keynesian focus on short-term issues have remained intact in this process. However the 'new economics of growth' extensions proposed by Lucas (1988) and Romer (1986) and attempts by Scott (1989) to explain technological progress, do not address Arrow's (1962) concerns or explain Kuznet's (1957) and Maddison's (1991) empirical telescoping of the economic growth experience of the last two hundred years.

This paper attempts to address some of these issues by developing a model which adopts Aghion and Howitt's (1992) suggestion to examine endogenous growth in the form of technological innovation in monopolistic capital goods production. Human capital and non-rival partially excludable technology are inputs to production, which may have non-constant returns to scale. Profit maximising behaviour is analysed in terms of a variable Tobin's q, which when greater than one, drives economic growth.

This approach differs from endogenous growth theory in that production is characterised as initially increasing returns to scale, which subsequently diminish as production expands until decreasing returns to scale are realised. Central to this model is the nonlinear set of dynamic saddlepath solutions for the production of new technology, which is important for three reasons. First, the solutions characterise the adoption of new technology as the process of Schumpeterian 'creative destruction'. Second, the possible speeds of adoption of new technology can vary significantly over time. Third, the families of possible saddlepaths define an endogenously evolving metaproduction function.

These analytic processes are briefly compared with some stylised historical evidence of changing technological leadership in observed long run economic growth processes.

KEYWORDS: Creative destruction, endogenous growth, technological innovation, Tobin's q, variable returns to scale, nonlinear saddlepaths.

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1. INTRODUCTION

The drivers of economic growth have been the subjects of intense debate among economists of differing schools of thought over the last two centuries.¹ The recent revival of interest in the form of the 'new economics of growth' has led to an exciting resurgence in economic theorising.² However, this process of scientific investigation is starting to indicate some shortcomings in theory and methodology. They include the interrelated difficulties of adequately specifying the growth process and explaining the dynamic essence of economic growth. Early empirical studies do not attempt to measure these things and mostly focus on testing for conditional convergence.³

This paper attempts to address some of these issues by proposing a four-sector model of long run economic growth. The approach adopts Aghion and Howitt's (1992) suggestion for further research by analysing economic growth in terms of Schumpeterian creative destruction, which explicitly incorporates technological change in physical and human capital. This is done by modeling endogenous growth in the form of Romer's (1990) technological innovation in monopolistic capital goods production. Human capital and non-rival partially excludable technology are inputs to production, which may have non-constant returns to scale. Profit maximising behaviour is analysed in terms of Tobin's q, which when greater than one, drives economic growth.

This approach differs from endogenous growth theory in that production is characterised as initially increasing returns to scale, which subsequently diminish as production expands until decreasing returns to scale are realised. Central to this model is the nonlinear set of dynamic saddlepath solutions for technological production, which is important for three reasons. First, the solutions characterise the adoption of new technology as the process of Schumpeterian 'creative destruction'. Second, the possible speeds of adoption of new technology can vary significantly over time. Third, the families of possible saddlepaths define an endogenously evolving metaproduction function.

The paper is organised into four sections. The next section presents the basic model with required static equilibrium relationships. Consumption is then introduced and the requirements for equilibrium growth are analysed in Section 3. The main contribution of this paper is in Section 4, which explores possible nonlinear dynamic saddlepath solutions for long run economic growth characterised as variable returns to scale. The model explicitly includes the process of Schumpeterian creative destruction and variable rates of adoption of new technology. A metaproduction function is then presented which stylizes Maddison's (1991) historical evidence of shifting technological leadership during the last two centuries. The final Section 5 presents a brief summary.

¹ See Marglin (1984) and Solow (1970) for competing views. For an excellent overview of the theories see Sen (1970), particularly the introductory chapter, Hahn (1971), and Abhamovitz (1989).

² Recent interest has been rekindled by Romer (1986, 1990) Lucas (1988), Ram (1986), Barro (1990, 1991), Basalla (1988), Scott (1989) among many others. The wisdom of Adam Smith on specialisation, Marshall's on external economies and Allyn Young's on increasing returns has been combined or formalised in seminal papers by Arrow (1962), Schults (1976), Romer (1986) and Lucas (1988). This literature is referred to as the 'new economics of growth'.

³ Empirical studies were led by Barro (1990) and Sala-i-Martin (1992). Subsequent analysis has been carried out by Quah (1993), Mankiw, Romer and Weil (1992), Levin and Renelt (1992), Levine and Servos (1993). Reservations about some of this work have been expressed by Pack (1994) and Solow (1994).

2. PRODUCTION

2.1 The Basic Production Model

The production side of the model comprises three sectors, namely, final goods, intermediate capital goods and research sectors. The production of the homogeneous final good, denoted by y, is characterised by perfect competition with simplified aggregate production function: ⁴

$$y = x^{\alpha} \qquad 0 < \alpha < 1 \tag{2.1}$$

A monopolist with production function produces the intermediate good x:

$$x = a^{\beta} h_{x}^{\gamma} \quad 0 < \gamma < 1, \ 0 < \beta + \gamma > 1$$
(2.2)

where *a* represents the stock of knowledge and h_x the stock of human capital used in producing the intermediate good *x*.⁵

The inverse demand curve is simply:

$$p_x = \alpha x^{\alpha - 1} \tag{2.3}$$

where p_x represents the price of x. Clearly the producer of x is a price maker for $\alpha < 1$, with price elasticity of demand:

$$\varepsilon_{p_x} = \frac{1}{\alpha - 1} \tag{2.4}$$

For neoclassical constant returns to scale in final goods production, the demand for the intermediate good is forced to be infinitely elastic with constant p_x set to unity.

The research sector is characterised by perfect competition with the production of knowledge resulting in new technology, denoted \dot{a} , by: ⁶

⁴ Production could be specified in terms of the expanded function, $y = f(x, h_{\ell}, \ell)$ where h_{ℓ} and ℓ represent human capital and unskilled labour inputs respectively. However, the simpler specification in equation (2.1) gives a more analytically tractable equation (2.3), which allows the analysis of the interaction between human capital and technology in the other two sectors.

