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Additional Notes on

A Model for

COMMUNICATING SEQUENTIAL PROCESSES

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Summary: These notes contain copies of the overhead projector slides presented at the Communicating Sequential Processes Symposium in Wollongong which were not included in the original preprint 80-1 issued at the Symposium.

ALGEBRAIC PROPERTIES.

$\square$ is associative	$P \square (Q \square R) = (P \square Q) \square R$
commutative	$P \square Q = Q \square P$
idempotent	$P \square P = P$
with unit:	$P \square ABORT = P$
and "zero":	$P \square (\bar{P})^* = (\bar{P})^*$
$\|\|$ is associative	$P \| (Q \| R) = (P \| Q) \| R$
commutative	$P \| Q = Q \| P$
with unit:	$P \| (\bar{P})^* = P$
and zero:	
; is associative	$P ; (Q ; R) = (P ; Q) ; R$
with unit:	$SKIP ; P = P$
and zero:	$ABORT ; P = ABORT$
and distributes:	$(a \rightarrow P) ; Q = a \rightarrow (P ; Q)$
	$(P \square Q) ; R = (P ; R) \square (Q ; R)$
$\gg$ is associative	$P \gg (Q \gg R) = (P \gg Q) \gg R$
and distributes:	

$$(\mathrm{ABORT} \gg \mathrm{ABORT}) = (\mathrm{ABORT} \gg (?x{:}T \to Q(x)))$$

$$= ((!x;\, P) \gg \mathrm{ABORT}) = \mathrm{ABORT}$$

$$(!v;\, P) \gg (?x{:}T \to Q(x)) = P \gg Q(v) \qquad \text{for } v \in T.$$

$$(\quad) \gg (\quad) = \mathrm{ABORT} \qquad \text{for } v \notin T$$

$$(!v;\, P) \gg (!x;\, Q) = !x;\, ((!v;\, P) \gg Q) \qquad ''$$

$$(?x{:}S \to P(x)) \gg (?y{:}T \to Q(y)) =$$

$$(?x{:}S \to (P(x) \gg (?y{:}T \to Q(y))))$$

$$(?v{:}S \to P(v)) \gg (!x;\, Q) =$$

$$(?v{:}S \to (P(v) \gg (!x;\, Q)))$$

$$\square\, (!x;\, ((?v{:}S \to P(v)) \gg Q))$$

$$(?v{:}S \to P(v)) = (?w{:}S \to P(w))$$

$$(?v{:}\{\} \to P(v)) = \mathrm{ABORT}.$$

RECURSION

Def A process with alphabet A is a non-empty prefix-closed subset of A^*

Thm. If P_i are processes with alphabet A for all i in T, then so are $\bigcup_{i \in T} P_i$ Thus processes form a complete lattice under $\subseteq$, with ABORT as bottom and A^* as top

Def. A function F from processes to processes is <u>distributive</u> if for all sets $\{P_i \mid i \in T\}$

$$F\left(\bigcup_{i \in T} P_i\right) = \bigcup_{i \in T} F(P_i)$$

Thm. $\rightarrow$, $\Box$, $\|$, $;$, $m:$, $\backslash$ are distributive (and hence continuous and monotonic) in each of their arguments.

Thm (Tarski, Scott). If F is continuous, then the least p satisfying

$$p = F(p) = (\ldots; p \ldots \| \ldots p \ldots p \ldots p \ldots)$$

is $\bigcup_{i \in NN} F^i(ABORT) \quad =_{df} \quad \mu p.\, F(p)$

where $F^0(q) = q$

$$F^{n+1}(q) = F^n(F(q))$$

When a process is defined by recursion, we intend it to be the least solution of its defining equation.

And the same is true for sets of mutually recursive equations.

Proof RHS $= F\left(\bigcup_{i \in NN} F^i(ABORT)\right) = \bigcup_{i \in NN} F(F^i(ABORT))$ [continuity

$= \bigcup_{i \in NN} F^{i+1}(ABORT)$ [def F^{i+1}

$= ABORT \cup \bigcup_{i > 0} F^i(ABORT)$ [ABORT $\leq$ any

$= \bigcup_{i \in NN} F^i(ABORT)$.

Unique Fixed Points.

If $F(p)$ is an expression in which p appears only to the right of $\rightarrow$, (and F does not contain localisation), then the solution to $p = F(p)$ is <u>unique</u>.

