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PROFIT-MAXIMISING AND NON-PROFIT NETWORKS' DENSITY, MEMBERSHIP FEES AND SERVICE IN MONOPOLISTIC AND DUOPOLISTIC FRAMEWORKS*

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ABSTRACT

Network density, membership fees and service are analytically related to factors such as the size of the target population, the distribution of ability to enjoy the network's service within the target population, the tax-rebate rate on membership payment, the service production costs, which are increased by congestion or reduced by agglomeration, and the network organisational costs. The analysis is conducted for the cases of monopolistic and duopolistic profit-maximising networks and for the case of a monopolistic budget-balancing and density-targeting network.

JEL classification: D20, L20

Keywords: Network density, membership fee and service.

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I. INTRODUCTION

The term network is used in this paper to indicate the affiliation of people with a common interest in a service to an organisation which makes that service equally accessible to members. The networks considered in this paper generate revenues by collecting a uniform membership fee for accessing their service. In addition to fixed costs, they incur the variable costs of producing their service, organisational costs of billing and communicating with members, and congestion costs and enjoy agglomeration benefits. The quality of the service depends on the networks' membership fees and objective. It is also affected by size externalities that can be positive in the case of a dominant agglomeration effect or negative in the case of a dominant congestion effect.[1]

These organisations of common interest groups can be generally classified as social networks or economic networks. Much of the literature on networks deals with social networks and is written from a sociological perspective. A survey of that strand of literature is provided by Wellman and Berkowitz (1988). A brief survey of the economic literature on networks is provided by Jackson and Wolinsky (1996), who analysed the stability and efficiency of social and economic networks within a cooperative game framework.[2]

The main contribution of this paper to the network literature is the theoretical derivation of possible relationships between networks' density, membership fees and service. Membership fees and the ability to enjoy the network's service are incorporated into the individual's decision of joining a network. A critical ingredient in modeling this decision, and consequently network density, is that people have different levels of ability to enjoy a network service. That is, despite being simultaneously and equally offered, a network service appeals to some people more than to others and hence a considerable variation in people's willingness to pay for an identical service can be expected.

The distribution of the ability to enjoy the network service in a given population, and factors such as membership fees and tax concessions, determine the network density. In turn, network density affects both the network revenues, organisational and congestion costs and agglomeration benefits. These interdependencies are conceptually integrated into the analysis of the determination of membership fees and service.

The analysis is focused on two specific types of networks: profit-maximising networks in a monopolistic and duopolistic frameworks, and budget-balancing and density-targeting

monopolistic networks. The ownership and management of profit-maximising networks are likely to be separated from their members, whereas the budget-balancing and density-targeting networks are usually owned by their members and managed by democratically elected committees. Examples of profit-maximising networks are Internet subscriber's networks, matching agencies and private medical funds. Examples of budget-balancing and density-targeting networks are trade unions, co-ops, and professional, social, cultural and ethnic clubs. Budget-balancing represents a possible financial policy for non-profit networks, and density-targeting may serve these networks for increasing their bargaining and lobbying power.[3]

The paper is organised as follows. The membership decision and the minimum ability to enjoy the network service required for joining the network are described in section II. The minimum ability to enjoy the network's service is subsequently used for analysing the density of a monopolistic network in section III. Profit-maximising membership fees and service and their properties are derived in section IV for the monopolistic network case. Membership fees and service are also generated for a budget-balancing and density-targeting monopolistic network in section V. The analysis is extended to the more complicated case of two non-identical rival networks in section VI. The effects of the a network's own choice, and the rival's choice, of membership fees and service on its density are analysed. The Cournot-equilibrium membership fees and service were described for profit-maximising networks. Conclusions are summarised in section VII.

