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Amnon Levy
University of Wollongong, levy@uow.edu.au

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Amnon Levy

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WOULD A RATIONAL LUCY TAKE-OFF WITHOUT ASSESSING THE PROBABILITY OF A CRASH LANDING?

by

AMNON LEVY

ABSTRACT

A random life expectancy and a positive relationship between the probability of dying and the degree of addiction are incorporated into a model of rational addiction. The Becker-Murphy equality between the addictive commodity's full price and marginal utility is modified by discounting the market price and marginal utility of the addictive commodity by the probability of survival. The individual's appreciation of the consumption capital stock is positive as long as the improved consumption enjoyment dominates the diminishing survival prospects. The rate of change of the shadow price of addiction is lower than that obtained when the effect of addiction on the probability of dying is ignored. (*JEL* classification: D91)

Affiliation and contact information: Amnon Levy is a member of the Department of Economics, University of Wollongong, Northfields Avenue, Wollongong, NSW 2522, Australia. (Tel: 61-2-42213658 or 61-2-42258244, Fax: 61-2-42213725, E-mail: amnon_levy@uow.edu.au).

I. INTRODUCTION

Stigler and Becker (1977), Iannaccone (1986), Becker and Murphy (1988) and many others construct models of rational addiction in which forward-looking utility maximization is used to explain observed addictive behavior. These rational addiction models propose that rational planning stemming from lifetime utility maximization and addiction are not incompatible, that when dealing with addictive goods unstable steady states are a common characteristic, that these unstable steady states imply that small deviations in current consumption can lead to large cumulative changes, and that addicts respond more to permanent than to temporary changes in price. By defining the full price of the addictive commodity as the current market price plus the present discounted benefit or harm of the increase in the stock of addiction, it was also shown that this full price rises faster with time than the market price, and hence the probability of addiction is lower, the lower the discount of future consumption, and the lower the depreciation rate of past consumption.

Models of rational addiction were subjected to empirical tests and applied to the analysis of the consumption of addictive commodities such as cigarettes (Chaloupka, 1991; Becker, Grossman and Murphy, 1994; Douglas, 1998), coffee (Olekalns and Bardsley, 1996) and alcohol (Waters and Sloan, 1995; Grossman, Chaloupka and Sirtlalan, 1998), and were extended to include updating of subjective beliefs about the harm inflicted by the consumption of addictive commodities and regret (Orphannides and Zervos, 1995).

When considering harmful addictive commodities many interpretations could be given to the present discounted harm of the additional units of the stock of addiction that Becker and Murphy (1988) consider. A natural extension to this harm is the possible negative effects of harmful addictive commodities on life-expectancy and the probability of survival. The concept of rational addiction is broadened in this paper to include these effects explicitly with an allegorical reference to Lucy, the heroine of The Beetles' hit "*Lucy in the sky with diamonds*". If Lucy were rationally high on LSD would not she assess the probability of a crash landing prior to taking-off?

Similarly to earlier papers on rational addiction the controversial assumption of forward looking utility maximizing consumption of an addictive harmful commodity underlines the present analysis. But unlike the common feature in these papers of a fixed lifetime, it is proposed in the present analysis that lifetime is random and that there is a positive relationship between the probability of dying and the degree of addiction.

Furthermore, lifetime budget constraint considerations are excluded from the present analysis. The underlying rationale is that the direct spending on many addictive commodities, legal ones in particular, constitutes a small share of their consumers' current income. When the addiction is to expensive drugs, such as in the case of Lucy, the adverse effects of addiction on health and life expectancy render the consideration of a lifetime budget constraint inappropriate. The greater the adverse effects of Lucy's addiction on her health and life expectancy the lower Lucy's incentive to save and ability to borrow. In the absence of saving and credit Lucy might resort to illegal and self degrading income-generating activities for financing an immediate purchase of drugs and her engagement in such activities might reinforce her need to consume these drugs.

Section II incorporates the adverse effects of addiction on life-expectancy, health and income into Lucy's decision on her consumption path of the addictive commodity within a stochastic framework of lifetime-utility maximization. The implications of these adverse effects for the optimality conditions of Lucy's rational consumption of the addictive commodity and the shadow price of her addiction are presented in section III.