Proof: $F(p)$ always does something before making the recursive call on p. So $F^n(p)$ does at least n things before calling on p; these are the same things as for $F^n(q)$. Suppose $p = F(p)$ & $q = F(q)$. Let $s \in p$ be of length n. Since $p = F^m(p)$ for all m, $s \in F^n(p)$. so $s \in F^n(q)$. Thus $p \subseteq q$. Similarly, $q \subseteq p$. $\therefore p = q$

The same is true for sets of mutually recursive equations.

RECURSION INDUCTION

$$COUNT_0 = (iszero \rightarrow COUNT_0 \;[]\; up \rightarrow COUNT_1)$$

$$COUNT_n = (down \rightarrow COUNT_n \;[]\; up \rightarrow COUNT_{n+2})$$

$$POS = (down \rightarrow SKIP \;[]\; up \rightarrow POS; POS)$$

$$ZERO = (iszero \rightarrow ZERO \;[]\; up \rightarrow POS; ZERO).$$

Theorem. $ZERO = COUNT_0$.

Proof. Define $C_0 = (iszero \rightarrow C_0 \;[]\; up \rightarrow C_1)$

$$C_{n+1} = POS; C_n$$

$$C_{n+1} = (down \rightarrow SKIP; C_n \;[]\; up \rightarrow (POS; POS); C_n) \quad \text{def } C_{n+1} \text{ \& distr } ;$$

$$= (down \rightarrow C_n \;[]\; up \rightarrow POS; (POS; C_n)) \quad \text{prop } ;$$

$$= (down \rightarrow C_n \;[]\; up \rightarrow POS; C_{n+1}) \quad \text{def } C_{n+1}$$

$$= (down \rightarrow C_n \;[]\; up \rightarrow C_{n+2}) \quad \text{def } C_{n+1}$$

$\therefore \quad C_n = COUNT_n$ for all n. ... (1)

but $C_0 = (iszero \rightarrow C_0 \;[]\; up \rightarrow POS; C_0)$

$\therefore \quad C_0 = ZERO$ ... (2)

Conclusion follows from (1), (2).

YET ANOTHER COUNT.

$$Z = (\text{iszero} \rightarrow Z \;[]\; \text{up} \rightarrow (p{:}Z \,||\, X); Z)$$

where

$$\begin{aligned} X = (\ & \text{up} \rightarrow p.\text{up} \rightarrow X \\ & []\,\text{down} \rightarrow (p.\text{iszero} \rightarrow \text{SKIP} \\ & \qquad\qquad []\,p.\text{down} \rightarrow X \\ &) \qquad\qquad) \end{aligned}$$

Theorem. $Z = \text{ZERO}$

RELATIONS.

Let $R : (\text{ins}(\alpha P))^* \leftrightarrow (\text{outs}(\alpha P))^*$

$P \ \underline{\text{sat}} \ R =_{df} \forall s \ (s \in P \rightarrow (\text{ins}(s), \text{outs}(s)) \in R)$

– at all times, the sequence of values input by P bears relation R to the sequence of values output by P.

e.g. let $f:$ be a <u>monotonic</u> function of traces.

let $R_f = \{(i, o) \mid o \leq f(i)\}$

If $P \ \underline{\text{sat}} \ R_f$, P is said to be a <u>pipe</u> for f.

We shall often represent R as a predicate on the variables "in" and "out"

eg. $P \ \underline{\text{sat}} \ (\text{out} \leq \text{in}) \quad = P \ \underline{\text{sat}} \ \{(\text{in}, \text{out}) \mid \text{out} \leq \text{in}\}$

– means that P is a <u>buffer</u>

i.e. a pipe for the identity function.

Theorem. $Q \ \underline{\text{sat}} \ R \ \& \ R \subseteq S \Rightarrow Q \ \underline{\text{sat}} \ S$

$(\forall i \ Q_i \ \underline{\text{sat}} \ R) \Rightarrow (\bigcup_i Q_i) \ \underline{\text{sat}} \ R$

If ABORT $\underline{\text{sat}}$ R & $\forall p.\ p \ \underline{\text{sat}} \ R \Rightarrow F(p) \ \underline{\text{sat}} \ R$

then $(\mu p.F(p)) \ \underline{\text{sat}} \ R$. (fixed point induction)

① $\text{ABORT}\ \underline{\text{sat}}\ R \equiv R^{in}_{\langle\rangle}\ {}^{out}_{\langle\rangle}$