II. Membership Decision And The Minimum Ability To Enjoy Network's Service

It is assumed that people make decisions on membership in a network by comparing the periodical membership fee set by the network to their evaluation of the periodical service provided by the network. In addition to this conventional wisdom, the modeling of the membership decision incorporates people's ability to enjoy the network service on a zero-one scale and the upper-bound on people's willingness to pay for that service. This approach to modeling the membership decision is chosen so as to facilitate the subsequent sections' analysis of network density.

Similarly to Levy (1998), let ***S*** denote a combined index of the quantity and quality of the periodical service offered by a network to its members. Let the willingness to pay for ***S*** of an individual member ***i*** be a fraction of the highest willingness to pay for that service within the population:

$$p_i = q_i p_{\max}(S) \tag{1}$$

where $q_i \in (0,1)$ indicates the ***i***-th member ability to enjoy the service provided by the network and $p_{\max}(S)$ the upper-bound on members' willingness to pay for the periodical service offered by the network. This upper-bound rises with ***S*** (i.e., $p'_{\max} > 0$). Despite being simultaneously and equally offered, the network service appeals to some people more than to others because of differences in preferences, incomes, familiarity, skills and attributes needed for using the network and strategic behaviour.[4]

Let ***m*** denote the periodical network membership fee, and ***t*** the tax-rebate rate applicable in some countries for membership in some networks (e.g., professional associations and trade unions) and assumed, for simplicity, to be the same for all members. We may expect an individual ***i*** to be a member of a network as long as his, or her, willingness to pay for S is not lower than the tax-rebate adjusted membership fee:

$$q_i p_{\max}(S) \geq (1-t)m. \tag{2}$$

This membership rule implies that the minimum level of ability to enjoy the network service within the group of the network members rises with the membership fee in accordance with the following linear equation

$$q_{\min} = \left[\frac{1-t}{p_{\max}(S)} \right] m \tag{3}$$

where $p_{\max}(S)$ is calibrated so that $0 \le q_{\min} \le 1$.

III. Network Density And The Feasible Membership Fee-Service Set

For simplicity, let us first consider the case of a monopolistic network. The network density is defined as the ratio of the number of the network's members (***M***) to the target population (***L***). Recalling equation 3, the density of the network can be rendered as

$$\frac{M}{L} = \int_{\left[\frac{1-t}{p_{\max}(S)}\right]m}^{1} f(q_i)\,di \tag{4}$$

where $f(q_i)$ is the probability density function of the ability to enjoy the network service within the population. Let this probability density function be uniform,[5] then the network's density can be expressed as

$$\frac{M}{L} = 1 - \left(\frac{1-t}{p_{\max}(S)} \right) m. \tag{5}$$

A hundred per cent network density is reached when either the membership fee goes to zero or the upper-bound on members' willingness to pay for the network service goes to infinity. The positive scalar $(1-t)/p_{\max}(S)$ can be interpreted as the aggregate propensity to withdraw the membership in the network as the membership-fee rises. This membership

withdrawal inclination is moderated by the tax-rebate rate and the upper-bound on the members' willingness to pay for the service provided by the network.

The network-density equation implies that the empty-network set, $\{(m,S) \in R_+^2 : M = 0\}$, comprises all the membership fees and service combinations that satisfy the following equality

$$S_{|M=0} = p_{\max}{}^{-1}((1-t)m). \tag{6}$$

Recalling the assumption that $p'_{\max} > 0$, the corresponding empty-network curve is positively sloped in the first orthan spanned by ***m*** and ***S***. To simplify the following sections' analyses, let the upper-bound on members' willingness to pay be linear in the network service

$$p_{\max}(S) = vS \tag{7}$$

where ***v*** is a positive scalar indicating the upper-bound on members' marginal willingness to pay for the service provided by the network. Then, the empty-network combinations of ***m*** and ***S*** should satisfy

$$S_{|M=0} = \left(\frac{1-t}{v}\right)m. \tag{8}$$

Consequently, the network's feasible set of membership fees and service is the region above this *empty-network line* (ENL) as depicted by Figure 1.