⁵ Adoption costs in terms of foregone production of intermediate good *x* can be included in the production function (2.2). Consider adoption costs in the form of x^{η} where $x = x^{\eta}a^{b}h_{x}^{c}$ for $0 < \eta < 1$ and $0 < \eta + b + c \leq 1$. Transforming gives, $x = a^{\beta}h_{x}^{\gamma}$ with $\beta = b(1-\eta)^{-1}$ and $\gamma = c(1-\eta)^{-1}$. For the inequality requirement, $\eta + b + c \leq 1$, that is, $b + c \leq 1 - \eta$, which gives $\beta + \gamma \leq 1$, as required. For the lower inequality, $\beta + \gamma > -\eta(1-\eta)^{-1}$ which for $0 < \eta < 1$ means that $-\eta(1-\eta)^{-1}$ will be negative. This paper will constrain $\beta + \gamma > 0$ for convenience. Alternatively, the production function could be defined as $x = a^{\beta}h_{x}^{\gamma} - \rho x$, where $\rho < 1$ represents the amount of *x* used in the adoption of *a*. The function becomes $x = (1-\rho)^{-1}a^{\beta}h_{x}^{\gamma}$ which only differs from equation (2.2) by the constant of proportionality $(1-\rho)^{-1}$.

⁶ The production of new technology, \dot{a} , is assumed to be adopted by the monopolist. The new technology may be embodied in physical or human capital or it could also be disembodied, for example in the form of information or computer software. The only requirement is its excludability. The externalities and the public good components are ignored to keep the analysis manageable.

$$\dot{a} = Ah_a^{\delta} a^{\varepsilon} \qquad 0 < \delta < 1, 0 < \delta + \varepsilon < 1$$
(2.5)

where h_a represents the stock of human capital employed in this sector. This specification incorporates Romer's (1990) non-rival technology with ε importantly not restricted to unity. We will initially set A=1 for convenience, then in Section 4 below we will redefine it as an endogenous systematic parameter.

2.2 Static Maximisation

The monopolist's current nominal profit, π_{x} , is given by:

$$\pi_x = p_x x - w_x h_x - p_a \dot{a} \tag{2.6}$$

where w_x is the nominal wage rate and p_a is the price paid for the adopted technology, \dot{a} in the current period. The inclusion of p_a is important because of the assumption of partial excludability of the new technology, which has to be purchased by the monopolist.⁷

Maximising profit for a risk neutral monopolist using equations (2.2), (2.3) and (2.5) gives:

$$\frac{\pi}{a} = \alpha^2 \beta a^{\alpha\beta-1} h_x^{\alpha\gamma} - p_a \varepsilon a^{\varepsilon-1} h_a^{\delta} = 0$$

Solving for the equilibrium price, p_a^* : ⁸

$$p_a^* = \frac{\alpha^2 \beta}{\varepsilon} a^{\alpha\beta-\varepsilon} \frac{h_x^{\alpha\gamma}}{h_a^{\delta}} \qquad \alpha\beta < \varepsilon$$
(2.7)

Inspection of equation (2.7) shows that p_a^* and *a* are inversely related for the monopsonist when $\alpha\beta < \varepsilon$. This is a very important condition for this model.

Rearranging (2.7) and substituting equations (2.2) and (2.5) obtains:

$$p_a^* = \frac{\alpha^2 \beta}{\varepsilon} \frac{x^{\alpha}}{\dot{a}}$$
(2.8)

Clearly an increase in the production of the final goody y, which in turn increases the demands for good x and for technology, a, will increase the equilibrium price of new technology, p_a^* . A reduction in the production of new technology, \dot{a} , will also increase p_a^* .

Let's now maximise monopolistic profit, π_x , with respect to the human capital, h_x , employed in the production of the intermediate capital good in order to determine the

⁷ The monopolist producer of capital good, *x*, acts as a monopsonist in purchasing the new technology, \dot{a} from the research sector.

⁸ Profit maximisation is assured when: ${}^{2}\pi_{x}/a^{2} = \alpha^{2}\beta(\alpha\beta - \varepsilon)a^{\alpha\beta - 2}h_{x}^{\alpha\gamma} < 0$ iff $\alpha\beta < \varepsilon$.

equilibrium wage rate, w_x^* . Using equations (2.2), (2.3), (2.5) and (2.7), calculate $\frac{\pi_x}{h_x} = 0$ for endogenous p_a to derive the equilibrium wage, w_x^* :⁹

$$w_{x}^{*} = \frac{\alpha^{2} \gamma}{\epsilon} \left(\epsilon - \alpha \beta \right) a^{\alpha \beta} h_{x}^{\alpha \gamma - 1} \qquad \alpha \gamma < 1, \ \alpha \beta < \epsilon$$
(2.9)

This wage rate is positive for $\alpha\beta < \varepsilon$ and it can be seen that the demand for h_x varies inversely with w_x^* when $\alpha\gamma < 1$.

The equilibrium equality between the economic and technical rates of substitution between \dot{a} and h_x is given by the ratio of equations (2.7) and (2.9):

$$\frac{w_x^*}{p_a^*} = \frac{\gamma}{\beta} \left(\varepsilon - \alpha\beta\right) a^{\varepsilon} \frac{h_a^{\delta}}{h_x}$$
(2.10)

Using equation (2.5) gives the required result, subject to the requirement that $\alpha\beta < \epsilon$:

$$\frac{w_x^*}{p_a^*} = \frac{\gamma}{\beta} \left(\varepsilon - \alpha\beta\right) \frac{\dot{a}}{h_x}$$
(2.11)

The knowledge and new technology producing research sector will now be considered. Current profit, π_a , is defined for the perfectly competitive firms as:

$$\pi_a = p_a^* \dot{a} - w_a h_a \tag{2.12}$$

where \dot{a} is the technology purchased by the monopolist acting as a monopsonist, p_a^* is the given price for each price taking competitive research firm and w_a is the wage rate for human capital, h_a employed in the research sector. Using equations (2.5), (2.7) and maximising current profit by setting $\frac{\pi_a}{h_a} = 0$ for risk neutral producers gives: ¹⁰

$$w_{a}^{*} = \frac{\alpha^{2}\beta\delta}{\varepsilon} a^{\alpha\beta} \frac{h_{x}^{\alpha\gamma}}{h_{a}}$$
$$= \frac{\alpha^{2}\beta\delta}{\varepsilon} \frac{x^{\alpha}}{h_{a}} \qquad \delta < 1 \qquad (2.13)$$

This relationship shows that an increase in the demand for intermediate capital good, x and therefore for \dot{a} and h_a will increase the equilibrium wage for human capital employed in research, w_a^* . Also a reduction in h_a will also increase w_a^* .

¹⁰ The maximisation condition is satisfied when: ${}^{2}\pi_{a}/h_{a}^{2} = p_{a}a^{\varepsilon}\delta(\delta-1)h_{a}^{\delta-2} < 0$ iff $\delta < 1$.