② $(!x;\ P)\ \underline{\text{sat}}\ R \equiv P\ \underline{\text{sat}}\ R^{out}_{\langle x\rangle out}$

where $R^{out}_{\langle x\rangle out} = \{(i,o) \mid (i, \langle x\rangle o) \in R\}$

i.e. replace "out" by "$\langle x\rangle$out" in R

③ $(?x{:}T \rightarrow P(x))\ \underline{\text{sat}}\ R \equiv \forall x{:}T.(\ P(x)\ \underline{\text{sat}}\ R^{in}_{\langle x\rangle in}\ )$

i.e. it works for all input values.

by ② $((!x;\ B)\ \underline{\text{sat}}\ out \le \langle x\rangle in) \equiv B\ \underline{\text{sat}}(\langle x\rangle out \le \langle x\rangle in)$

$\equiv B\ \underline{\text{sat}}\ out \le in.$

by ③ $((?x{:}T \rightarrow !x;\ B)\ \underline{\text{sat}}\ out \le in)$

$\equiv \forall x{:}T\ ((!x;B)\ \underline{\text{sat}}\ out \le \langle x\rangle in)$

$\equiv B\ \underline{\text{sat}}\ out \le in$ (just proved)

by ① ABORT $\underline{\text{sat}}$ $out \le in.$ (because $outs(\langle\rangle) = ins(\langle\rangle) = \langle\rangle$)

$\therefore$ if $B =_{df.} (?x{:}T \rightarrow (!x;\ B))$

then B is a buffer. (Fixed point induction)

$$T = \text{outs}(\alpha P) = \text{ins}(\alpha Q)$$

$$(P \ \underline{\text{sat}}\ R) \ \& \ (Q \ \underline{\text{sat}}\ S) \Rightarrow$$

$$(P \gg Q) \ \underline{\text{sat}}\ \exists s\,(R^{out}_{s} \ \& \ S^{in}_{s})$$

which is $(R;S)$ — relational composition of R and S.

Proof. $t \in (P \gg Q)$, $P \ \underline{\text{sat}}\ R$, $Q \ \underline{\text{sat}}\ S$ — assume

$\therefore \exists u \ \ t = u \upharpoonright T \ \ \& \ \exists v \in P \ \exists w \in Q$

(A) $\text{ins}(v) = \text{ins}(u) \ \& \ \text{outs}(w) = \text{outs}(u) = \text{outs}(t)$

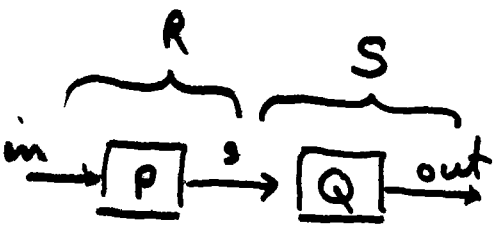
$\& \ \text{outs}(v) = \text{ins}(w) = u \upharpoonright T$

$\therefore (\text{ins}(v), \text{outs}(v)) \in R \ \& \ (\text{ins}(w), \text{outs}(w)) \in S$

$\Rightarrow (\text{ins}(u), u \upharpoonright T) \in R \ \& \ (u \upharpoonright T, \text{outs}(u)) \in S$

$\Rightarrow \exists s \ (\text{ins}(t), s) \in R \ \& \ (s, \text{outs}(t)) \in S$

$\Rightarrow (\text{ins}(t), \text{outs}(t)) \in (R;S)$.



~~If~~ f and g are monotonic:

$(P \underline{sat}\ outs \le f(ins))\ \&\ (Q\ \underline{sat}\ outs \le g(ins))$

$\Rightarrow (P \gg Q)\ \underline{sat}\ \exists s\ \ s \le f(ins)\ \&\ outs \le g(s))$

$\Rightarrow (P \gg Q)\ \underline{sat}\ outs \le g(f(ins))$

theorem If P is a pipe for f and Q for g

then $P \gg Q$ is a pipe for $g \circ f$.

If P and Q are buffers, so is $P \gg Q$

(a buffer is a pipe for the identity function).

since $B1 = (?x{:}T \to !x;\ B1)$ is a buffer.

so is $B_{nos} = B_n \gg B_1$ for all $n \ge 1$.

Proof. induction on n

WARNING <u>sat</u> defines only a form of partial correctness - does not prove absence of deadlock. eg. the following are buffers.