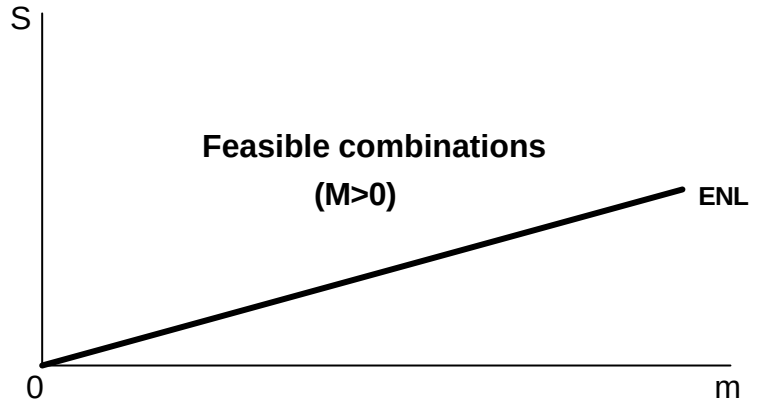

Figure 1. *The empty-network line and feasible combinations*

IV. Membership Fees And Service Of A Profit-Maximising Monopolistic Network

The network considered in this section is economic, has a monopoly on providing a service, and sets its membership fee and service level so as to maximise profit: the difference between the sum of the members' periodical membership payments and the network's periodical operational costs. In addition to a fixed cost C_0 the network has two types of variable costs: the service-production costs, C_1, and the organisational costs (e.g., billing and communicating with members), C_2.

Suppose that the service-production costs are quadratic in the level of service and affected by the network size (i.e., network externalities) as displayed by the second term on the right-hand side of the following equation

$$C_1 = c_1 S^2 + c_2 M \tag{9}$$

where c_1 is a positive scalar, and c_2 is a scalar indicating the marginal network externalities, which is positive if the provision of the service to member is adversely affected by the network membership size because of a dominant congestion effect, but negative if the provision of the service is enhanced by a dominant agglomeration effect, as in the case of matching networks.

Suppose also that the organisational costs are proportional to the number of members

$$C_2 = c_3 M \tag{10}$$

where c_3 is a positive scalar.

Then, the monopolistic network's profit equation is

$$\Pi = (m - c_2 - c_3) M - c_1 S^2 - C_0 . \tag{11}$$

By substituting the network's density equation 5 for ***M*** and equation 7 for $p_{\max}$ into equation 11 the profit equation can be rendered as

$$\Pi = (m - c_2 - c_3)[1 - \left(\frac{1-t}{vS}\right) m] L - c_1 S^2 - C_0 . \tag{12}$$

By differentiating Π with respect to ***m*** and setting the derivative to be equal to zero, the monopolistic profit-maximising membership fee for a given network service is

$$m = 0.5\left(\frac{v}{1-t}\right) S + 0.5(c_2 + c_3) . \tag{13}$$

In other words, for any given network's service level the profit-maximising membership fee is equal to half the marginal network's externalities and organisational costs plus half the upper-bound on members' willingness to pay for the network's service deflated by the effective cost of a dollar spent as membership payment (i.e., one minus the tax-rebate rate). This relationship is portrayed in Figure 4 by the *profit-maximising membership-fee line* (PMMFL).

By substituting equation 13 for ***m*** into equation 12 and rearranging terms, the network's profit function can be concentrated on ***S***

$$\Pi = 0.25\left(\frac{vL}{1-t}\right)S + 0.25\left(\frac{(1-t)L(c_2+c_3)^2}{v}\right)\frac{1}{S} - c_1 S^2$$
$$-0.5(c_2+c_3)L - C_0 \qquad .(14)$$

The monopolistic profit-maximising service should obey the first-order condition

$$\frac{d\Pi}{dS} = \frac{vL}{1-t} - \frac{(1-t)L(c_2+c_3)^2}{vS^{*2}} - 8c_1 S^* = 0. \qquad (15)$$

The second-order condition for maximum profit is satisfied as

$$\frac{d^2\Pi}{dS^2} = \frac{-2(1-t)(c_2+c_3)^2}{vS^{*3}} - 8c_1 < 0. \qquad (16)$$

The monopolistic profit-maximising network service, S^*, is obtained by solving the polynomial 15. The corresponding membership fee is obtained by substituting S^* into equation 13. This profit-maximising combination of the monopolistic network's service and membership fee is indicated in Figure 2 by E .