⁹ Profit maximisation also requiries: ${}^{2}\pi_{x}/h_{x}^{2} = \alpha^{2}\gamma\epsilon^{-1}(\alpha\gamma - 1)(\epsilon - \alpha\beta)a^{\alpha\beta}h_{x}^{\alpha\gamma - 2} < 0$ iff $\alpha\gamma < 1$ and $\alpha\beta < \epsilon$.

Given the static equilibrium, human capital wage rates in the intermediate sector and the knowledge and new technology producing research sector, the static equilibrium levels of employment, h_a^* and h_x^* can be determined. Setting $w_a^* = w_x^*$ and using equations (2.9), (2.13) and the fact that $h_a + h_x = h$:

$$h_a^* = \frac{\beta \delta}{\beta \delta + \gamma \tau} h = \Theta h \tag{2.14}$$

and:

Good S

$$h_x^* = \frac{\gamma \tau}{\beta \delta + \gamma \tau} h = (1 - \theta)h$$
(2.15)

(2.17)

(2.18)

where $\tau = \varepsilon - \alpha\beta > 0$ and $0 < \theta < 1$.

In summary, the equilibrium levels of production in the three sectors are given by the following relationships:

Research Sector
$$\dot{a}^* = \frac{\beta\delta}{\beta\delta + \gamma\tau} \delta^{\delta} a^{\epsilon} h^{\delta} \qquad 0 < \delta < 1, \ 0 < \delta + \epsilon < 1$$
 (2.16)

Intermediate Capital
$$x^* = \frac{\gamma \tau}{\beta \delta + \gamma \tau} a^{\delta} a^{\beta} h^{\gamma} \qquad 0 < \gamma < 1, \ 0 < \beta + \gamma < 1$$

Good Sector

 $y^* = \frac{\gamma \tau}{\beta \delta + \gamma \tau} a^{\alpha \beta} h^{\alpha \gamma} \quad 0 < \alpha < 1, \ 0 < a (\beta + \gamma) > 1$ Final Good Sector

These structural equations include both direct and indirect links between the sectors.

Our three sector model bears some resemblance with Marxian two sector models and also neoclassical two sector models to the extent that the final goods sector is linked with the other two sectors through profit maximising behaviors of self interested rational production agents (employer and workers). It differs in some important respects, particularly returns to scale. We introduce a fourth sector, namely consumption in the following section to derive equilibrium conditions linking the sectors in static and dynamic settings.

3. CONSUMPTION AND EQUILIBRIUM GROWTH

3.1 Consumption

To close the model, consider risk averse households who select the time path of consumption, c, to maximise utility, u(c):

$$u(c) = \int_{0}^{0} u[c(t)]e^{-rt}dt$$
(3.1)

where r is the discount rate. The instantaneous felicity function, u[c(t)], is assumed to have constant intertemporal elasticity of substitution, for example:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \quad \sigma > 0, \ \sigma = 1$$

The resource constraint is given by:

$$\dot{a} + x = y - c \tag{3.2}$$

where savings flows, y - c, are used to create new technology, \dot{a} , and intermediate capital goods, x.

Utility can be maximised via equation (3.1) subject to equation (3.2) by defining the current value Hamiltonian, $H = \frac{c^{1-\sigma}}{1-\sigma} + \zeta \dot{a}$, with transversality condition, $\lim_{t} [a(t)\zeta(t)] = 0$. Solving for $\frac{H}{c} = 0$ and using equations (2.5), (3.2) and the Euler equation, $\dot{\zeta} = -\frac{H}{a}$ gives the desired result:

$$\frac{\dot{c}}{c} = \frac{\varepsilon}{\sigma} h_a^{\delta} a^{\varepsilon - 1}$$
(3.3)

Increases in h_a , δ and ε increase the production of new technology, \dot{a} , which allows increased growth in consumption. Note that higher stocks of technology, a, will increase (decrease) consumption growth if and only if $\varepsilon > 1$, ($\varepsilon < 1$).

3.2 Equilibrium Growth

The balanced growth paths for the variables of interest can be easily calculated. From equation (2.5):

$$\frac{\dot{a}^*}{a} = h_a^{*\delta} a^{\varepsilon - 1} \tag{3.4}$$

so that increases in h_a , δ and ε unambiguously increase the growth in technology. Using the equilibrium condition for h_a^* given by equation (2.14) shows:

$$\frac{\dot{a}^*}{a} = \frac{\beta\delta}{\beta\delta + \gamma\tau} \,^{\delta} h^{\delta} a^{\varepsilon-1} \tag{3.5}$$

Equation (2.5) details how an increase in δ will increase the productivity of human capital used in producing new technology, \dot{a} . An increase in β (which also lowers τ) will increase the productivity of the stock of knowledge, a, in producing the intermediate good, x via equation (2.2), which will increase the demand for new technology by the monopolist. According to equation (2.2), a decrease in γ will mean that human capital will be less productive in producing x and so the demand for h_x will fall. Given the equilibrium wage condition, $w_a^* = w_x^*$, the employment of human capital in the research sector, h_a^* , will increase, causing \dot{a} to increase as well.

The growth in equilibrium consumption is given by combining equations (3.3) and (3.4):

$$\frac{\dot{c}^*}{c} = \frac{\varepsilon}{\sigma} \frac{\dot{a}^*}{a}$$
(3.6)

The growth in the equilibrium intermediate capital good is: ¹¹

$$\frac{\dot{x}^*}{x} = \beta \frac{\dot{a}^*}{a} + \gamma \frac{\dot{h}}{h}$$
(3.7)

The growth in the equilibrium final good is: ¹²

$$\frac{\dot{y}^*}{y} = \alpha \beta \frac{\dot{a}^*}{a} + \alpha \gamma \frac{\dot{h}}{h}$$
(3.8)

Equations (3.5) to (3.8) show the central importance of the stocks of human capital, h, and knowledge, a, to the equilibrium growth process of this simple four sector economy. Zero growth in human capital, $\dot{h} = 0$ and unitary elasticity of substitution between consumption at two points in time, $\sigma = 1$, gives:

$$\frac{\dot{y}^*}{v} = \frac{\dot{x}^*}{x} = \frac{\dot{c}^*}{c} = \frac{\dot{a}^*}{a} \quad \text{iff} \quad \alpha = \beta = \varepsilon = 1$$
(3.9)

This result is equivalent to Romer's (1990) steady-state growth rate and shows that growth is unbounded and equal to $(\theta h)^{\delta}$. Unlike Romer, this model does not set β and γ equal to unity, which equations (3.5) to (3.8) show are crucial assumptions for the behaviour of $\frac{\dot{a}}{a}$ and therefore the equilibrium growth path of the economy. In this sense, setting $\beta = \varepsilon = 1$ gives a mechanistic evolution of the economy, along the lines of the early exogenous technological change growth models. Consequently, now consider the dynamics of growth with particular focus on the role of the parameter β , for increasing, decreasing and constant returns to scale in the production of the intermediate capital good, *x*.