ABORT, $;(?x{:}\{3\} \to !3;\ B_1)_g\ B_{\langle\rangle}$ where

$B_s = (?x{:}T \to (?y{:}T \to !s_0;\ B_{s'\langle x\rangle\langle y\rangle}))$

COMMUNICATIONS.

A communications protocol consists of a transmitting process P and a receiving process Q such that $P \gg Q$ is a buffer. i.e its outputs are at all times an initial segment of its inputs

Theorem. If for all $x:T$, $P_x \gg Q_x$ is a buffer

then so is $(?x:T \rightarrow (P_x \gg (!x; Q_x)))$ ..... (1)

Proof. Let t be a trace of (1)

then $\text{first}(\text{ins}(t)) = \text{first}(\text{outs}(t))$. ..... (2)

Let t' be formed from t by omitting its first input and its first output. t' must be a trace of $P_x \gg Q_x$, which is a buffer.

$\therefore \quad \text{outs}(t') \leq \text{ins}(t')$ (3)

but $\text{ins}(t) = \langle \text{first}(\text{ins}(t)) \rangle \text{ins}(t')$ (4)

and $\text{outs}(t) = \langle \text{first}(\text{outs}(t)) \rangle \text{outs}(t')$ (5)

$\therefore \quad \text{outs}(t) \leq \text{ins}(t)$ from (2, 3, 4, 5)

If for all $x:T$

$$P_x \gg Q_x = (?y:T \rightarrow P_y \gg (!y\,;\, Q_y))$$

then $P_x \gg Q_x$ is a buffer for all $x:T$.

Proof. induction on length of trace of $P_x \gg Q_x$.

t is OK $\equiv_{df}$ $outs(t) \leq ins(t)$, so $\langle\rangle$ is OK.

assume all t of length $\leq n$ in $P_x \gg Q_x$ are OK (for all x)

now let $t' \in P_x \gg Q_x$ be ~~longer than~~ of length $n+1$.

If t' is all inputs, it's OK

otherwise $t' \in RHS$, so on removal of its first input and output (which are equal), it is still in $P_y \gg Q_y$ for some y. By induction hypothesis, its still OK

If $P_1 \gg Q_1$ and $P_2 \gg Q_2$ are buffers then so ~~same~~ is $(P_1 \gg P_2) \gg (Q_2 \gg Q_1)$

(Composition of protocols).

Phase encoding.

$$P = (?x:\{0,1\} \rightarrow (!x;\ !(1-x);\ P))$$

$$(Q = ?x:\{0,1\} \rightarrow (?y:\{1-x\} \rightarrow (!x;\ Q)))$$

$$(!0;\ !1;\ R) \gg P = !0;\ !1;((!1;R) \gg P)$$

$$= !0;\ !1;\ !1;\ !0;(R \gg P)$$

Theorem. $P \gg Q$ is a buffer.

Proof $P \gg Q =$

$$= ?x:B \rightarrow (!x;\ !(1-x);P) \gg (?y:B \rightarrow ?z:\{1-y\} \rightarrow (!y;\ Q))$$

$$= ?x:B \rightarrow ((!(1-x);\ P) \gg (?z:\{1-x\} \rightarrow (!x;\ Q)))$$

$$= ?x:B \rightarrow (P \gg (!x;\ Q))$$

$\therefore$ $P \gg Q$ is a buffer.

NRZ Protocol $\qquad P_0 \gg Q_0$

$P_0 = ?x:\{0,1\} \rightarrow !x;\ P_x$

$P_1 = ?x:\{0,1\} \rightarrow !(1-x);\ P_x$

x records previous input bit.

$$(!1;\ 0!;!0;\ !1;\ R) \gg P_0 = !1;((!0;!0;1;R) \gg P_1)$$
$$= !1;\ !1;((!0;!1;R) \gg P_0)$$
$$= !1;\ !1;\ !0;\ ((!1;R) \gg P_0)$$
$$= !1;\ !1;\ !0;\ !1;\ (R \gg P_1)$$

P copies first bit
then outputs 0 if input value remains same
1 if input value changes.

$Q_0 = ?x:\{0,1\} \rightarrow !x;\ Q_x$

$Q_1 = ?x:\{0,1\} \rightarrow !(1-x);\ Q_{(1-x)}$

$$(!1;!1;\ !0;!1;\ R) \gg Q_0 = !1;\ !0;\ !0;\ !1;\ (R \gg Q_1)$$

Q copies first bit
then copies if previous output was 0
inverts if previous output was 1.