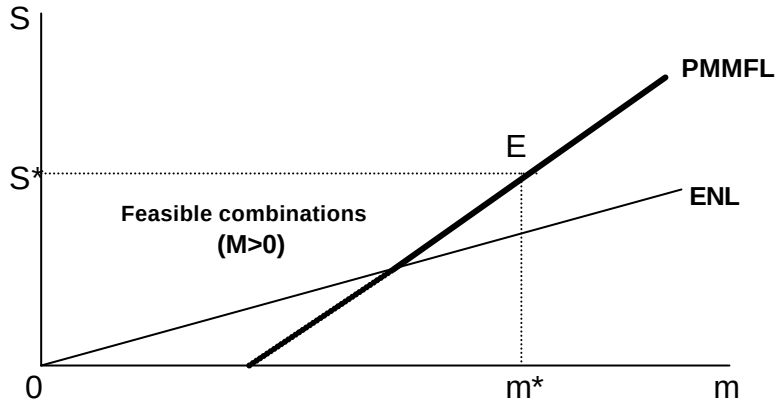

Figure 2. *The monopolistic profit-maximising service and membership fee*

The total differentiation of the first-order condition 15 and the negative sign of the second-order condition lead to the following claims about the properties of the monopolistic profit-maximising service. (See the Appendix for proofs.)

Claim 1: The effects of the maximum marginal willingness to pay (***v***) and the tax-rebate rate (***t***) on the service offered by the profit-maximising network are positive.

The underlying rationale is that the higher the upper-bound on the marginal willingness to pay and the tax-rebate rate on membership payment, the lower the overall membership withdrawal coefficient and hence the greater the network's revenues and profit and its incentive to provide a higher quality service.

Claim 2: The effect of the target population size on the monopolistic profit-maximising service depends on the product of the marginal network externalities plus the marginal organisational costs and the membership withdrawal coefficient. If initially this product is larger (smaller) than the profit-maximising service, an increase in the target population size would raise (lower) the level of the monopolistic profit-maximising service.

Note that in the case of a dominant agglomeration effect (i.e., $c_2 < 0$) it is less likely that an increase in L will raise the profit-maximising service level than in the case of a dominant congestion effect.

Claim 3: The higher the service-production costs' coefficient, the lower the monopolistic profit-maximising service.

Claim 4: The higher the marginal costs of the network externalities and organisation, the lower the monopolistic profit-maximising service.

Claim 5: The effects of changes in the model's parameters on the monopolistic profit-maximising membership fee have the same direction as the claimed effects on the monopolistic profit-maximising service, and are amplified by the upper-bound on the marginal willingness to pay and the tax-rebate rate.

V. Membership Fees And Service Of A Budget-Balancing And Density-Targeting Monopolistic Network

Let us now consider the membership fee and service levels set by a non-profit monopolistic network so as to cover the costs of servicing its members and ensure a predetermined membership percentage (i.e., density) target. This case is interesting as it is likely to be the policy of some social networks, professional associations and trade unions.