¹¹ From equation (2.2), $x^* = a^{\beta} h_x^{*\gamma}$ and taking Naperian logs gives $\ln x^* = \beta \ln a + \gamma \ln h_x^*$. Differentiating with respect to time gives $\dot{x}^*/x = \beta(\dot{a}^*/a) + \gamma(\dot{h}_x^*/h)$ and using equation (2.14) gives $\dot{x}^*/x = \beta(\dot{a}^*/a) + \gamma(\dot{h}/h)$.

¹² Taking Naperian logs of equation (2.1), $y^* = x^{*\alpha}$ gives $\ln y^* = \alpha \ln x^*$, such that $\dot{y}^* / y = \alpha (\dot{x}^* / x)$. Substituting using equation (3.7) gives $\dot{y}^* / y = \alpha \beta (\dot{a}^* / a) + \alpha \gamma (\dot{h} / h)$.

4. THE DYNAMICS OF CAPITALIST ECONOMIC GROWTH

4.1 Tobin's q

The monopolist's task is to determine the profit maximising time path of employment of human capital, h_x , and adoption of new technology, \dot{a} . This problem requires maximising the net present value, $_0 \pi_x(t)e^{-rt}dt$, subject to the constraint, $\dot{a} = h_a^\delta a^\varepsilon$, where r is the discount rate. The Hamiltonian is defined as:

$$H = \pi_x e^{-rt} + \zeta \dot{a}$$

with transversality condition $\lim_{t} [a(t)\zeta(t)] = 0$. For convenience, set the shadow price $\zeta = q_a e^{-rt}$ which gives the present value in terms of q_a . The variable q_a is Tobin's q, which is defined as the marginal valuation of technology relative to its replacement cost. For values of $q_a > 1$ the monopolist will purchase the new technology at price p_a , whereas for $q_a < 1$, the monopolist will use the stock of existing technology and therefore not innovate. In this sense, the solution of the Hamiltonian problem will determine the optimum adoption of technology in terms of the present value of this new technology's contribution to the monopolist's profits. Consider the Euler equation for a risk neutral monopolist, $\zeta = -\frac{H}{a}$. Substituting equation (2.6) and $\zeta = q_a e^{-rt}$:

$$\frac{\left(q_{a}e^{rt}\right)}{t} = -\frac{\pi_{x}}{a}e^{-rt}$$
$$\left(\dot{q}_{a} - rq_{a}\right)e^{rt} = -\frac{\pi_{x}}{a}e^{-rt}$$

which solves to the well-known and important result:

$$\dot{q}_a = rq_a - \frac{\pi_x}{a} \tag{4.1}$$

The last term in equation (4.1) represents the marginal profit obtained by the monopolist from the marginal technology. It can be determined by substituting equations (2.2), (2.3), (2.8), (2.9) and (2.15) into the profit equation (2.6) and differentiating with respect to a:

$$\frac{-\pi_x}{a} = \frac{\alpha^2 \beta}{\epsilon} (1 - \alpha \gamma) (\epsilon - \alpha \beta) a^{\alpha \beta - 1} (\theta h)^{\alpha \gamma} \qquad \alpha \gamma < 1, \ \alpha \beta < \epsilon, \ \beta \delta < \gamma |\epsilon - \alpha \beta|$$
(4.2)

For $\alpha\beta < 1$, $\frac{\pi_x}{a}$ is an inverse function of the stock of technology, *a*. ¹³ Solving equation (4.1) gives the standard result:

¹³ When $1 < \alpha\beta < \varepsilon$ then there is a positive relationship between π_x/a and a.

$$q_a = \int_0^\infty \frac{\pi_x}{a} e^{-r(s-t)} ds \tag{4.3}$$

which shows that q_a is the net present value of all future marginal monopoly profits due to the marginal adoption of new technology. For values of Tobin's $q_a > 1$, the monopolist will willingly adopt new technology, which implies $\dot{a} > 0$. However, when $q_a = 1$ the monopolist will be indifferent between existing and new technology, such that $\dot{a} = 0$. When Tobin's $q_a < 1$, the monopolist will have to rely on existing technology. To the extent this causes disinvestment in existing technology then $\dot{a} < 0$. These important effects can be incorporated into the new technology equation (2.5), $\dot{a} = Ah_a^\delta a^\varepsilon$ by replacing the initially fixed parameter A by the divergence of the endogenous Tobin's q_a from unity. That is, by setting $A = q_a -1$:

$$\dot{a} = (q_a - 1)h_a^\delta a^\varepsilon \tag{4.4}$$

Now $q_a \gtrsim 1$ implies $\dot{a} < 0$, as required.

Note that this extension fundamentally differentiates the approach of this model from others. For example, in Romer's (1990) model, \dot{a} is always positive in steady state via equation (3.5) which forces positive growth in consumption, c, the intermediate capital good, x and the final good, y, via equations (3.6) to (3.9). ¹⁴ In this model, positive economic growth is only obtain when the marginal valuation of new technology (representing the net present value of all future marginal monopoly profits) is greater than unity, that is when $q_a > 1$. The steady state equation (3.4) needs to be replaced with:

$$\frac{\dot{a}^*}{a} = (q_a - 1)h_a^{*\delta}a^{\varepsilon - 1}$$

$$\tag{4.5}$$

Importantly, the value of q_a and therefore \dot{a} , can vary endogenously in this model. This allows the analysis of the behaviour of the monopolist in terms of the dynamic solution for q_a , \dot{a} and therefore a. Linearising the equations of motion (4.1) and (4.4) around the steady state, $a = a^*$ and $q_a = 1$ when $\dot{a} = \dot{q} = 0$, gives the system of equations:

$$\dot{a}_{q_{a}} = \begin{array}{c} 0 & -\dot{a}_{q_{a}} \\ q_{a} = \frac{2\pi_{x}}{a^{2}} \Big|_{a=a^{*}} & r \end{array}$$
(4.6)

The general solution for initial values a_0 and q_0 is given by:

¹⁴ In this subset of the model, the steady state properties require the value of q to be unity and $\dot{q} = 0$. Solving equation (4.1) gives $q_a^* = \frac{\pi_x / a}{r} = 1$, so the steady state discount rate, r, can be interpreted as the marginal monopoly profit from adopting new technology, π_x / a . This relationship for the marginal value of new technology, q_a^* , can be compared with Romer's (1990) equation (6) on page S87, rearranged to: $p_A = \frac{\pi}{r}$.

$$a - a^* = (a - a_0)e^{\lambda t}$$
(4.7)

$$q_a - 1 = (q_a - q_0)e^{\lambda t}$$
(4.8)

where the negative characteristic root, λ , is chosen to ensure the locally stable, globally unstable saddlepath:

$$\lambda = \frac{1}{2} - r r^{2} - 4 \frac{2\pi_{x}}{a^{2}}\Big|_{a=a^{*}} \times \frac{\dot{a}}{q_{a}}\Big|_{q_{a}=1}$$
(4.9)

From equation (4.2):

$$\frac{{}^{2}\pi_{x}}{a^{2}} = \frac{\alpha^{2}\beta}{\varepsilon} (1 - \alpha\gamma)(\varepsilon - \alpha\beta)(\alpha\beta - 1)a^{\alpha\beta - 2}(\theta h)^{\alpha\gamma}$$
(4.10)

Now $\frac{{}^{2}\pi_{x}}{a^{2}} < 0$ for $\alpha\gamma < 1$, $\alpha\beta < \varepsilon$ and $\alpha\beta < 1$. Combining equations (2.14) and (4.5) gives:

$$\frac{\dot{a}}{q_a} = a^{\varepsilon} \left(\Theta h \right)^{\delta} > 0 \tag{4.11}$$

which ensure a real value general solution in equation (4.9).

Solving the system of equations (4.6) for q_a gives the required equation for the saddlepath:

$$q_{a} = 1 + \frac{a - a^{*}}{r - \lambda} \frac{{}^{2}\pi_{x}}{a^{2}}$$
(4.12)

This is shown as the SS schedule in Figure 1 for $\alpha\beta < 1$, which gives $\frac{2\pi}{a^2} < 0$. Inspection of equation (4.12) shows that if the stock of technology is below the optimum level, $a < a^*$, then Tobin's $q_a > 1$, which causes the stock of technology to grow, $\dot{a} > 0$. Conversely, $a > a^*$ implies $q_a < 1$ which forces $\dot{a} < 0$. The economy will therefore move along the locally stable saddlepath, SS, in the direction of the arrows until the steady state point $(a = a^*, q_a = 1)$ is reached.



FIGURE 1.

4.2 Schumpeterian Creative Destruction

Now consider a shock to the equilibrium in the form of an increase in the marginal productivity of technology, *a*, or human capital, h_x , used to produce the intermediate capital good, *x*. This is represented by an increase in the parameters β or γ . According to equations (4.10) and (4.12), $\frac{{}^2\pi_x}{a^2}$ will increase and the SS schedule will shift upwards to S₁S₁ in Figure 2. ¹⁵ Since q_a is now greater than unity, the increased marginal valuation of profit due to the new technology adopted by the monopolist, causes $\dot{a} > 0$ so that the level of technology will increase until the new steady state a_1^* is reached (consistent with $q_a = 1$ again).



¹⁵ The increase in productivity would increase profits, π_x , according to equation (4.2) and increase the demand and therefore the price, p_a , of the new technology.

This simple example can be interpreted in terms of Schumpeterian creative destruction in that higher profits associated with the new adopted technology at point B in Figure 2 "destroys" the previous profits and existing technology at point A. However diminishing returns, in the form of $\alpha\beta < 1$, ensure that q_a will return to unity at point C on the new saddlepath, S₁S₁.

Indeed the model has the potential to explain interesting and important behaviour in terms of the monopolist's adoption of new technology. For example, equation (4.7):

$$a-a^*=(a-a_0)e^{\lambda t}$$

shows the speed of adoption of technology by the monopolist is a positive function of λ . When there are diminishing returns, $\alpha\beta < 1$, then $\lambda < 0$ and so the rate of adoption of new technology is positive but diminishing exponentially. The value of λ determined in equation (4.9) is affected by the parameters in equations (4.10) and (4.11). For example, the speed of adoption, λ , will increase if human capital, *h*, increases, its productivity, δ , increases, or the productivity of technology, ε , increases.

A further behavioral extension of the model is to redefine the monopolist to have instantaneous felicity function:

$$u(\pi_x) = \frac{\pi_x^{1-\upsilon}}{1-\upsilon} \qquad \upsilon > 1$$

and to select the time path of output to maximise profits and utility:

$$_{0} u\left[\pi_{x}(t)\right]e^{-rt}dt$$

subject to the constraint, $\dot{a} = A h_a^{\delta} a^{\varepsilon}$. The Hamiltonian:

$$H = \frac{\pi_x^{1-\upsilon}}{1-\upsilon} e^{-rt} + \zeta \dot{a}$$

 $q_a = \frac{\pi_x / a}{\pi_x^{\upsilon}} e^{-r(s-t)} ds.$

has general solution:

Consider the consequences of this general solution for a monopolist who prefers risk, denoted by $\upsilon < 0$. Whilst Tobin's q_a is still a positive function of $\frac{\pi_x}{a}$ it is also a positive function of $\pi_x^{[\nu]}$. This new second term implies an accelerator effect in that higher levels of profit, π_x , in addition to the marginal profit gains, $\frac{\pi_x}{a}$, cause q_a to further increase. The adoption of new technology, \dot{a} , will therefore increase by a larger amount, via equation (4.4).

This model of Schumpeterian creative destruction, with or without the behavioral extensions, contrasts with earlier approaches. Romer 's (1990) model has $\dot{a} > 0$ always, which forces the relevant SS schedule to shift rightwards. Whilst this process is endogenous, it describes the adoption of new technology as mechanical. The endogenous growth literature, by defining constant returns to scale in the form of restricting α , β and γ to unity, forces

the uninteresting result that $q_a = 1$ always. These models do not generally explicitly include the possibility increasing returns to scale. This outcome is usually excluded in economic analysis because of the unappealing property that the mathematical solution is unbounded. If increasing returns to scale exist in the form of $\alpha\beta > 1$ then $\frac{2\pi_x}{a^2} > 0$, which gives the locally unstable saddlepath solution, TT, given in equation (4.12) and shown in Figure 1. Values of *a* above (below) a^* means that $q_a > 1$ ($q_a < 1$) and so *a* will increase, $\dot{a} > 0$ (decrease, $\dot{a} < 0$) without bound. ¹⁶ This possibility is explored in the next sub-section.