Prove that $P_x >> Q_x$ is a buffer. for $x = 0, 1$

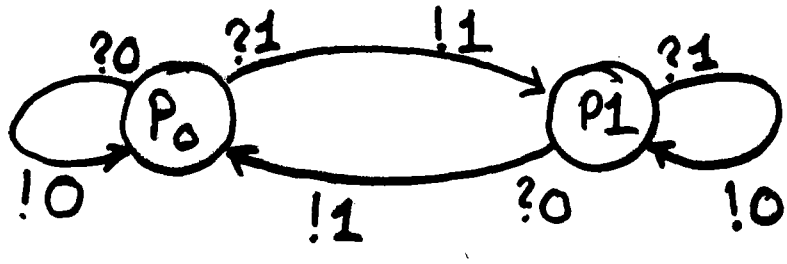
$$P_0 >> Q_0 = ?x:\{0,1\} \rightarrow ((!x;\ P_x) >> Q_0)$$
$$= ?x:\{0,1\} \rightarrow (P_x >> (!x;\ Q_x))$$

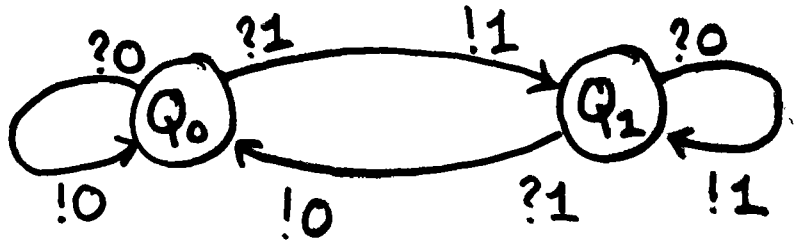
$$P_1 >> Q_1 = ?x:\{0,1\} \rightarrow ((!1-x;\ P_x) >> Q_1)$$
$$= ?x:\{0,1\} \rightarrow P_x >> (!(1-(1-x));\ Q_{(1-(1-x))})$$
$$= ?x:\{0,1\} \rightarrow P_x >> (!x;\ Q_x)$$

So $P_y >> Q_y = (?x:\{0,1\} \rightarrow P_x >> (!x;\ Q_x))$

for $y = 0, 1$

Therefore they are buffers.





A MODEL OF NON-DETERMINISM IN COMMUNICATING SEQUENTIAL PROCESSES.

with thanks to
Steve Brooks, Bill Roscoe

March 1980

The problem

Consider $R = (x \rightarrow a \rightarrow P \;[]\; y \rightarrow b \rightarrow Q) \setminus \{x, y\}$

Clearly, on its first step, it can accept "a", and it can accept "b". BUT also, it <u>can refuse</u> "a" (if "y" happened) and it <u>can refuse</u> "b" (if "x" happened). In our simple model, $R = (a \rightarrow P \;[]\; b \rightarrow Q)$, and the possibilities of refusal have not been represented. We need a more complex model.

Let P be a process with <u>finite</u> alphabet A. Let traces(P) be the subset of A^* denoting traces of the possible behaviour of P.

So traces(P) is nonempty & prefix-closed.

Define $P^\circ = \{a \mid \langle a \rangle \in \text{traces}(P)\}$

P° is the set of events possible for P on the very first step.

Let X be a subset of A denoting the events possible for the <u>environment</u> of P.

"P <u>canrefuse</u> X" means that P can deadlock in this environment.

So $\quad P$ <u>canrefuse</u> $\{\ \}$

$$P \ \underline{\text{canrefuse}}\ X \Rightarrow P \ \underline{\text{canrefuse}}\ X \cap Y$$

$$P \ \underline{\text{canrefuse}}\ X \Rightarrow P \ \underline{\text{canrefuse}}\ X \cup (A - P^0).$$

$(A - P^0)$ is a set which P <u>must</u> refuse

Let s be in traces (P). Then "P <u>after</u> s" denotes the future behaviour of P if s is a trace of its past behaviour.

So $\quad P \ \underline{\text{after}} \ \langle\rangle = P$

$$P \ \underline{\text{after}}\ st = (P \ \underline{\text{after}}\ s) \ \underline{\text{after}}\ t$$

Proposition: A process is defined by what it can <u>do</u> and what it can <u>refuse</u>.