Recalling the fixed costs, service production costs, network externalities' costs and organisational costs specified in the previous section, the balanced-budget constraint can be displayed as:

$$C_0 + c_1 S^2 + c_2 M + c_3 M = mM \,. \tag{17}$$

Consequently, the relationship between the service provided by the network and the budget-balancing membership fee is

$$S = \sqrt{\frac{(m - c_2 - c_3)M - C_0}{c_1}} \quad . \tag{18}$$

Moreover, in recalling the network-density equation 5 and equation 7, this relationship can be rendered as

$$S = \sqrt{\frac{(m - c_2 - c_3)[1 - \left(\frac{1-t}{vS}\right)m]L - C_0}{c_1}}$$

(19)

or, equivalently, as

$$S^3 - \left(\frac{(m - c_2 - c_3 - C_0)L}{c_1}\right)S + \frac{(m - c_2 - c_3)(1-t)L}{c_1 v} = 0 \,. \tag{20}$$

Consequently, the budget-balancing membership fee for any service level is given by

$$m = \frac{\frac{c_1}{L}S^3 + (c_2 + c_3 + C_0)S - (c_2 + c_3)\frac{1-t}{v}}{S - \frac{1-t}{v}} \,. \tag{21}$$

By differentiating equation 21 it can be shown that the budget-balancing combinations of ***m*** and ***S*** are located on a U-shaped *balanced-budget curve* as depicted in Figure 3. That is, up to a critical membership fee the budget-balancing service level decreases but then rises.

If in addition to balancing its budget the network sets the level of its service so as to achieve a membership rate ***x***, and if the ability to enjoy its service is uniformly distributed within the unit interval, then the critical member has an ability of ***1-x***. Recalling equations 2 and 7, the network ensures that a share of ***x*** of the target population joins its ranks by setting the membership fee to be

$$m = \left(\frac{(1-x)v}{1-t} \right) S \,. \tag{22}$$

The set of all combinations of ***m*** and ***S*** ensuring **x**-membership rate is depicted by the *x-membership-rate line* in Figure 3.

The combinations of ***m*** and ***S*** satisfying simultaneously both the balanced-budget constraint and the x-membership-rate target can be found by equating the terms on the right-hand sides of equations 21 and 22. These combinations are found in the intersection between the *balanced-budget curve* and the *x-membership-rate line* as indicated by **A** and **B** in Figure 3. From the members' perspective, the positive service differential between **B** and **A** is a compensation for the membership-fee differential between these combinations.

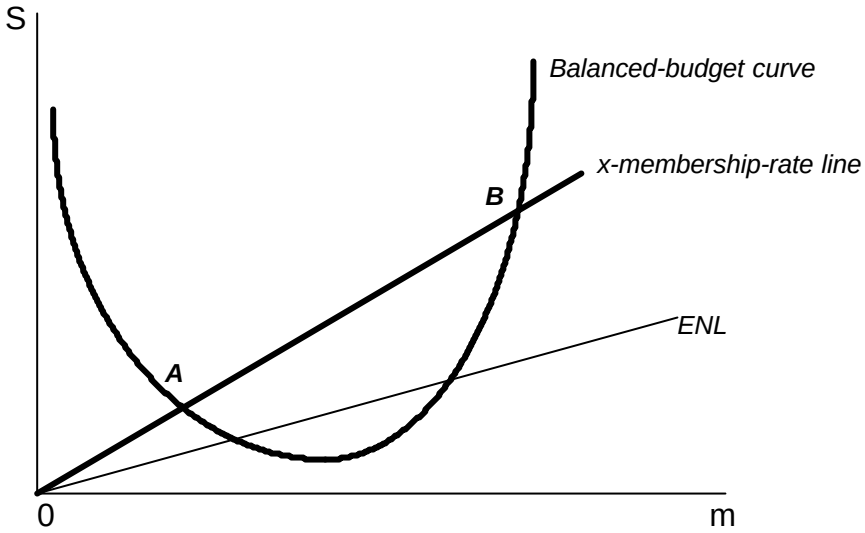

Figure 3. *Budget-balancing and x-membership-rate combinations*

VI. Extension To Duopolistic Networks: Density and the Cournot Equilibrium

Let us now analyse networks' density in an industry comprising two non-identical networks and a population of potential customers that for reasons such as habit, loyalty and snobbism might appreciate the service of the networks in a biased manner. Recalling equations 5 and 7, the number of members affiliated to *network 1* can be expressed as:

$$M_1 = (L - M_2)\left[1 - \frac{1-t}{v_1 S_1} m_1\right] \tag{23}$$

and the number of members affiliated to *network 2* is

$$M_2 = (L - M_1)\left[1 - \frac{1-t}{v_2 S_2} m_2\right] \tag{24}$$

where v_1 is not necessarily equal to v_2 because of the aforementioned service-evaluation biases.

By substituting equation 24 into equation 23 for M_2, solving for M_1 and dividing by L, the density of *network 1* is:

$$\frac{M_1}{L} = 1 - \left[\frac{\dfrac{m_1}{v_1 S_1}}{\dfrac{m_1}{v_1 S_1} - \dfrac{(1-t)m_1 m_2}{v_1 v_2 S_1 S_2} + \dfrac{m_2}{v_2 S_2}} \right] \tag{25}$$

and, by symmetry, the density of *network 2* is:

$$\frac{M_2}{L} = 1 - \left[\frac{\dfrac{m_2}{v_2 S_2}}{\dfrac{m_1}{v_1 S_1} - \dfrac{(1-t)m_1 m_2}{v_1 v_2 S_1 S_2} + \dfrac{m_2}{v_2 S_2}} \right]. \tag{26}$$

As can be intuitively expected, the differentiation of equations 25 and 26 implies that the density of each network increases with its service level, its service assessment coefficient, and the membership fee set by its rival and decreases with is own membership fee, the service level of its rival, and the assessment coefficient of the service of its rival. The curves NET1 and NET2 in Figure 4 display the relationship between network density and membership fee-service ratio for *network 1* and *network 2*, respectively for the case where $v_1 > v_2$.

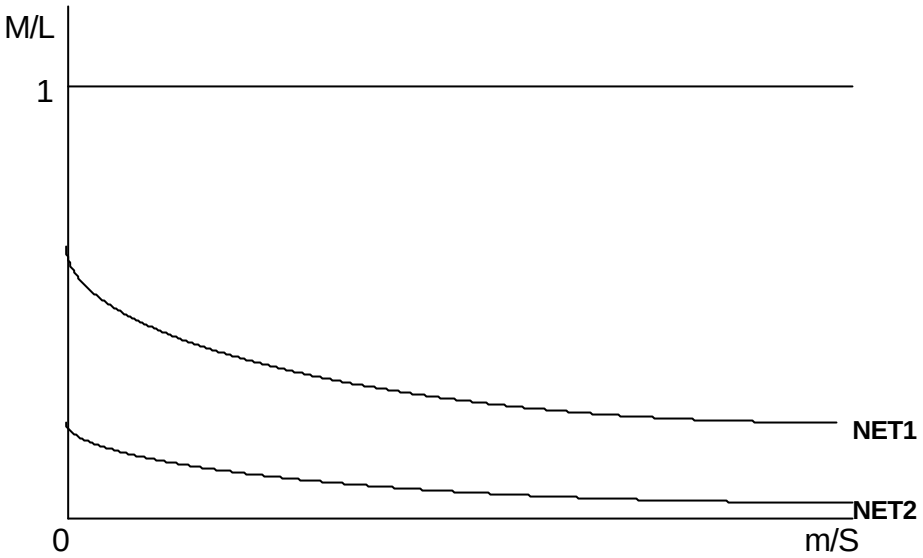

Figure 4. *Networks' density where* $v_1 > v_2$

In recalling equations 11 and 25 the profit function of *network 1* is

$$\Pi_1 = (m_1 - c_{21} - c_{31})\left[1 - \frac{\dfrac{m_1}{v_1 S_1}}{\dfrac{m_1}{v_1 S_1} - \dfrac{(1-t)m_1 m_2^e}{v_1 v_2 S_1 S_2^e} + \dfrac{m_2^e}{v_2 S_2^e}}\right] L - c_{11} S_1^2 - C_{01} \tag{27}$$