4.3 Variable Returns to Scale: An Example

Production is hypothesised to demonstrate initially increasing returns to scale, which subsequently diminish as production expands until decreasing returns to scale are realised. This approach, common to textbook theories of the firm and to well-known rates of adoption models, can be characterised by the Logistic and Gompertz functions.¹⁷ By way of example, replace the simple production function in equation (2.2) with the Logistic function, *L*. Production of the intermediate capital good, *x*, is a function of adopted technology, *a*, and human capital, h_x , with parameters β_1 , β_2 and β_3 :

$$x = L(a, h_x^{\gamma}; \beta_1, \beta_2, \beta_3)$$

= $\frac{\beta_1}{1 + \beta_2 e^{-\beta_3 a}} h_x^{\gamma}$ (4.13)

where the initial value, $x_0 = \frac{\beta_1}{1 + \beta_2}$, the limiting value, $\lim_{a} x = \beta_1$ and the point of inflection (x_1, a_1) is given by, $x_1 = \frac{\beta_1}{2} h_x^{\gamma}$ and $a_1 = \frac{\ln \beta_2}{\beta_3}$.¹⁸

From equation (2.6), π_x is a function of $p_x x$, which is given by equation (2.3) as $p_x x = \alpha x^{\alpha}$. Substituting and differentiating gives:

$$\frac{{}^{2}\pi_{x}}{a^{2}} = \frac{{}^{2}}{x^{2}} \left(\alpha x^{\alpha} \right) \frac{{}^{2}x}{a^{2}}$$
$$= \alpha^{2} \left(\alpha - 1 \right) x^{\alpha - 2} \frac{{}^{2}x}{a^{2}}$$
(4.14)

¹⁷ *Vide*: Rogers and Shoemaker (1971).

¹⁶ This result requires $|\lambda| < r$ in equation (4.12). Equation (4.9) shows that this must be the case for ${}^{2}\pi_{x}/{a^{2}} > 0$. Note that this requirement also allows the interesting possibility of a complex solution to the locally unstable saddlepath solution, TT. This would cause the unbounded behaviour of *a* and *q_a* to exhibit increasing oscillations.

¹⁸ The point of inflexion is found by setting, ${}^{2}x/a^{2} = \beta_{1}\beta_{2}\beta_{3}^{2}e^{\beta_{3}a}(\beta_{2} - e^{\beta_{3}a})(\beta_{2} + e^{\beta_{3}a})^{-3}h_{x}^{\delta} = 0$. The non-trivial solution requires $\beta_{2} = e^{\beta_{3}a}$ and solving gives $a_{I} = (\ln\beta_{2})/\beta_{3}$. Substituting this result in equation (4.13) gives $x_{I} = (\beta_{1}/2)h_{x}^{\gamma}$ as required.

Now $\alpha^2 (\alpha - 1) x^{\alpha - 2} \stackrel{<}{_{>}} 0$ for $0 < \alpha \stackrel{<}{_{>}} 1$ so that:

$$\frac{a^2 x}{a^2} \stackrel{>}{<} 0 \text{ iff } a \stackrel{\leq}{>} \frac{\ln \beta_2}{\beta_3} \text{ and } x \stackrel{\leq}{>} \frac{\beta_1}{2} h_x^{\gamma}$$

$$(4.15)$$

That is, there are increasing (decreasing) returns to scale before (after) the point of inflection (x_l, a_l) .

If equations (4.1) and (4.4) are linearised in an appropriate neighborhood around values $a = \hat{a}$ and $q_a = \hat{q}_a$, the new system of dynamic equations become:

$$\dot{a}_{\dot{q}_{a}} = \frac{-\dot{a}_{a}}{-\frac{2\pi_{x}}{a^{2}}} \begin{vmatrix} \dot{a}_{a} & \dot{a}_{a} \\ \dot{q}_{a} & -\dot{a}_{a} \end{vmatrix}$$

$$(4.16)$$

The general solution is:

$$a - \hat{a} = (a - a_0)e^{\mu t}$$
$$q_a - \hat{q}_a = (q_a - q_0)e^{\mu t}$$

with:

$$\mu_{1,2} = \frac{1}{2} \left[\omega \pm \left(\omega^2 - 4\psi \right)^{\frac{1}{2}} \right]$$
(4.17)

and where:

$$\omega = r + \frac{\dot{a}}{a}\Big|_{\hat{a}} \tag{4.18}$$

$$\Psi = r \frac{\dot{a}}{a}\Big|_{\hat{a}} + \frac{2\pi_x}{a^2}\Big|_{\hat{a}} \times \frac{\dot{a}}{q_a}\Big|_{\hat{q}_a}$$
(4.19)

Now consider the neighborhood to be in the increasing returns to scale range of the production of the intermediate capital good, *x*. That is, $\hat{x} < x_I$ and $\hat{a} < a_I$, which gives from equation (4.15), $\frac{2}{a^2} \Big|_{\hat{a}} > 0$ and therefore from equation (4.14), $\frac{2\pi_x}{a^2} \Big|_{\hat{a}} > 0$. If the production of the final good, *y*, also exhibits increasing returns to scale in the form of $\alpha > 1$ then equation (4.11) shows $\frac{\dot{a}}{q_a} \Big|_{\hat{q}_a} > 0$. These inequalities therefore describe the locally unstable saddlepath solution TT, as shown in Figure 1.

Selecting a neighborhood around (\tilde{a}, \tilde{q}_a) on the other side of the inflexion point, in the Logistic function (4.13), that is $\tilde{x} > x_I$ and $\tilde{a} > a_I$, gives $\frac{|a_i^2|}{|a_i^2|} < 0$ and $\frac{|a_i^2|}{|a_i^2|} < 0$ by

equations (4.15) and (4.14). If $\psi > 0$ then $\mu > 0$ and the unstable solution prevails. However, higher levels of new technology, *a*, will ultimately decrease $\frac{{}^{2}x}{a^{2}}$ and therefore $\frac{{}^{2}\pi_{x}}{a^{2}}$ to levels such that $\psi < 0$ will give $\mu < 0$ and the locally stable form of the saddlepath, SS in Figure 1.