So if $\text{alphabet}(P) = \text{alphabet}(Q)$

and $\text{traces}(P) = \text{traces}(Q)$

and $\forall X\ (P\ \underline{\text{canrefuse}}\ X \equiv Q\ \underline{\text{canrefuse}}\ X)$

and $\forall a\ (a \in P^0 \Rightarrow P\ \underline{\text{after}}\ \langle a \rangle = Q\ \underline{\text{after}}\ \langle a \rangle)$

then $P = Q$

We therefore <u>define</u> a process P as a relation:

for s in A^* and $X \subseteq A$

$(s, X) \in P$ <u>means</u> $s \in \text{traces}(P)\ \&\ (P\ \underline{\text{after}}\ s)\ \underline{\text{canrefuse}}\ X$.

So $\text{traces}(P) =_{df} \{s \mid (s, \{\}) \in P\}$

$P\ \underline{\text{canrefuse}}\ X =_{df} (\langle\rangle, X) \in P$

$P\ \underline{\text{after}}\ s =_{df} \{(t, X) \mid (st, X) \in P\}$

traces (P) must be nonempty & prefix-closed

$\{X \mid P\ \underline{\text{canrefuse}}\ X\}$ must be nonempty & left closed and closed under union with $\text{traces}(P)^0$

EXAMPLES. with alphabet A.

$STOP_A$ can't do anything
must refuse everything.

$$STOP_A =_{df} \{(\langle\rangle, X) \mid X \subseteq A\}$$

RUN_A can do anything
can't refuse anything.

$$RUN_A =_{df} \{(s, \{\}) \mid s \in A^*\}$$

$CHAOS_A$ can do anything
can refuse anything.

$$CHAOS_A =_{df} \{(s, X) \mid s \in A^* \ \& \ X \subseteq A\}$$

for all $s \in A^*$: RUN_A <u>after</u> s $= RUN_A$

$CHAOS_A$ <u>after</u> s $= CHAOS_A$.

$\longrightarrow$

Let F be a function from A to processes.
Let $B \subseteq A$. Then

$(x : B \to F(x))$ first accepts any x in B, and then behaves like $F(x)$

$$(x : B \to F(x)) =_{df} \{(\langle\rangle, X) \mid X \subseteq A - B\}$$
$$\cup \{(\langle x \rangle s, X) \mid x \in B \ \& \ (s, X) \in F(x)\}$$

$(b \to P)$ is short for $(x : \{b\} \to P)$

$(x : B \to F(x))$ <u>after</u> $b = F(b)$ for all $b \in B$.

$(x : \{\} \to F(x)) = STOP_A$

$(x : B \to F(x)) = (y : B \to F(y))$

PARALLEL COMPOSITION.

P and Q have same alphabet A.

P||Q can accept anything acceptable to <u>both</u> P and Q and if P can refuse X and Q can refuse Y, P||Q can refuse $X \cup Y$

$$(P \| Q) =_{df.} \{(s, X \cup Y) \mid (s, X) \in P \ \& \ (s, Y) \in Q\}$$

$$\text{traces}(P \| Q) = \text{traces}(P) \cap \text{traces}(Q)$$

$$(P \| Q)\ \underline{\text{after}}\ s = (P\ \underline{\text{after}}\ s) \| (Q\ \underline{\text{after}}\ s) \quad \text{for } s \in \text{traces}(P \| Q)$$

|| is associative, & commutative, with unit RUN_A and zero $STOP_A$.

$$(x : B \to F(x)) \| (y : C \to G(y)) = (z : B \cap C \to (F(z) \| G(z)))$$

NONDETERMINISM.

$P \sqcap Q$ behaves non-deterministically, either like P or like Q.

It can do anything that P <u>or</u> Q can do.

It can refuse anything that P <u>or</u> Q can refuse.

$$P \sqcap Q =_{df} P \cup Q$$

$\sqcap$ is associative, commutative, and idempotent with zero $CHAOS_A$.

$$\text{traces}(P \sqcap Q) = \text{traces}(P) \cup \text{traces}(Q)$$

$$\begin{aligned}
(P \sqcap Q)\ \underline{\text{after}}\ s &= P\ \underline{\text{after}}\ s && \text{if } s \in \text{traces}(P) - \text{traces}(Q) \\
&= Q\ \underline{\text{after}}\ s && \text{if } s \in \text{traces}(Q) - \text{traces}(P) \\
&= (P\ \underline{\text{after}}\ s) \sqcap (Q\ \underline{\text{after}}\ s) \\
& && \text{if } s \in \text{traces}(P) \cap \text{traces}(Q)
\end{aligned}$$

If we admit the EMPTY relation as a process, it wd be the unit of $\sqcap$.