and in recalling equation 26 the profit function of *network 2* is

$$\Pi_2 = (m_2 - c_{22} - c_{32})\left[1 - \frac{\dfrac{m_2}{v_2 S_2}}{\dfrac{m_1^e}{v_1 S_1^e} - \dfrac{(1-t)m_1^e m_2}{v_1 v_2 S_1^e S_2} + \dfrac{m_2}{v_2 S_2}}\right] L - c_{12} S_2^2 - C_{02} \tag{28}$$

where the superscript *e* denotes expected values of the rival network's control variables. The Cournot-equilibrium membership fees and service levels are obtained by solving the combined set of first-order conditions for *network 1* and *network 2* simultaneously under the assumption

that the expected values are confirmed in equilibrium. That is, m_1^*, m_2^*, S_1^* and S_2^* satisfying simultaneously

$$\frac{\P\Pi_1}{\P m_1} = M_1 + (m_1 - c_{21} - c_{31})\frac{\P M_1}{\P m_1} = 0 \tag{29a}$$

$$\frac{\P\Pi_1}{\P S_1} = (m_1 - c_{21} - c_{31})\frac{\P M_1}{\P S_1} - 2c_{11}S_1 = 0 \tag{29b}$$

$$\frac{\P\Pi_2}{\P m_2} = M_2 + (m_2 - c_{22} - c_{32})\frac{\P M_2}{\P m_2} = 0 \tag{29c}$$

$$\frac{\P\Pi_2}{\P S_2} = (m_2 - c_{22} - c_{32})\frac{\P M_2}{\P S_2} - 2c_{12}S_2 = 0 \tag{29d}$$

where,

$$M_1 = \left[1 - \frac{\dfrac{m_1}{v_1 S_1}}{\dfrac{m_1}{v_1 S_1} - \dfrac{(1-t)m_1 m_2}{v_1 v_2 S_1 S_2} + \dfrac{m_2}{v_2 S_2}}\right] L \tag{30a}$$

and

$$M_2 = \left[1 - \frac{\dfrac{m_2}{v_2 S_2}}{\dfrac{m_1}{v_1 S_1} - \dfrac{(1-t)m_1 m_2}{v_1 v_2 S_1 S_2} + \dfrac{m_2}{v_2 S_2}}\right] L. \tag{30b}$$

Possible comparative statics' results can be obtained through simulations.

VII. CONCLUSION

Membership fees and ability to enjoy a network service are incorporated into the individual's decision to join the network. The analysis started with a monopolistic network. The network density was related to the distribution of the ability to enjoy the network service, membership fees and tax concessions. It was shown that the network density declines linearly with the membership fee. The decline in the network density stemming from an increase in the membership fee is moderated by the upper bound on individuals' willingness to pay for the network service and the tax-rebate rate.

As the network density affects both the network revenues and organisational costs and generates negative externalities due to congestion and positive ones due to greater opportunities for interaction among members, its determination was integrated into the analysis of the choice of membership fee and service by a profit-maximising monopolistic network and, alternatively, by a non-profit monopolistic network that balances its budget and seeks the affiliation of a desired share of the target population.