With this in mind, consider a relatively large set of *n* possible values of adopted technologies, *a*, denoted $\{a_i\}_{i=1}^n$. Each of these $\{a_i = \hat{a}_i\}_{i=1}^n$ can then be nominated in turn to sequentially calculate the relevant linearised saddlepath in the relatively small non-overlapping neighborhoods around each a_i , defined as $(\hat{a}_i \pm \phi)$. For a larger number of smaller neighborhoods the set becomes:

$$\lim_{n} \lim_{\phi} \left\{ a_i = \hat{a}_i \pm \phi \right\}_{i=1}^n = \bigcup_i a_i$$
(4.20)

This set comprises three adjoint subsets:

$$\bigcup_{i} a_{i} = \left\{a_{j}\right\} \cup \left\{a_{k}\right\} \cup \left\{a_{l}\right\}$$

$$(4.21)$$

which each representing increasing, constant and decreasing returns to scale respectively for the Logistic function:

$$a_j; \ \frac{a_j}{a_j^2} > 0, \ \frac{a_k}{a_j^2} > 0, \ a_k; \ \frac{a_k}{a_k^2} = \frac{a_k}{a_k^2} = 0, \ a_l; \ \frac{a_l}{a_l^2} < 0, \ \frac{a_k}{a_l^2} < 0$$

Applying equation (4.3) gives the values of Tobin's q for the three ranges and an assumed continuous characterisation of this set is shown as VV in Figure 3. This curve is drawn with local instability around the steady state a_0^* in the first set and local stability around a_1^* in the third set. Curve VV must be concave to the a axis because the Logistic production function for the intermediate capital good, x, given by equation (4.13) demonstrates increasing returns which become decreasing after point B. Growth is characterised in this model for $q_a > 1$ and the arrows show the dynamics of motion on VV between points A and D. The rate of adoption of technology is increasing between points A and B since $\frac{2\pi_x}{a^2} > 0$ giving $\mu > 0$ in equation (4.17), $a - \hat{a} = (a - a_0)e^{\mu t}$, for the locally linearised portion of VV around \hat{a} , where $a_0^* < \hat{a} < a^*$.

For \tilde{a} occurring after the point B, that is where $a^* < \tilde{a} < a_1^*$, then $\frac{-2\pi_x}{a^2} < 0$ giving $\psi < 0$ in equation (4.19) and therefore $\mu < 0$ in equation (4.17). This ensures that the rate of adoption of new technology will fall until point D is eventually reached.



This process of adoption can be illustrated by modifying the Logistic production function of equation (4.13) to include both a short run variable technology factor of production, a_1 , and a long run technology factor of production, a_2 , which is assumed to be fixed in the short run. That is:

$$x = L(a_{1}, a_{2}^{\beta_{4}}, h_{x}^{\gamma})$$

= $\frac{\beta_{1}}{1 + \beta_{2}e^{-\beta_{3}a_{1}}} a_{2}^{\beta_{4}}h_{a}^{\delta} \qquad \beta_{4} > 0$ (4.22)

The short run rate of adoption of new technology is unchanged from equation (4.4):

$$\dot{a}_1 = (q_a - 1)h_a^\delta a_1^\varepsilon \tag{4.23}$$

However, the adoption of technology in the long run is defined as:

$$\dot{a}_{2}(t) = \begin{array}{cc} 0 & t < t_{\tau} \\ [q_{a}(t) - 1]h_{a}^{\gamma}(t)a_{2}^{\nu}(t) & t & t_{\tau} \end{array}$$
(4.24)

Equation (4.24) shows that $a_2(t) = \overline{a}_2$ for $t < t_{\tau}$ whilst it will be increasing for $t = t_{\tau}$. The slope of VV is given by equation (4.14) such that an increase in a_2 at time t_{τ} may increase $\frac{2\pi_x}{a^2}$ and therefore q_a .¹⁹ Given that the slope of the curve will become steeper and the

¹⁹ Differentiating equation (4.22) with respect to the variable technology, a_1 holding a_2 and h_a fixed in the short run gives, ${}^2x/a_1^2 = \beta_1\beta_2\beta_3^2e^{\beta_3a_1}(\beta_2 - e^{\beta_3a_1})(\beta_2 + e^{\beta_3a_1})^{-3}a_2^{\beta_4}h_a^{\delta}$. An increase in a_2 will therefore increase ${}^2x/a_1^2$, ${}^2\pi_x/a_1^2$ and therefore q_a .

starting point is later in time reflecting a higher starting level of technology, $a > a_0^*$, then the new curve denoted WW must cut the initial curve VV from below. This effect is also shown in Figure 3.



FIGURE 5.

Indeed a family of possible curves may exist and a monopolist having adopted new technology and moving towards point C along the curve VV will experience diminishing returns and reduced marginal profits indicated by falling q_a . The existing monopolist or new entrant, realising that the new production process characterised by WW exhibits increasing returns to scale, will maximise profits by switching at (or near) point C to the new process characterised by WW. It is also important to note that the rate of adoption of this newer process will be faster since $\mu_{WW} > \mu_{VV}$ at point C. This switching to the new higher technology path, which dominates the older technology in terms of relatively higher Tobin's q and speed of adoption, characterises the process of creative destruction.

Maximising over all possible production technologies describes the possible envelopes MM in Figures 4 and 5. These envelopes describe the intertemporal adoption of new technology, which clearly resemble the spatial metaproduction function described by Hyami and Ruttan (1971).²⁰ The characterised shape of this metaproduction function also bears remarkable similarities with the stylized facts reported in Maddison (1991) and reproduced in Figure 6. The measure of productivity growth is GDP per man-hour in 1985 constant \$US for the period 1580-1989. The shifts in productivity leadership from the Netherlands to the U.K. during the period 1820-1840, and then to the USA around 1890, are noteworthy.²¹ Maddison (1991) also examines the catching up of Japan, particularly during the last four decades. The Japanese success in closing the gap, in terms of per capita GDP growth reproduced in Figure 7, is remarkable. The use of new technology, produced in the global research sector as well as in Japan, seems to be driving this process.



FIGURE 6.—Changes in Productivity Leadership, 1580-1989 (GDP per man hour in 1985 \$US)

Source: Maddison, A. (1991), *Dynamic Forces in Capitalist Development*, Oxford University Press, New York, p. 31.

²⁰ Details are reported in Chaudhri and Wilson (1995).

²¹ Baumol *et.al.* (1988) examination of the US leadership corroborates this evidence for recent decades.