UNION

$P \,[\!]\, Q$ behaves like P or like Q; the choice can be influenced by its environment, but only on the first step.

It can do anything P <u>or</u> Q can do.
It can refuse anything that <u>both</u> P and Q can refuse.

$$P \,[\!]\, Q = \{(\langle\rangle, X) \mid (\langle\rangle, X) \in P \cap Q\}$$
$$\cup \{(s, X) \mid s \neq \langle\rangle \;\&\; (s, X) \in (P \cup Q)\}$$

$[\!]$ is associative, commutative and idempotent with unit $STOP_A$.

$$\text{traces}(P \,[\!]\, Q) = \text{traces}(P) \cup \text{traces}(Q)$$

$$(x : B \to F(b)) \,[\!]\, (y : C \to G(c)) = (z : B \cup C \to$$
$$\underline{\text{if}}\ z \in B - C\ \underline{\text{then}}\ F(z)\ \underline{\text{else}}\ \underline{\text{if}}\ z \in C - B\ \underline{\text{then}}\ G(z)$$
$$\underline{\text{else}}\ F(z) \sqcap G(z))$$

LIMITS.

$P \sqsubseteq Q$ means Q is more deterministic than P.
& therefore more predictable, controllable, useful.

Everything Q can do so can P
Everything Q can refuse, so can P.

$P \sqsubseteq Q \;=_{df}\; Q \subseteq P$ or $P \sqcap Q = P$

e.g. $P \sqcap Q \sqsubseteq P [] Q$ $\qquad$ $CHAOS_A \sqsubseteq P$

$P \sqcap (P [] Q [] R) \sqsubseteq P [] Q$.

If $P_i \sqsubseteq P_{i+1}$ for all i, then we write

$$\bigsqcup_i P_i \;=_{df}\; \bigcap_i P_i$$

The relation $\sqsubseteq$ is a complete partial order with $CHAOS_A$ as its bottom.

If we add EMPTY, processes form a complete lattice with EMPTY as an isolated top.

SYMBOL CHANGE.

Let F be a total function from alphabet B onto alphabet A

Let P have alphabet A.

Then $f^{-1}(P)$ can do b (in B) whenever P can do $f(b)$, and can refuse X ($\subseteq B$) whenever P can refuse $f(X) = \{f(x) \mid x \in X\}$

$f^{-1}(P) = \{(s, X) \mid (f(s), f(X)) \in P\}$ with alphabet B.

$\text{traces}(f^{-1}(P)) = \{s \mid f(s) \in \text{traces}(P)\}$

$(f^{-1}(P))^{\circ} = f^{-1}(P^{\circ})$

$(f^{-1}(P))\ \underline{\text{after}}\ s = f^{-1}(P\ \underline{\text{after}}\ f(s))$

$f^{-1}(x : C \to F(x)) =$

$f^{-1}(P \| Q) =$

$f^{-1}(P \,[\!]\, Q) =$

If f is one-one, write $f(P)$ for $(f^{-1})^{-1}(P)$

ALPHABET EXTENSION

Let P be a process with alphabet A then P <u>ext</u> B is a process with alphabet $A \cup B$, which behaves like P, except that it is always prepared for any event in $B - A$, which it then ignores.

$$P \underline{\text{ext}}\ B = \{(s, X) \mid s \in (A \cup B)^* \ \&\ (s_A, X) \in P\}$$

where s_A is formed from s by omitting all symbols outside A.

$$(P \underline{\text{ext}}\ B)^\circ = P^\circ \cup (B - A)$$

$$(P \underline{\text{ext}}\ B)\ \underline{\text{convefuse}}\ X \equiv P\ \underline{\text{convefuse}}\ X$$

$$(P \underline{\text{ext}}\ B)\ \underline{\text{after}}\ s = (P\ \underline{\text{after}}\ s_A)\ \underline{\text{ext}}\ B$$

$$P\ \underline{\text{ext}}\ A = P, \quad (P\ \underline{\text{ext}}\ B)\ \underline{\text{ext}}\ C = P\ \underline{\text{ext}}\ (B \cup C)$$

If Q has alphabet B then

$$P \| Q =_{df} (P\ \underline{\text{ext}}\ B) \| (Q\ \underline{\text{ext}}\ A)$$

$\|$ is associative and commutative, etc.