The substitution of the aforementioned linear relationship between network density and membership fee into a profit-maximising objective function implied that for any given network service level the profit-maximising membership fee is equal to half the marginal network externalities and organisational costs plus half the upper-bound on members' willingness to pay for the network service deflated by the effective cost of a dollar spent on membership payment. The effect of the overall membership withdrawal coefficient on the level of service offered by a profit-maximising network was found to be negative. Moreover, the higher the members' ability to enjoy the network service and the higher the tax-rebate rate on membership payment, the greater the profit-maximising service. The effect of the target population size on the level of the profit-maximising service was found to depend on the product of the marginal organisational costs and the membership withdrawal coefficient. It was argued that if initially the aforementioned product is larger (smaller) than the profit-maximising service an increase in the target population size would raise (lower) the profit-maximising service level. It was shown that the higher the marginal service-production costs and the marginal organisational costs the lower the profit-maximising service level. It was also argued that the effects of changes in the model's

parameters on the profit-maximising membership fee have the same directions as the aforementioned effects on the profit-maximising service and are amplified by the upper-bound willingness to pay coefficient and the tax-rebate rate.

By substituting the linear relationship between the network density and membership fee into a budget-balancing objective it was shown that the budget-balancing combinations of membership fee and service are located on a U-shaped curve. That is, up to a critical level the positive effect of an increase in the budget-balancing membership fee on the network revenues is outweighed by the negative effect of diminishing network density on the network revenues and hence forcing the network to reduce costs by lowering the service quality. Beyond that critical level the positive effect of an increase in the budget-balancing membership fee on the network revenues outweighs the negative effect of declining network density on the network's revenues and hence enables the budget-balancing network to raise the quality of service. In turn, an improvement in the network's service is necessary for increasing the target population's willingness to pay. It was argued that if the network sets the level of its service so as to achieve a target membership rate, the locus of all combinations of membership fee and service rate will be a positively sloped line. The intersection of this line with the U-shaped balanced-budget curve indicates the combinations of membership fee and service quality satisfying both the target network density and balanced budget.

The analysis was extended to the more complicated case of two non-identical rival networks. The effects of the a network's own choice, and the rival's choice, of membership fees and service on its density were analysed. The Cournot-equilibrium membership fees and service were described for profit-maximising networks.

FOOTNOTES

1. Usually networks are characterised by positive size externalities commonly called "network externalities", indicating that the benefits stemming from the addition of an extra node or member exceed the private benefits accruing to that node or member.

2. Dutta and Mutuswami (1997) analysed further whether the tension between stability and efficiency can be resolved.

3. See for example McDonald and Suen (1992) and Levy (1998), who have analysed the relationship between trade union bargaining power and density.

4. For example, Cohen and Zilberman (1997) argue that lack of familiarity with a new technology and strategic behaviour led to underestimates of farmers' actual willingness to pay for the Californian statewide network of weather information.

5. The use of the normal distribution as an approximation of f could introduce interesting factors such as the mean and variance of the capacity to enjoy the network service. However, the incorporation of the normal distribution is inconsistent with the requirement that $q_i \in (0,1)$.

APPENDIX

Proof of claim 1:

$$\frac{dS^*}{d\left(\frac{1-t}{v}\right)} = \frac{[((c_2+c_3)^2 / S^{*2}) + (v^2 / (1-t)^2)]L}{d^2\Pi / dS^2} < 0.$$

Proof of claim 2:

$$\frac{dS^*}{dL} = \left\{ \frac{\frac{(c_2+c_3)^2(1-t)}{vS^2} - \frac{v}{1-t}}{\frac{d^2\Pi}{dS^2}} \right\} \begin{matrix} > \\ = \\ < \end{matrix} 0 \text{ as } S^* \begin{matrix} < \\ = \\ > \end{matrix} \left\{ (c_2+c_3)\frac{(1-t)}{v} \right\}.$$

Proof of claim 3: $$\frac{dS^*}{dc_1} = \frac{8S^*}{d^2\Pi / dS^2} < 0.$$

Proof of claim 4:

$$\frac{dS^*}{d(c_2+c_3)} = \frac{\frac{2(1-t)L(c_2+c_3)}{vS^{*2}}}{\frac{d^2\Pi}{dS^2}} < 0.$$

Proof of claim 5: By virtue of equation 13 the profit-maximising membership fee is positively related to the network's service level.

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