II. RATIONAL ADDICTION MODEL WITH RANDOM LIFE-EXPECTANCY

The present analysis of rational addiction with random life-expectancy is based on the following assumptions. First, Lucy derives utility from consuming an addictive commodity and a non-addictive good. Second, Lucy's level of satisfaction from consuming the addictive commodity is enhanced by the degree of addiction, but with a loss of health and income. Third, Lucy assesses that her probability of dying at any given time increases with her degree of addiction.

To facilitate the analysis of the full price of the addictive commodity it is assumed that Lucy's instantaneous utility function is additively separable and her marginal utility from the consumption of the non-addictive good is constant. Taking the market price of, and Lucy's marginal utility from, the non-addictive good as a numeraire, its current consumption level and the associated satisfaction level are given by the difference between Lucy's current income and her spending on the addictive commodity.

These assumptions are formalized as follows. Lucy's instantaneous utility function is

$$u(t) = x(t)c(t)^{a} + \{[1 - x(t)]y - p(t)c(t)\} - x(t)v \qquad (1)$$

where,

$x(t)$ = Lucy's degree of addiction at time t, $x \in (0,1)$,

a = a positive scalar smaller than 1,

$c(t)$ = Lucy's consumption of the addictive commodity at time t,

y = a positive scalar representing the upper bound on Lucy's income,

$p(t)$ = the price of the addictive commodity at time t, and

v = a non-negative scalar representing the upper bound on loss of health.

In this framework, Lucy's satisfaction from consuming the addictive commodity, forgone instantaneous income and loss of health are proportional to her degree of addiction which, in addition to indicating Lucy's capacity to consume the addictive commodity, records the accumulated effect of consuming the addictive commodity on her physical and mental conditions.

Lucy's lifetime utility function is assumed to be additively separable and the stream of instantaneous utility over her lifetime is exponentially discounted by a fixed positive rate of time preference, $\boldsymbol{r}$, indicating Lucy's degree of impatience and reflecting her time-consistent preferences.

Lucy's decision problem is postulated as choosing the trajectory of c so as to maximize her expected lifetime utility from consuming the addictive commodity and the non-addictive good and from having good health subject to the evolution of her degree of addiction. The likelihood that Lucy dies at t is depicted by a probability density function $f(t;T,x)$ reflecting, as specified explicitly below, the existence of an upper bound, $\boldsymbol{T}$, on Lucy's lifetime and diminishing prospects of survival as her degree of addiction increases.

Lucy's objective is formally portrayed as

$$\max_{c} \int_{0}^{T} f(t;T,x)[\int_{0}^{t} e^{-rt} u(t)dt]dt \qquad (2)$$

subject to the motion equation of her degree of addiction (or consumption capital)

$$\dot{x}(t) = c(t) - dx(t) \qquad (3)$$

and her initial degree of addiction

$$x(0) = x_0 \qquad (4)$$

where $\boldsymbol{d}$, a non-negative scalar, indicates Lucy's rehabilitation rate.

Integrating by parts, Lucy's objective function can be rendered as

$$\max_{c} \int_{0}^{T} e^{-rt} u(t)[1 - F(t;T,x)]dt \qquad (5)$$

where $F(t;T,x)$ is the cumulative density function associated with $f(t;T,x)$ and indicating Lucy's probability of dying by t .

It is reasonable to assume that Lucy's probability of dying by time t increases as t converges to $\boldsymbol{T}$ and that this increase is amplified by Lucy's degree of addiction as the latter factor records the cumulative adverse effect of consuming the addictive commodity on her health. This assumption is incorporated into the analysis by using the following exponential cumulative distribution function:

$$F(t) = e^{-b(1 - x(t))(T - t)} \qquad (6)$$

where b is a positive scalar reflecting Lucy's assessment of the effect of an infinitesimal decline in her degree of addiction on the rate of change of her probability of dying by t. The larger b, *ceteris paribus*, the higher Lucy's probability of living up to her oldest possible age $\boldsymbol{T}$.

The present-value Hamiltonian associated with Lucy's intertemporal decision problem is

$$H(x, l, c, t) = e^{-rt}u(t)[1 - F(t)] + l(t)[c(t) - dx(t)] \qquad (7)$$

where the costate variable $\mathbf{l}$ indicates Lucy's shadow price of her degree of addiction, and u and F are as specified earlier.