ALPHABET CONTRACTION

Let P have alphabet A. Let B be a set of events to be regarded as internal to P. Then P\B is the process which behaves like P, but events in B may occur whenever they are possible, without participation or even the knowledge of the environment of P.

$(P\backslash B)^\circ \supseteq P^\circ - B$

P <u>can refuse</u> X & $X \cap B = \{\}$ $\Rightarrow$ (P\B) <u>can refuse</u> X

$s \in traces(P) \Rightarrow s_{A-B} \in traces(P\backslash B)$

$(P\backslash B)$ <u>after</u> s_{A-B} $\sqsubseteq$ (P <u>after</u> s)\B.

These properties are satisfied by

$$\boxed{P\backslash B} = \{(s_{A-B}, X) \mid X \cap B = \{\} \ \& \ (s, X \cup B) \in P\}$$

But $\{(b^n, X) \mid X \subseteq \{a\}\} \backslash \{b\}$ is empty, i.e.

<u>not a process</u>

The trouble lies in the <u>infinite</u> trace consisting of hidden symbols. The process <u>may</u> choose to follow this path forever and <u>never</u> engage in any further external interactions. or it may not. But you can't rely on anything. It's as bad as CHAOS. So let's make it so.

$$P\backslash B = \boxed{P\backslash B} \cup \{(st, X) \mid \{u \mid u \in P \;\&\; u_{A-B} = s\} \text{ is infinite}\}$$

$$P\backslash\{\} = P$$

$$(P\backslash B)\backslash C = P\backslash(B \cup C)$$

NOTE – we rely on <u>finitude</u> of alphabets

MONOTONICITY

$P \sqsubseteq R$ means that for all purposes R is better than P. Let F be a function on processes. Regard $F(P)$ as an assembly into which P has been plugged. We would like that replacement of P by a better component can only improve the assembly. For this, F must be monotonic, i.e.

$$F(P) \sqsubseteq F(R) \quad \text{whenever} \quad P \sqsubseteq R.$$

All functions defined so far are monotonic.

DISTRIBUTIVITY.

Let F be a monotonic function of processes. Suppose we wish to implement

$$F(P) \sqcap F(Q)$$

An easy way to do this may be first to implement $(P \sqcap Q)$ and then apply F to the result. This is valid only if F is <u>distributive</u>. i.e.

$$F(P) \sqcap F(Q) = F(P \sqcap Q)$$

All functions defined so far are distributive.

RECURSION.

A function F from processes to processes is <u>continuous</u> if for all ascending chains

$$\{P_i \mid i \in NN \ \& \ \forall i \ P_i \sqsubseteq P_{i+1}\}$$

$$F(\bigsqcup_i P_i) = \bigsqcup_i F(P_i).$$

If F is continuous, the least solution of

$$p = F(p)$$

is given by $p = \bigsqcup_i F^i(\mathrm{CHAOS}_A)$

where F^i is the i-fold composition of F

All functions defined so far are continuous.

THE UNIVERSITY OF WOLLONGONG

DEPARTMENT OF COMPUTING SCIENCE

symposium on

COMMUNICATING SEQUENTIAL PROCESSES

PROGRAMME — Pentagon Lecture Theatre 2.

Saturday March 22, 1980.

8.30 - 9.30	Registration		
9.30 - 10.30		Hoare C.A.R.	CSP Lecture I
10.30 - 11.00	Morning Tea		
11.00 - 12.00		Hoare C.A.R.	CSP Lecture II
12.00 - 13.00		Dromey R.G.	Text Searching
13.00 - 14.00	Lunch		
14.00 - 15.00		Hoare C.A.R.	CSP Lecture III
15.00 - 15.30	Afternoon Tea		
15.30 - 16.30		Stanton R.B.	Primitives for Concurrency
16.30 - 17.30		Barter C.J.	CSP Language and Implementation
17.30 - 18.00	Happy Hour		
18.00 - 21.00	Dinner	Reinfelds J.	Software Science

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Sunday March 23, 1980.

9.30 - 10.30		Hoare C.A.R.	CSP Lecture IV
10.30 - 11.00	Morning Tea		
11.00 - 12.00		Hoare C.A.R.	CSP Lecture V
12.00 - 13.00		Mateti P.	Correctness Proof of Indenting Program
13.00 - 14.00	Lunch		
14.00 - 15.00		Hoare C.A.R.	CSP Lecture VI
15.00 - 15.30	Afternoon Tea		
15.30 - 16.30		Tobias J.M.	Single User Multi Processor Around High Level Language

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