FIGURE 7.—Comparisons of Per Capita GDP Growth, 1890-1989 (Logarithm of GDP per capita in 1985 \$US)

5. CONCLUSIONS AND IMPLICATIONS

This paper develops a four-sector model of long run capitalist economic growth. The approach adopts Aghion and Howitt's (1992) suggestion to model economic growth in terms of creative destruction, which explicitly incorporates technological change in physical and human capital. This is done by modeling endogenous growth in the form of Romer's (1990) technological innovation in monopolistic capital goods production, where human capital and non-rival, partially excludable, technology are inputs to production. Abstracting from short term concerns and business cycles gives a better theoretical approximation to the functioning of the capitalist system and the drivers of long run economic growth. The model not only incorporates Schumpeterian ideas of creative destruction as the guiding spirit of economic development, it also accommodates the increasing returns ideas of Young (1928) and Schultz (1990).

The contribution of this paper is the inclusion of variable returns to scale in capital goods production. An example characterises production as initially increasing returns to scale, which subsequently diminish as production expands until decreasing returns to scale are realised. The profit maximising monopolist's behaviour is analysed in terms of a variable Tobin's *q*, which derive possible saddlepath solutions. These solutions explain the adoption of new technology as the process of creative destruction, which drive long run economic growth. The set of saddlepath solutions are modelled to form a nonlinear class reflecting the changing returns to scale. The rate of adoption of new technology is consequently variable with important variations over time. Finally, the class of solutions define an endogenously evolving Hyami and Ruttan (1971) style metaproduction function. This specification provides for stable long run growth with output and productivity growth alternatively accelerating and decelerating in non-mechanical ways. The model stylizes Maddison's (1991) historical evidence of shifting technological leadership in capitalist economic growth processes during the last two centuries.

Source: Maddison, A. (1991), *Dynamic Forces in Capitalist Development*, Oxford University Press, New York, p. 31.

REFERENCES

- AGHION, P. AND P. HOWITT (1992): "A Model of Growth Through Creative Construction", *Econometrica*, 60, 323-351.
- ARROW, K.J. (1962): "The Economic Implications of Learning by Doing", *Review of Economic Studies*, 29 155-173.
- BARRO, R.J. (1990): "Government Spending in a Simple Model of Endogenous Growth", Journal of Political Economy, 98, S103-125.
- BARRO, R.J. AND X. SALA-I-MARTIN (1992) "Convergence", *Journal of Political Economy*, 100, 223-251.
- BASALLA, G. (1988): The Evolution of Technology, Cambridge University Press, Cambridge.
- BAUMOL,W.J. et.al. (1991): Productivity and American Leadership: The Long View, The MIT Press, Cambridge, Mass.
- BLANCHARD, O. (1981): "Output, the Stock Market, and Interest Rates", American Economic Review, 71, 132-143.
- BLANCHARD, O. AND S. FISCHER (1990): "Lectures on Macroeconomics", *MIT Press*, Cambridge, MA.
- CHAUDHRI, D.P. AND E. WILSON (1995): "Sources of Modern Economic Growth: Theoretical and Empirical Explorations with Asian Focus", mimeo.
- CHIANG, A. (1992): "Elements of Dynamic Optimisation", McGraw-Hill, Sydney.
- CHRISTENSEN, L., D. JORGENSON AND L. LAU (1973): "Transcendental Logarithmic Production Functions", *Review of Economics and Statistics*, 55, 28-45.
- HAHN, F.H. AND R.C.O. MATHEWS (1964): "The Theory of Economic Growth: A Survey", *Economic Journal*, 74, 779-902.
- HYAMI, Y. AND V. RUTTAN (1971): Agricultural Development: An International Perspective. John Hopkins University Press, Baltimore.
- JONES, L. AND R. MANUELLI (1990): "A Convex Model of Equilibrium Growth: Theory and Policy Implications", *Journal of Political Economy*, 98, 1008-1038.
- JORGENSON, D. (1989): "Productivity and Economic Growth" in Berndt, E.R. and J.E. Triplett (ed) *Fifty Years of Economic Measurement*, NBER, University of Chicago Press, Chicago.
- KALDOR, N. (1957): "A Model of Economic Growth", The Economic Journal, 67, 591-624.
 - (1960): Essays on Economic Stability and Growth, Duckworth, London.
- KALDOR, N. AND J.A. MIRRLEES (1961): "Growth Model with Induced Technical Progress" *Review of Economic Studies*, 29, 176-90.
- LUCAS, R. (1988): On the Mechanics of Economic Development", Journal of Monetary Economics, 22, 3-42.

- MADDISON, A. (1991): Dynamic Forces in Capitalist Development, Oxford University Press, New York.
- MANDELHAUM, M.K. (1945): The Industrialisation of Backward Areas, Blackwell, Oxford.
- MARGLIN, S. (1984): *Growth, Distribution and Prices*, Harvard University Press, Cambridge, Massachussets.
- MUNDLAK, Y. (1993): "On the Empirical Aspects of Economic Growth Theory", *American Economic Review*, Papers and Proceedings, May, 415-420.
- REBELO, S. (1991): "Long Run Policy Analysis and Long Run Growth", *Journal of Political Economy*, 99, 500-521.
- ROGERS, E.M. AND F.F. SHOEMAKER (1971): *Diffusion of Innovations*, The Free Press, New York.
- ROMER, P.M. (1986): "Increasing Returns and Long Run Growth", Journal of Political Economy, 94, 1002-1037.

(1990): "Endogenous Technical Change", *Journal of Political Economy*, 98, S71-102.

SCHULTZ, T.W. (1976): "Ability to Deal with Disequilibria", *Journal of Economic Literature*, 13, 827-846.

(1990): *Restoring Economic Equilibrium: Human Capital in the Modern Economy*, Basil Blackwell, Cambridge, Mass.

SCHUMPETER, J. (1939): Business Cycles, McGraw Hill, New York.

SCOTT, M. F. G. (1989): A New View of Economic Growth, Clarendon Press, Oxford.

(1992): "A New Theory of Endogenous Economic Growth", *Oxford Review* of Economic Policy, Winter, 8, 29-42.

(1993): "Explaining Economic Growth", American Economic Review, 83, 421-425.

- SEN, A.K. (ed.) (1971): Growth Economics, Penguin Modern Economics Readings, London.
- SOLOW, R. (1994): "Perspectives on Growth", Journal of Economic Perspectives, 8, 45-54.
- SWAN, T. (1956): "Economic Growth and Capital Accumulation", *The Economic Record*, 32, 334-361.
- YOUNG, A. (1928): "Increasing Returns and Economic Progress", *The Economic Journal*, XXXVIII.