III. Properties Of The Random Life-Expectancy Augmented Rational Addiction

Since u is concave in c and F is convex in x the following conditions are necessary and sufficient for Lucy's maximum expected lifetime utility:

$$\dot{l} = -\{u_x[1 - F] - F_x u\}e^{-rt} + dl \qquad (8.1)$$

$$l = -u_c[1 - F]e^{-rt} \qquad (8.2)$$

$$\dot{x}(t) = c(t) - dx(t) \qquad (8.3)$$

where,

$$u_x = c^a - (y + v) \qquad (9)$$

$$u_c = axc^{a-1} - p \qquad (10)$$

and

$$F_x = b(T - t)e^{-b(1-x)(T-t)}. \qquad (11)$$

Similarly to Becker and Murphy (1988) the optimality condition 8.2 requires equality between the full price of the addictive commodity and Lucy's direct marginal utility from consuming this commodity at every instance:

$$l + pe^{-rt}[1-F] = axc^{a-1}e^{-rt}[1-F]. \qquad (12)$$

However, both the market price and Lucy's direct marginal utility from consuming the addictive commodity are discounted not only by Lucy's rate of time preference but also by her probability of living at least until t. Moreover, by rearranging terms,

$$l = (axc^{a-1} - p)e^{-rt}[1-F] \qquad (13)$$

implying that when life expectancy aspects are taken into account the effect of Lucy's degree of addiction on her appreciation of her consumption capital (**l**) is not clear. On the one hand, a rise in Lucy's degree of addiction enhances her ability to enjoy the consumption of the addictive commodity at t. On the other hand, it diminishes Lucy's probability of surviving until t.

The optimality condition and the adjoint equation 8.1 imply further that along Lucy's optimal consumption path of the addictive commodity the rate of change of the shadow price of her degree of addiction is equal to:

$$\frac{\dot{l}}{l} = d + \frac{u_x}{u_c} - \frac{uF_x}{u_c[1-F]} \ . \qquad (14)$$

Since the marginal effect of Lucy's degree of addiction on the probability of dying by t is positive, the rate of change of the shadow price of her addiction is lower than that obtained when the adverse effect of addiction on her life expectancy is ignored ($F_x = 0$): namely, Lucy's rate of rehabilitation plus her marginal rate of substitution between x and c (i.e., the first two terms on the right hand side of equation 14).

In recalling equations 9-11, the difference between the present analysis' rate of change of the shadow price of Lucy's degree of addiction and that obtained with rational addiction models ignoring the possible effect of Lucy's degree of addiction on her life expectancy and probability of dying can be expressed as

$$\frac{\dot{l}}{l} - \left[d + \frac{u_x}{u_c} \right] =$$

$$- \frac{\{xc^{a} + [(1-x)y - pc] - xv\} b(T-t) e^{-b(1-x)(T-t)}}{(axc^{a-1} - p)[1 - e^{-b(1-x)(T-t)}]} \quad . \qquad (15)$$

This expression reveals that the difference between the theoretical rates of change of the shadow price of addiction generated by the two different approaches to life expectancy (i.e, the random versus the fixed) diminishes as t increases and converges to the upper bound on the possible lifetime T.

IV. CONCLUSION

The effect of addiction on life expectancy and probability of dying is incorporated into the analysis of rational addiction. It is shown that like in the seminal paper by Becker and Murphy (1988) rational addiction leads to equality between the addictive commodity's full price and the direct marginal utility from its consumption at any instance. However, both the market price and direct marginal utility of the addictive commodity are discounted not only by the rate of time preference but also by the probability of living at least until that instance, and the rate of change of the shadow price of addiction is lower than that obtained when the adverse effect of addiction on life expectancy is ignored. Furthermore, the effect of the degree of addiction on the appreciation of the consumption capital stock is not clear due to the conflict between satisfaction enhancement and prospects of survival. Hence, a rational Lucy weighs the benefits and costs of her degree of addiction and moderates her degree of addiction so as to enjoy not only exciting take-offs but also safer landings.

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